VIII — Mathematics Assignment — Factors

Basic Concepts

In factorization, we express a polynomial as the product of two polynomials.

e.g., $x^2+5x+6 = (x+3)(x+2)$

So we have expressed the polynomial x^2+5x+6 as the product of $(x+3)$ and $(x+2)$

So $(x+3)$ and $(x+2)$ are the factors of x^2+5x+6.

Thus, the process of writing an algebraic expression as the product of two or more algebraic expressions, are called factorization.

Each expression occurring in the product is called a factor of the given expression.

There are various method of factorization in various case.

Cont-Pg-2 →

Case 1.

When each term of the given expression contains a common monomial factor.

eg,
$$6a^2 + 5ab - a$$
$$= a(6a + 5b - 1)$$

Case 2.

When a polynomial is a common multiplier of each term of the given expression.

eg,
$$2a(a+b) + 2b(a+b)$$
$$= (a+b)(2a+2b)$$

Case 3. When the given expression is the difference of two squares.

In this case, one has to use the identity
$$A^2 - B^2 = (A+B)(A-B)$$

eg,
$$9x^2 - 16y^2$$
$$= (3x)^2 - (4y)^2$$
$$= (3x+4y)(3x-4y)$$

Cont-Pg-3

Case 4. When the given expression is a perfect square

eg, $4x^2 + 12xy + 9y^2$

$= (2x)^2 + 2.(2x).(3y) + (3y)^2$

$= (2x+3y)^2$

Case 5. When the given expression is a perfect cube

In this case, one can use the identity

(i) $(A+B)^3 = A^3 + B^3 + 3A^2B + 3AB^2$

(ii) $(A-B)^3 = A^3 - B^3 + 3AB^2 - 3A^2B$

eg, $8x^3 + 27y^3 + 36x^2y + 54xy^2$

$= (2x)^3 + (3y)^3 + 3.\,4x^2.3y + 3.\,2x.9y^2$

$= (2x+3y)^3$

Cont-Pg-4

Factorization of Quadratic Trinomials

Case 1. When the expression is of the form $x^2 + px + q$

In this case, one can factorise q in such a way that the sum (or difference) of factors is p. then brake p into sum (or difference) then by making the grouping, factorise

eg, $x^2 + 5x + 6$

Factors of 6 are 1, 2, 3, 6

Now choose 2, 3 $\because$ $2+3 = 5$ (Coeff of x)

i.e. now $x^2 + 2x + 3x + 6$

$= x(x+2) + 3(x+2)$

$= (x+2)(x+3)$

Case 2. When the expression is of the form $ax^2 + bx + c$.

$a \times c$ = ac = (Factorise such that the sum (or diff) of factors is b.

then brake 'b' and by grouping, factorise.

Cont-Pg5

eg, Factorise $2x^2 + x - 3$

$2 \times -3 = -6 = +3 \times -2$

So $2x^2 + 3x - 2x - 3$

$= x(2x+3) - 1(2x+3)$

$= (2x+3)(x-1)$

Some more Cases of Factorization

Case 1.

When the given expression is expressible as the Sum of two Cubes.

eg, $64x^3 + 125y^3$

In such Cases, one has to use the following identity.

$$A^3 \pm B^3 = (A \pm B)(A^2 + B^2 \mp AB)$$

So Write $64x^3 + 125y^3$ as

$(4x)^3 + (5y)^3$

$= (4x+5y)((4x)^2 + (5y)^2 - 4x.5y)$

$= (4x+5y)(16x^2 + 25y^2 - 20xy)$

Cont:- Pg-6

Example of difference of two cubes

$$27x^3 - 125y^3$$

$$= (3x)^3 - (5y)^3$$

$$= (3x-5y)(9x^2+25y^2+15xy)$$

Case II When the given expression is of the form of

$$a^3+b^3+c^3-3abc$$

Its identity is

$$a^3+b^3+c^3-3abc = (a+b+c)(a^2+b^2+c^2-ab-bc-ca)$$

Example —

Factorise

$$8x^3+27y^3+64z^3-72xyz$$

$$= (2x)^3+(3y)^3+(4z)^3-3\times 2x\times 3y\times 4z$$

$$= (2x+3y+4z)(4x^2+9y^2+16z^2-6xy-12yz-8xz)$$

Case III — When the given expression is of the form $a^3+b^3+c^3$ under the given condition that $a+b+c=0$

Cont-Pg-7 →

In this case, the identity is

$$a^3+b^3+c^3 = abc$$ Subject to the Condition that $a+b+c=0$

Example–

Factorise,

$$(x-y)^3+(y-z)^3+(z-x)^3$$

let
$$A = x-y$$
$$B = y-z$$
$$C = z-x$$

$$\Rightarrow A+B+C = x-y+y-z+z-x$$
$$= 0$$

Since $A+B+C=0 \Rightarrow$
$$A^3+B^3+C^3 = 3ABC$$

Replacing the value of A, B, C, we get,

$$(x-y)^3+(y-z)^3+(z-x)^3 = 3(x-y)(y-z)(z-x)$$

So, till now, we have discussed almost the maximum cases of factorization, Now, I am writing a Consolidated list of formulae.

Cont → Pg-8

List of Formulae

1. $(a+b)^2 = a^2 + b^2 + 2ab$
2. $(a-b)^2 = a^2 + b^2 - 2ab$
3. $(a+b)^3 = a^3 + b^3 + 3ab(a+b)$
   OR $a^3 + b^3 + 3a^2b + 3ab^2$
4. $(a-b)^3 = a^3 - b^3 - 3ab(a-b)$
   OR $a^3 - b^3 - 3a^2b + 3ab^2$
5. $(a+b+c)^2 = a^2 + b^2 + c^2 + 2(ab+bc+ca)$
6. $a^2 - b^2 = (a-b)(a-b)$
7. $a^3 - b^3 = (a-b)(a^2+b^2+ab)$
8. $a^3 + b^3 = (a+b)(a^2+b^2-ab)$
9. $a^4 - b^4 = (a+b)(a-b)(a^2+b^2)$
10. $a^3 + b^3 + c^3 - 3abc = (a+b+c)(a^2+b^2+c^2-ab-bc-ca)$
    OR $(a+b+c)(a^2+b^2+c^2-ab-bc-ca)$
11. If $a+b+c=0$ then $a^3+b^3+c^3 = 3abc$