SECOND TERM (SA-II)
MATHEMATICS
(With Solutions)
CLASS X

Time Allowed : 3 Hours] *[Maximum Marks : 80*

General Instructions :

(i) *All questions are compulsory.*
(ii) *The question paper consists of 34 questions divided into four sections A, B, C and D. Section A comprises of 10 questions of 1 mark each, Section B comprises of 8 questions of 2 marks each, Section C comprises of 10 questions of 3 marks each and Section D comprises of 6 questions of 4 marks each.*
(iii) *Question numbers 1 to 10 in Section A are multiple choice questions where you are to select one correct option out of the given four.*
(iv) *There is no overall choice. However, internal choice has been provided in 1 question of two marks, 3 questions of three marks each and 2 questions of four marks each. You have to attempt only one of the alternatives in all such questions.*
(v) *Use of calculators is not permitted.*

Section 'A'

Question numbers 1 to 10 are of one mark each.

1. The roots of the equation $x^2 + x - p(p + 1) = 0$, where p is a constant, are

(a) $p, p + 1$ (b) $-p, p + 1$
(c) $p, -(p + 1)$ (d) $-p, -(p + 1)$

Solution. Choice (c) is correct.

$$x^2 + x - p(p + 1) = 0$$
$\Rightarrow \quad x^2 + [(p + 1) - p]x - p(p + 1) = 0$
$\Rightarrow \quad x^2 + (p + 1)x - px - p(p + 1) = 0$
$\Rightarrow \quad [x^2 + (p + 1)x] - [px + p(p + 1)] = 0$
$\Rightarrow \quad x[x + (p + 1)] - p[x + (p + 1)] = 0$
$\Rightarrow \quad [x + (p + 1)][x - p] = 0$
$\Rightarrow$ Either $x = -(p + 1)$ or $x = p$

Hence, $-(p + 1)$ and p are the roots of the given equation.

2. In an A.P., if $d = -2$, $n = 5$ and $a_n = 0$, then the value of a is

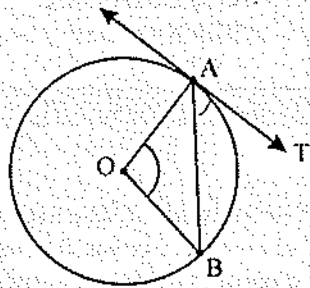
(a) 10 (b) 5
(c) -8 (d) 8

Solution. Choice (d) is correct.

Given : $d = -2$, $n = 5$ and $a_n = 0$

$\therefore \quad a_n = 0$
$\Rightarrow \quad a + (n - 1)d = 0$
$\Rightarrow \quad a + (5 - 1)(-2) = 0$
$\Rightarrow \quad a - 8 = 0$
$\Rightarrow \quad \mathbf{a = 8.}$

3. In figure, O is the centre of a circle, AB is a chord and AT is the tangent at A. If $\angle AOB = 100°$, then $\angle BAT$ is equal to



(*a*) 100° (*b*) 40°
(*c*) 50° (*d*) 90°

Solution. Choice (*c*) is correct.

In ΔOAB, we have

$\angle AOB + \angle OAB + \angle OBA = 180°$ [Sum of three $\angle s$ of a $\Delta = 180°$]

$\Rightarrow \quad 100° + \angle OAB + \angle OBA = 180°$

$\Rightarrow \quad \angle OAB + \angle OBA = 80°$ ...(1)

In ΔOAB, we have

$OA = OB$ [Each = radii of a circle]

$\Rightarrow \quad \angle OBA = \angle OAB$ ...(2) [$\angle s$ opposite to equal sides of a Δ are equal]

From (1) and (2), we have

$2\angle OAB = 80°$

$\Rightarrow \quad \angle OAB = 40°$ ...(3)

Since $OA \perp AT$, therefore

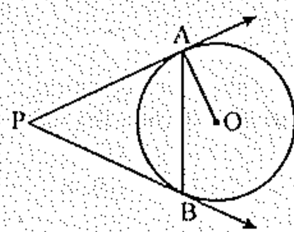
$\angle OAT = 90°$

$\Rightarrow \quad \angle OAB + \angle BAT = 90°$

$\Rightarrow \quad 40° + \angle BAT = 90°$ [using (3)]

$\Rightarrow \quad \angle BAT = 50°$

4. In figure, PA and PB are tangents to the circle with centre O. If $\angle APB = 60°$, then $\angle OAB$ is



(*a*) 30° (*b*) 60°
(*c*) 90° (*d*) 15°

Solution. Choice (*a*) is correct.

In ΔPAB, we have

$PA = PB$ [Length of the tangents drawn from an external point to a circle are equal]

So, PAB is an isosceles triangle.

In ΔPAB, we have

$$\angle PAB + \angle PBA + \angle APB = 180^\circ$$ [Sum of three $\angle s$ of a Δ is 180°]

$\Rightarrow \quad 2\angle PAB + 60^\circ = 180^\circ$ [$\angle APB = 60^\circ$ (given)]

$\Rightarrow \quad 2\angle PAB = 120$

$\Rightarrow \quad \angle PAB = 60^\circ$

But $\quad \angle OAP = 90^\circ$ [Angle between the tangent and the circle is 90°]

Now, $\quad \angle OAB = \angle OAP - \angle PAB$

$$= 90^\circ - 60^\circ$$

$$= 30^\circ$$

5. The radii of two circles are 4 cm and 3 cm respectively. The diameter of the circle having area equal to the sum of the areas of the two circles (in cm) is

(*a*) 5 (*b*) 7

(*c*) 10 (*d*) 14

Solution. Choice (*c*) is correct.

Let R cm be the radius of the required circle, then

Area of a circle of radius R cm.

= Area of a circle of a radius 4 cm + Area of a circle of a radius 3 cm

[According to the given condition of the question]

$\Rightarrow \quad \pi R^2 = \pi(4)^2 + \pi(3)^2$

$\Rightarrow \quad \pi R^2 = \pi[16 + 9]$

$\Rightarrow \quad R^2 = 25 = (5)^2$

$\Rightarrow \quad R = 5$ cm

Hence, the diameter of the required circle is **10 cm.**

6. A sphere of diameter 18 cm is dropped into a cylindrical vessel of diameter 36 cm, partly filled with water. If the sphere is completely submerged, then the water level rises (in cm) by

(*a*) 3 (*b*) 4

(*c*) 5 (*d*) 6

Solution. Choice (*a*) is correct.

Diameter of a sphere $= 2R = 18$ cm

$\Rightarrow$ Radius of a sphere $= R = 9$ cm

Volume of a sphere $= \frac{4}{3}\pi R^3$

$$= \frac{4}{3}\pi(9)^3 \quad \ldots(1)$$

Diameter of a cylindrical vessel $= 2r$ cm $= 36$ cm

$\Rightarrow$ Radius of a cylindrical vessel $= r = 18$ cm

Let the water level rises (in cm) by h, then

Volume of the cylindrical vessel $= \pi r^2 h$

$$= \pi(18)^2 \times h \quad \ldots(2)$$

According to the given question, we have

$$(1) = (2)$$

$\Rightarrow \quad \frac{4}{3}\pi(9)^3 = \pi \times (18)^2 \times h$

$$\Rightarrow \qquad h = \frac{\frac{4}{3}\pi(9\times9\times9)}{\pi\times18\times18}$$

$$\Rightarrow \qquad h = 3 \text{ cm}$$

Hence, the water level rises in cylindrical vessel (in cm) by **3**.

7. The angle of elevation of the top of a tower from a point on the ground which is 30 m away from the foot of the tower is 45°. The height of the tower (in metres) is

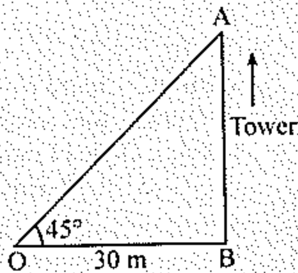
(*a*) 15 (*b*) 30

(*c*) $30\sqrt{3}$ (*d*) $10\sqrt{3}$

Solution. Choice (*b*) is correct.

Let O be a point 30 m away from the foot of the tower $AB = h$ m.

In right ΔOBA, we have



$$\tan 45° = \frac{AB}{OB}$$

$$\Rightarrow \qquad 1 = \frac{h}{30}$$

$$\Rightarrow \qquad h = 30 \text{ m}$$

Thus, the height of the tower in metres is **30**.

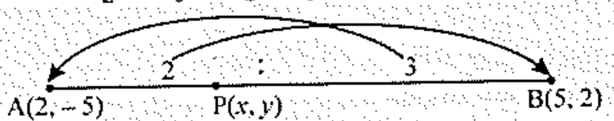
8. The point P which divides the line segment joining the points $A(2, -5)$ and $B(5, 2)$ in the ratio 2 : 3 lies in the quadrant

(*a*) I (*b*) II

(*c*) III (*d*) IV

Solution. Choice (*d*) is correct.

Let $P(x, y)$ divides the line segment joining the points $A(2, -5)$ and $B(5, 2)$ in the ratio 2 : 3, then



$$\therefore \qquad x = \frac{3\times2+2\times5}{2+3} = \frac{16}{5} \quad \text{and} \quad y = \frac{3\times(-5)+2\times2}{2+3} = \frac{-11}{5}$$

$\therefore$ The coordinates of P are $\left(\frac{16}{5}, \frac{-11}{5}\right)$. Hence the point P lies in the **IV quadrant.**

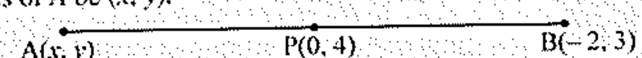
9. The mid-point of segment AB is the point $P(0, 4)$. If the coordinates of B are $(-2, 3)$ then the coordinates of A are

(*a*) (2, 5) (*b*) (– 2, – 5)

(*c*) (2, 9) (*d*) (– 2, 11)

Solution. Choice (*a*) is correct.

Let the coordinates of A be (x, y).



It is given that the mid-point of segment AB is the point $P(0, 4)$, therefore

$$\frac{x-2}{2} = 0 \quad \text{and} \quad \frac{y+3}{2} = 4$$

$$\Rightarrow \quad x = 2 \quad \text{and} \quad y + 3 = 8$$

$$\Rightarrow \quad x = 2 \quad \text{and} \quad y = 5$$

Hence, the coordinates of A are **(2, 5).**

10. Which of the following cannot be the probability of an event ?

(*a*) 1.5 (*b*) $\frac{3}{5}$

(*c*) 25% (*d*) 0.3

Solution. Choice (*a*) is correct.

The probability of an event lies between 0 and 1 *i.e.*, $0 \leq P \leq 1$

$\therefore 1.5 > 1 \Rightarrow$ **1.5** cannot be the probability of an event.

Section 'B'

Question numbers 11 to 18 carry 2 marks each.

11. Find the value of p so that the quadratic equation $px(x-3) + 9 = 0$ has two equal roots.

Solution. The given quadratic equation is :

$$px(x-3) + 9 = 0$$

$$\Rightarrow \quad px^2 - 3px + 9 = 0$$

Here, $a = p$, $b = -3p$, $c = 9$.

For equal roots :

$$D = b^2 - 4ac = 0$$

$$\Rightarrow \quad (-3p)^2 - 4(p) \times 9 = 0$$

$$\Rightarrow \quad 9p^2 - 36p = 0$$

$$\Rightarrow \quad p^2 - 4p = 0$$

$$\Rightarrow \quad p(p-4) = 0$$

$\Rightarrow$ Either $p = 0$ or $p = 4$

Hence, the value of p is **4** and reject $p = 0$.

12. Find whether – 150 is a term of the A.P. 17, 12, 7, 2,... ?

Solution. Let the nth term of the A.P. be – 150, then

$$-150 = a + (n-1)d$$

$$\Rightarrow \quad -150 = 17 + (n-1)(-5) \qquad [\because a = 17, d = 12 - 17 = -5]$$

$$\Rightarrow \quad -150 = 17 - 5n + 5$$

$$\Rightarrow \quad -150 = 22 - 5n$$

$$\Rightarrow \quad 5n = 22 + 150$$

$$\Rightarrow \quad 5n = 172$$

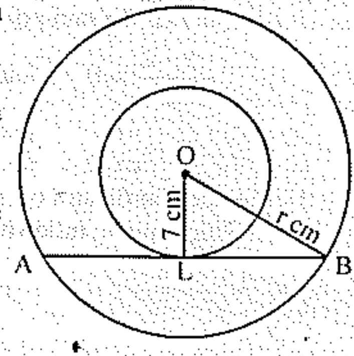
$$\Rightarrow \quad n = 172 \div 5$$

$$\Rightarrow \quad n = 34\frac{2}{5}$$

Hence, **– 150 is not** a term of the A.P. 17, 12, 7, 2, ...

13. Two concentric circles are of radii 7 cm and r cm respectively, where $r > 7$. A chord of the larger circle, of length 48 cm, touches the smaller circle. Find the value of r.

Solution. Let O be the centre of two concentric circles and AB be a chord of the larger circle touching the smaller circle at L.



Join OL.

Since OL (= 7 cm) is the radius of the smaller circle and AB is a tangent to this circle at a point L.

$\therefore \quad OL \perp AB.$

We know that the perpendicular drawn from the centre of a circle and AB is a tangent of the circle bisects the chord at L.

$\Rightarrow \quad AL = BL$

$\therefore \quad AL + BL = 48$ cm $\Rightarrow BL + BL = 48$ cm $\Rightarrow 2BL = 48$ cm

$\Rightarrow \quad BL = 24$ cm

In right ΔOLB, we have

$$LB^2 = OB^2 - OL^2$$

$\Rightarrow \quad (24)^2 = (r)^2 - (7)^2$

$\Rightarrow \quad 576 = r^2 - 49$

$\Rightarrow \quad r^2 = 576 + 49$

$\Rightarrow \quad r^2 = 625 = (25)^2$

$\Rightarrow \quad r = 25$ cm

Hence the value of r is **25 cm.**

14. Draw a line segment of length 6 cm. Using compasses and ruler, find a point P on it which divides it in the ratio 3 : 4.

Solution. Steps of construction :

Step 1. Draw a line segment $BC = 6$ cm.

Step 2. Draw any ray BY making an acute angle with BC.

Step 3. Locate $7 (= 3 + 4)$ points $B_1, B_2, B_3, B_4, B_5, B_6$ and B_7 on BY.

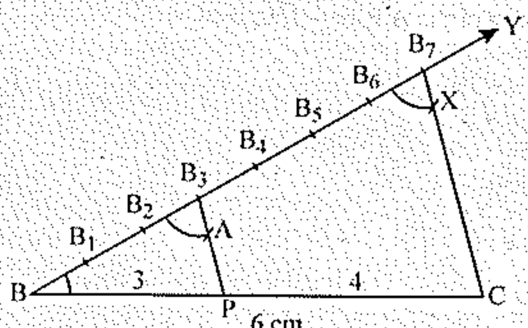
So that $BB_1 = B_1B_2 = B_2B_3 = B_3B_4 = B_4B_5 = B_5B_6 = B_6B_7$.

Join B_7C.

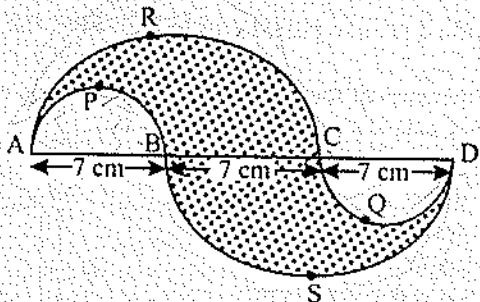
Step 4. With B_7 as centre mark an arc cutting B_7C at X.

Step 5. Through B_3 draw a line B_3P parallel to B_7C making an angle equal to BB_7C at B_3 intersecting BC at a point P.

The point P so obtained is the required point which divides BC internally in the ratio 3 : 4.



15. In figure, APB and CQD are semi-circles of diameter 7 cm each, while ARC and BSD are semi-circles of diameter 14 cm each. Find the perimeter of the shaded region. $\left[\text{Use } \pi = \frac{22}{7}\right]$



Solution. In figure, diameters of semi-circles APB and CQD and ARC and BSD are $AB = 7$ cm and $CD = 7$ cm and $AC = 14$ cm and $BD = 14$ cm respectively.

$\therefore$ Perimeter of the shaded region

= Perimeter of the semi-circle on AB as diameter
+ Perimeter of the semi-circle on AC as diameter
+ Perimeter of the semi-circle on BD as diameter
+ Perimeter of the semi-circle on CD as diameter

$$= \pi\left(\frac{\text{Diameter } AB}{2}\right) + \pi\left(\frac{\text{Diameter } AC}{2}\right) + \pi\left(\frac{\text{Diameter } BD}{2}\right) + \pi\left(\frac{\text{Diameter } CD}{2}\right)$$

$= \pi/2[\text{Diameter } AB + \text{Diameter } AC + \text{Diameter } BD + \text{Diameter } CD]$

$= \pi/2[7 \text{ cm} + 14 \text{ cm} + 14 \text{ cm} + 7 \text{ cm}]$

$= \pi/2[42] \text{ cm}$

$= \pi(21) \text{ cm}$

$= \frac{22}{7} \times 21 = \mathbf{66\ cm.}$

Or

Find the area of a quadrant of a circle, where the circumference of circle is 44 cm.

Solution. Let r be the radius of a circle whose circumference is 44 cm (given).

$\therefore$ Circumference of a circle = 44 cm

$\Rightarrow \quad 2\pi r = 44$

$\Rightarrow \quad 2 \times \frac{22}{7} \times r = 44$

$\Rightarrow \quad r = \frac{44 \times 7}{44} = 7 \text{ cm}$

Area of a quadrant of a circle of radius 7 cm

$= \frac{90°}{360°} \times \pi r^2$

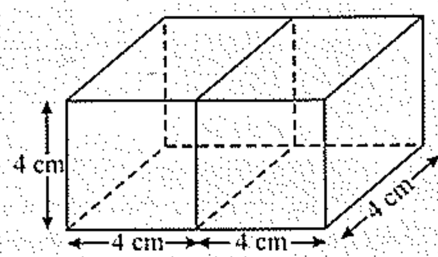
$= \frac{1}{4} \times \frac{22}{7} \times (7)^2 \text{ cm}^2$

$= \frac{77}{2} \text{ cm}^2$

$= \mathbf{38.5\ cm^2}$

16. Two cubes, each of side 4 cm are joined end to end. Find the surface area of the resulting cuboid.

Solution. We have the dimension of the cuboid formed when two edges of two cubes are joined.



Length of the cuboid (l) = (4 + 4) = 8 cm

Breadth of the cuboid (b) = 4 cm

Height of the cuboid (h) = 4 cm

Surface area of the cuboid formed

$$= 2(lb + bh + hl)$$
$$= 2[8 \times 4 + 4 \times 4 + 4 \times 8] \text{ cm}^2$$
$$= 2[32 + 16 + 32] \text{ cm}^2$$
$$= 2[80] \text{ cm}^2$$
$$= \mathbf{160 \text{ cm}^2}.$$

17. Find that value(s) of x for which the distance between the points $P(x, 4)$ and $Q(9, 10)$ is 10 units.

Solution. Here $P(x, 4)$ and $Q(9, 10)$ be the given points. Then

$$PQ = 10 \text{ units (given)}$$

$$\Rightarrow \sqrt{(9-x)^2 + (10-4)^2} = 10$$
$$\Rightarrow (9-x)^2 + 36 = 100$$
$$\Rightarrow 81 + x^2 - 18x + 36 = 100$$
$$\Rightarrow x^2 - 18x + 117 = 100$$
$$\Rightarrow x^2 - 18x + 17 = 0$$
$$\Rightarrow x^2 - 17x - x + 17 = 0$$
$$\Rightarrow x(x-17) - (x-17) = 0$$
$$\Rightarrow x - 17 = 0 \text{ or } x - 1 = 0$$
$$\Rightarrow \mathbf{x = 17 \text{ or } x = 1}$$

18. A coin is tossed two times. Find the probability of getting at least one head.

Solution. When a coin is tossed two times, the total possible outcomes are four *viz.*, {HH, HT, TH and TT} which are equally likely.

The outcome favourable to the event E, "getting at least one head" are : {HH, HT and TH }

So, the number of outcomes favourable to event E is 3.

∴ Probability of getting at least one head

$$= P(E) = \mathbf{\frac{3}{4}}.$$

Section 'C'

Question numbers 19 to 28 carry 3 marks each.

19. Find the roots of the following quadratic equation :

$$\mathbf{2\sqrt{3}x^2 - 5x + \sqrt{3} = 0}$$

Solution. Given : $2\sqrt{3}x^2 - 5x + \sqrt{3} = 0$

Here $a = 2\sqrt{3}$, $b = -5$ and $c = \sqrt{3}$

$$\therefore \quad D = b^2 - 4ac$$
$$\Rightarrow \quad D = (-5)^2 - 4(2\sqrt{3})(\sqrt{3})$$

$\Rightarrow \quad D = 25 - 24$

$\Rightarrow \quad D = 1$

$\therefore \quad x = \dfrac{-b \pm D}{2a}$

$\Rightarrow \quad x = \dfrac{5 \pm 1}{2 \times 2\sqrt{3}}$

$\Rightarrow \quad x = \dfrac{5 \pm 1}{4\sqrt{3}}$

$\Rightarrow \quad x = \dfrac{5+1}{4\sqrt{3}} \quad \text{or} \quad x = \dfrac{5-1}{4\sqrt{3}}$

$\Rightarrow \quad x = \dfrac{6}{4\sqrt{3}} \quad \text{or} \quad x = \dfrac{4}{4\sqrt{3}}$

$\Rightarrow \quad x = \dfrac{\sqrt{3}}{2} \quad \text{or} \quad x = \dfrac{1}{\sqrt{3}}$

Hence the roots of given quadratic equation are : $\dfrac{\sqrt{3}}{2}$ and $\dfrac{1}{\sqrt{3}}$.

Alternative Method :

Given : $\quad 2\sqrt{3}x^2 - 5x + \sqrt{3} = 0$

$\Rightarrow \quad 2\sqrt{3}x^2 - 3x - 2x + \sqrt{3} = 0$

$\Rightarrow \quad (2\sqrt{3}x^2 - 3x) - (2x - \sqrt{3}) = 0$

$\Rightarrow \quad \sqrt{3}x(2x - \sqrt{3}) - (2x - \sqrt{3}) = 0$

$\Rightarrow \quad (2x - \sqrt{3})(\sqrt{3}x - 1) = 0$

$\Rightarrow \quad 2x - \sqrt{3} = 0 \quad \text{or} \quad \sqrt{3}x - 1 = 0$

$\Rightarrow \quad x = \dfrac{\sqrt{3}}{2} \quad \text{or} \quad x = \dfrac{1}{\sqrt{3}}$

Hence the roots of the given quadratic equation are : $\dfrac{\sqrt{3}}{2}$ and $\dfrac{1}{\sqrt{3}}$.

20. Find the value of the middle term of the following A.P. :

$-6, -2, 2, \ldots, 58.$

Solution. Given A.P. is $-6, -2, 2, \ldots, 58$

The above A.P. whose first term (a) is -6, and

Common difference $(d) = -2 - (-6)$

$= -2 + 6$

$= 4$

Let the number of terms in the given A.P.

$-6, -2, 2, \ldots, 58$ be n, then

$t_n = 58$

$\Rightarrow \quad a + (n-1)d = 58$

$\Rightarrow -6 + (n-1)(4) = 58$

$\Rightarrow \quad (n-1)4 = 58 + 6$

$\Rightarrow \quad (n-1)4 = 64$

$\Rightarrow \quad n - 1 = 16$

$\Rightarrow \quad n = 16 + 1 = 17$

So, the middle term is 9th term

$\therefore \quad t_9 = a + (9 - 1)d$

$\Rightarrow \quad t_9 = -6 + (8)(4)$

$\Rightarrow \quad t_9 = -6 + 32$

$\Rightarrow \quad t_9 = 26$

Hence, the value of the middle term is **26.**

Or

Determine the A.P. whose fourth term is 18 and the difference of the ninth term from the fifteenth term is 30.

Solution. Let the first term and common difference of the A.P. be 'a' and 'd', respectively.

Let t_4, t_9 and t_{15} denote the 4th term, the 9th term and the 15th term of the A.P. then,

$t_4 = a + (4 - 1)d$ $\quad [\because n = 4]$

$\Rightarrow \quad 18 = a + 3d$ $\quad$ ...(1) $[\because t_4 = 18\text{(given)}]$

It is given that the difference of the 9th term from the fifteenth term is 30.

$\therefore \quad t_{15} - t_9 = 30$

$\Rightarrow \quad [a + (15 - 1)d] - [a + (9 - 1)d] = 30$

$\Rightarrow \quad (a + 14d) - (a + 8d) = 30$

$\Rightarrow \quad 6d = 30$

$\Rightarrow \quad d = 5$ $\quad$ ...(2)

From (1) and (2), we get

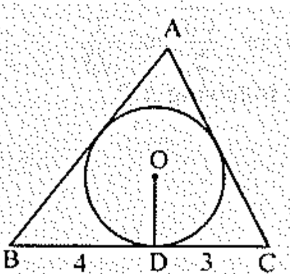
$a + 3 \times 5 = 18$

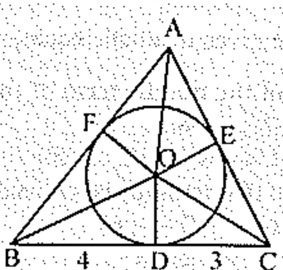
$\Rightarrow \quad a + 15 = 18$

$\Rightarrow \quad a = 18 - 15 = 3$

Thus, the A.P. is **3, 8, 11, 16...**

21. In figure, a triangle ABC is drawn to circumscribe a circle of radius 2 cm such that the segments BD and DC into which BC is divided by the point of contact D are of lengths 4 cm and 3 cm respectively. If area of $\Delta ABC = 21$ cm^2, then find the lengths of sides AB and AC.





Solution. Let a triangle ABC be drawn to circumscribe a circle with centre O of radius 2 cm such that the segments BD and DC into which BC is divided by the point D are of lengths 4 cm and 3 cm respectively.

Since the tangents drawn from an external point to a circle are equal in length.

$\therefore \quad AF = AE$ $\quad$ ...(1) [Tangents from A]

$BF = BD$ $\quad$ ...(2) [Tangents from B]

$CE = CD$ $\quad$ ...(3) [Tangents from C]

It is given that $BD = 4$ cm, $DC = 3$ cm therefore, from (2) and (3), we have : $BF = 4$ cm and $CE = 3$ cm.

Let $AF = AE = x$ cm, then

$AB = AF + BF = (x + 4)$ cm, $BC = BD + DC = (4 + 3) = 7$ cm, $CA = CE + AE = (3 + x)$ cm

Join OA, OB and OC and draw OE and OF perpendiculars on AC and AB respectively.

From figure

$$\text{Area of } \Delta ABC = \text{Area of } \Delta OAB + \text{Area of } \Delta OBC + \text{Area of } \Delta OCA$$

$$\Rightarrow \quad \text{(given) } 21 \text{ cm}^2 = \frac{1}{2} \times AB \times \text{radius of a circle} + \frac{1}{2} \times BC \times \text{radius of a circle} + \frac{1}{2} \times CA \times \text{radius of a circle}$$

$$\Rightarrow \quad 21 = \frac{1}{2} \times (x+4) \times 2 + \frac{1}{2} \times 7 \times 2 + \frac{1}{2} \times (3+x) \times 2$$

$$\Rightarrow \quad 21 = (x + 4) + 7 + (3 + x)$$

$$\Rightarrow \quad 21 = 2x + 14$$

$$\Rightarrow \quad 2x = 21 - 14$$

$$\Rightarrow \quad 2x = 7$$

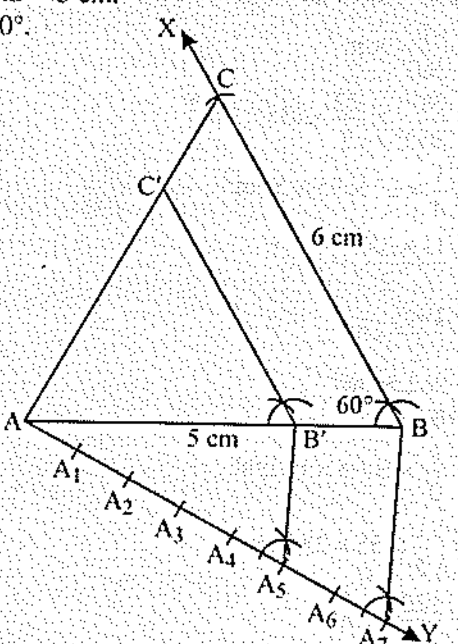
$$\Rightarrow \quad x = 3.5 \text{ cm}$$

Hence, the lengths of sides AB and AC are $(4 + 3.5) =$ **7.5 cm**, $(3 + 3.5) =$ **6.5 cm** respectively.

22. Draw a triangle ABC in which $AB = 5$ cm, $BC = 6$ cm and $\angle ABC = 60°$. Then construct a triangle whose sides are $\frac{5}{7}$ times the corresponding sides of ΔABC.

Solution. Steps of construction :

1. Draw a line segment $AB = 5$ cm.
2. At B make $\angle ABX = 60°$.

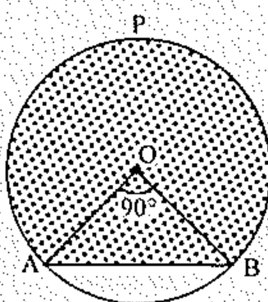


3. With B as centre and radius equal to 6 cm draw an arc intersecting BX at C.
4. Join AC. Then ABC is the required triangle.
5. Draw any ray AY making an acute angle with AB on the opposite side of the vertex C.
6. Locate 7 points $\left(\text{the greater of 5 and 7 in } \frac{5}{7}\right)$ $A_1, A_2, A_3, A_4, A_5, A_6$ and A_7 on AY so that

$$AA_1 = A_1A_2 = A_2A_3 = A_3A_4 = A_4A_5 = A_5A_6 = A_6A_7.$$

7. Join A_7B and draw a line through A_5 $\left(\text{the 5th point, 5 being smaller of 5 and 7 in } \frac{5}{7}\right)$ parallel to A_7B intersecting AB at B'.
8. Draw a line through B' parallel to the line BC to intersect AC at C'. Then $AB'C'$ is the required triangle.

23. Find the area of the major segment APB, in figure, of a circle of radius 35 cm and $\angle AOB = 90°$.



Solution. Area of the sector OAB of an angle $\angle AOB = 90°$ in a circle of radius 35 cm

$$= \frac{\theta}{360°} \times \pi r^2$$
$$= \frac{90°}{360°} \times \frac{22}{7} \times (35)^2 \text{ cm}^2$$
$$= \frac{1}{4} \times 22 \times 35 \times 5 \text{ cm}^2$$
$$= \frac{11 \times 35 \times 5}{2} \text{ cm}^2$$
$$= \frac{1925}{2} \text{ cm}^2$$
$$= 962.5 \text{ cm}^2$$

Area of a right triangle OAB

$$= \frac{1}{2} \times \text{Base} \times \text{Height}$$
$$= \left(\frac{1}{2} \times 35 \times 35\right) \text{ cm}^2$$
$$= \frac{1225}{2} \text{ cm}^2$$

$= 612.5 \text{ cm}^2$

Area of minor segment = Area of sector – Area of a right triangle

$= (962.5 - 612.5) \text{ cm}^2$

$= 350 \text{ cm}^2$

Area of the major segment APB of a circle of radius 35 cm and $\angle AOB = 90°$

= Area of a circle – Area of a minor segment

$= \pi r^2 - 350 \text{ cm}^2$

$= \frac{22}{7} \times (35)^2 - 350 \text{ cm}^2$

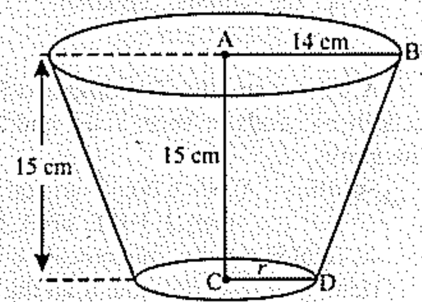
$= 22 \times 35 \times 5 \text{ cm}^2 - 350 \text{ cm}^2$

$= (3850 - 350) \text{ cm}^2$

$= \mathbf{3500 \text{ cm}^2}$

24. The radii of the circular ends of a bucket of height 15 cm are 14 cm and r cm ($r < 14$ cm). If the volume of bucket is 5390 cm^3, then find the value of r. $\left[\textbf{Use } \pi = \frac{22}{7}\right]$

Solution. Let 'r' and 'R' be the radii of the bottom and top circular ends of a bucket and 'h' be the height of the bucket.



$\therefore$ Volume of the bucket

$$= \frac{1}{3}\pi h(R^2 + r^2 + Rr)$$

$$\Rightarrow \quad 5390 \text{ (given)} = \frac{1}{3} \times \frac{22}{7} \times 15 \times (14^2 + r^2 + 14r)$$

$[\because h = 15 \text{ cm}, R = 14 \text{ cm}]$

$$\Rightarrow \quad 5390 = \frac{22}{7} \times 5 \times (196 + r^2 + 14r)$$

$$\Rightarrow \quad \frac{5390 \times 7}{22 \times 5} = 196 + r^2 + 14r$$

$\Rightarrow \quad 49 \times 7 = 196 + r^2 + 14r$

$\Rightarrow \quad 343 = 196 + r^2 + 14r$

$\Rightarrow \quad r^2 + 14r + 196 - 343 = 0$

$\Rightarrow \quad r^2 + 14r - 147 = 0$

$\Rightarrow \quad r^2 + 21r - 7r - 147 = 0$

$\Rightarrow \quad r(r + 21) - 7(r + 21) = 0$

$\Rightarrow \quad (r + 21)(r - 7) = 0$

$\Rightarrow$ Either $r + 21 = 0$ or $r - 7 = 0$

$\Rightarrow$ Either $r = -21$ or $r = 7$

Reject $r = -21$. Therefore, $r = 7$.

Hence the value of r is **7 cm.**

25. Two dice are rolled once. Find the probability of getting such numbers on two dice, whose product is a perfect square.

Solution. When two dice are rolled once, then the possible outcomes of the experiment are listed in the table :

	1	2	3	4	5	6
1	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
2	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
3	(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
4	(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
5	(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
6	(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

So, the number of possible outcomes $= 6 \times 6 = 36$

Let A be the event of getting such numbers on two dice, whose product is a perfect square.

There are 8 perfect square numbers, namely

$$A = \{(1, 1)\ (2, 2)\ (3, 3)\ (4, 4)\ (5, 5)\ (6, 6), (1, 4), (4, 1)\}$$

$\therefore$ Number of outcomes favourable to event $A = 8$

Hence the required probability : $P(A) = \frac{8}{36} = \frac{2}{9}$.

Or

A game consists of tossing a coin 3 times and noting its outcome each time. Hanif wins if he gets three heads or three tails, and loses otherwise. Calculate the probability that Hanif will lose the game.

Solution. When a coin is tossed 3 times, possible outcomes are

{HHH, HHT, THH, HTH, HTT, THT, TTH, TTT}

$\therefore$ Total number of outcomes = 8

Hanif will lose the game if all the tossed do not give the same result, *i.e.*, all heads or all tails.

So, favourable outcomes are

{HHT, THH, HTH, HTT, THT, TTH}

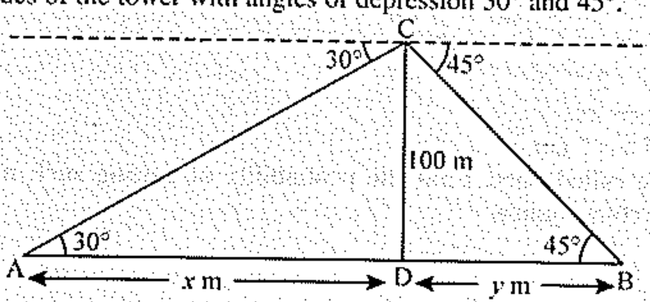
$\therefore$ Favourable number of outcomes are 6

Hence, P(Hanif will lose the game) $= \frac{6}{8} = \frac{3}{4}$

26. From the top of a tower 100 m high, a man observes two cars on the opposite sides of the tower with angles of depression 30° and 45° respectively. Find the distance between the cars.

[Use $\sqrt{3} = 1.73$]

Solution. Let $CD = 100$ m be the height of a tower. Let C be the position of a man observes two cars on the opposite sides of the tower with angles of depression 30° and 45°.



$\therefore \quad \angle CAD = 30°$ and $\angle CBD = 45°$

Let $AD = x$ m and $DB = y$ m

In right triangle ADC, we have

$$\tan 30° = \frac{CD}{AD}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{100}{x}$$

$$\Rightarrow \quad x = 100\sqrt{3} \text{ m} \quad \quad ...(1)$$

In right triangle BDC, we have

$$\tan 45° = \frac{CD}{DB}$$

$$\Rightarrow \quad 1 = \frac{100}{y}$$

$$\Rightarrow \quad y = 100 \text{ m} \quad \quad ...(2)$$

The distance between the two cars is AB *i.e.*,

$$\begin{aligned} AB &= AD + DB \\ &= x + y \\ &= (100\sqrt{3} + 100) \text{ m} \\ &= (100 \times 1.73 + 100) \text{ m} \\ &= (173 + 100) \text{ m} \\ &= 273 \text{ m} \end{aligned}$$

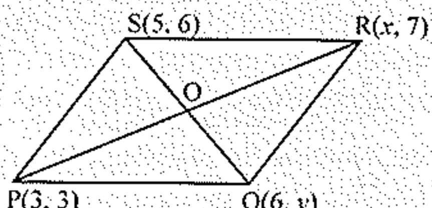
Thus, the distance between the two cars is **273 m.**

27. If (3, 3), (6, *y*), (*x*, 7) and (5, 6) are the vertices of a parallelogram taken in order, find the values of *x* and *y*.

Solution. We know that the diagonal of a ||gm bisect each other.

Let $P(3, 3)$, $Q(6, y)$, $R(x, 7)$ and $S(5, 6)$ be the vertices of a ||gm taken in order.

$\therefore$ Mid-point of diagonal PR = Mid-point of diagonal QS



$$\Rightarrow \quad \left(\frac{3+x}{2}, \frac{3+7}{2}\right) = \left(\frac{6+5}{2}, \frac{y+6}{2}\right)$$

$$\Rightarrow \quad \frac{3+x}{2} = \frac{11}{2} \quad \text{and} \quad \frac{10}{2} = \frac{y+6}{2}$$

$$\Rightarrow \quad 3 + x = 11 \quad \text{and} \quad 10 = y + 6$$

$$\Rightarrow \quad x = 11 - 3 \quad \text{and} \quad y = 10 - 6$$

$$\Rightarrow \quad x = 8 \quad \text{and} \quad y = 4$$

Hence the values of x and y are **8** and **4** respectively.

28. If two vertices of an equilateral triangle are (3, 0) and (6, 0), find the third vertex.

Solution. Let the two vertices of an equilateral triangle are $P(3, 0)$ and $Q(6, 0)$ respectively. Let the third vertex R be (x, y).

For an equilateral triangle, we have

$$PQ = QR = RP$$

Consider $PQ = QR$, then

$$PQ^2 = QR^2$$

$\Rightarrow \quad (6-3)^2+(0-0)^2=(x-6)^2+(y-0)^2$

$\Rightarrow \quad 9+0=(x-6)^2+y^2$

$\Rightarrow \quad (x-6)^2+y^2=9 \qquad \ldots(1)$

Again, consider $PQ = PR$, then

$$PQ^2 = PR^2$$

$\Rightarrow \quad (6-3)^2+(0-0)^2=(x-3)^2+(y-0)^2$

$\Rightarrow \quad 9+0=(x-3)^2+y^2$

$\Rightarrow \quad (x-3)^2+y^2=9 \qquad \ldots(2)$

Subtracting (2) from (1), we get

$[(x-6)^2+y^2]-[(x-3)^2+y^2]=9-9$

$\Rightarrow \quad (x-6)^2-(x-3)^2=0$

$\Rightarrow (x^2-12x+36)-(x^2-6x+9)=0$

$\Rightarrow \quad (-12x+6x)+(36-9)=0$

$\Rightarrow \quad -6x+27=0$

$\Rightarrow \quad x=\frac{27}{6}$

$\Rightarrow \quad x=\frac{9}{2}$

Further, substituting $x=\frac{9}{2}$ in (1), we get

$$\left(\frac{9}{2}-6\right)^2+y^2=9$$

$\Rightarrow \quad \left(\frac{9-12}{2}\right)^2+y^2=9$

$\Rightarrow \quad \frac{9}{4}+y^2=9$

$\Rightarrow \quad y^2=9-\frac{9}{4}$

$\Rightarrow \quad y^2=\frac{36-9}{4}$

$\Rightarrow \quad y^2=\frac{27}{4}$

$\Rightarrow \quad y=\pm\frac{3\sqrt{3}}{2}$

Hence, the third vertex of an equilateral triangle is $\left(\frac{9}{2}, \frac{3\sqrt{3}}{2}\right)$ or $\left(\frac{9}{2}, \frac{-3\sqrt{3}}{2}\right)$.

Or

Find the value of k, if the points $P(5, 4)$, $Q(7, k)$ and $R(9, -2)$ are collinear.

Solution. Since the given points P, Q and R are collinear, therefore, the area of the triangle PQR formed by them must be zero, *i.e.*,

$$\frac{1}{2}[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)]=0$$

Here $x_1 = 5, y_1 = 4, x_2 = 7, y_2 = k, x_3 = 9, y_3 = -2$

$$\Rightarrow \quad \frac{1}{2}[5(k+2) + 7(-2-4) + 9(4-k)] = 0$$

$$\Rightarrow \quad \frac{1}{2}[5k + 10 - 42 + 36 - 9k] = 0$$

$$\Rightarrow \quad \frac{1}{2}[-4k + 4] = 0$$

$$\Rightarrow \quad -4k + 4 = 0$$

$$\Rightarrow \quad k = 1$$

Verification : Area of $\Delta PQR = \frac{1}{2}[5(1+2) + 7(-2-4) + 9(4-1)]$

$$= \frac{1}{2}[15 - 42 + 27]$$

$$= \frac{1}{2}[42 - 42] = 0$$

Section 'D'

Question numbers 29 to 34 carry 4 marks each.

29. A motor boat whose speed is 20 km/h in still water, takes 1 hour more to go 48 km upstream than to return downstream to the same spot. Find the speed of the stream.

Solution. Let the speed of the stream be x km/h.

and the speed of the motor boat in still water = 20 km/h.

$\therefore$ Speed of the motor boat upstream = $(20 - x)$ km/h

and speed of the motor boat downstream = $(20 + x)$ km/h

Thus, time taken in going 48 km upstream = $\frac{48}{20-x}$ hours

and time taken in going 48 km downstream = $\frac{48}{20+x}$ hours

But the motor boat takes 1 hour more in going 48 km upstream and returning back 48 km downstream.

According to the given condition, we have

$$\frac{48}{20-x} - \frac{48}{20+x} = 1\text{h (given)}$$

$$\Rightarrow \quad 48 \times \left[\frac{1}{20-x} - \frac{1}{20+x}\right] = 1$$

$$\Rightarrow \quad 48 \times \left[\frac{20 + x - 20 + x}{(20-x)(20+x)}\right] = 1$$

$$\Rightarrow \quad 48 \times 2x = (20)^2 - x^2$$

$$\Rightarrow \quad 96x = 400 - x^2$$

$\Rightarrow \quad x^2 + 96x - 400 = 0$

$\Rightarrow \quad x^2 + 100x - 4x - 400 = 0$

$\Rightarrow \quad x(x + 100) - 4(x + 100) = 0$

$\Rightarrow \quad (x + 100)(x - 4) = 0$

$\Rightarrow$ Either $x + 100 = 0$ or $x - 4 = 0$

$\Rightarrow$ Either $x = -100$ or $x = 4$

Since x is the speed of the stream, it cannot be negative. So, we ignore the root $x = -100$.

Therefore, $x = 4$ gives the speed of the stream = **4 km/h**

Or

Find the roots of the equation $\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30}$, $x \neq -4, 7$.

Solution. We have

$$\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30}$$

As $x \neq -4, 7$ multiplying the equation by $(x + 4)(x - 7)$, we get

$$(x - 7) - (x + 4) = \frac{11}{30}(x + 4)(x - 7)$$

$\Rightarrow \quad -11 = \frac{11}{30}(x + 4)(x - 7)$

$\Rightarrow \quad -30 = (x + 4)(x - 7)$

$\Rightarrow \quad -30 = x^2 - 7x + 4x - 28$

$\Rightarrow \quad x^2 - 3x + 2 = 0$

So the given equation reduces to $x^2 - 3x + 2 = 0$, which is a quadratic equation.

Here, $a = 1$, $b = -3$, $c = 2$

So, $b^2 - 4ac = (-3)^2 - 4(1)(2)$

$= 9 - 8$

$= 1 > 0$

Therefore, $x = \frac{3 \pm \sqrt{1}}{2} = \frac{3 \pm 1}{2}$ *i.e.*, $x = 2$ or $x = 1$

So, the roots are **1** and **2.**

30. If the sum of first 4 terms of an A.P. is 40 and that of first 14 terms is 280, find the sum of its first n terms.

Solution. Let a and d be the first term and common difference of an A.P. $a, a + d, a + 2d, \ldots$

It is given that :

The sum of first 4 terms of an A.P. = 40 *i.e.*, $S_4 = 40$ ...(1)

and the sum of first 14 terms of an A.P. = 280 *i.e.*, $S_{14} = 280$ ...(2)

Rewriting (1) and (2) as

$$\frac{4}{2}[2a + (4 - 1)d] = 40$$

$\Rightarrow \quad 2[2a + 3d] = 40$

$\Rightarrow \quad 2a + 3d = 20$ ...(3)

and $\frac{14}{2}[2a + (14 - 1)d] = 280$

$\Rightarrow \quad 7[2a + 13d] = 280$

$\Rightarrow \quad 2a + 13d = 40 \quad \ldots(4)$

Subtracting (3) from (4), we get

$(2a + 13d) - (2a + 3d) = 40 - 20$

$\Rightarrow \quad 10d = 20$

$\Rightarrow \quad d = 2$

Substituting $d = 2$ in (3), we get

$2a + 6 = 20$

$\Rightarrow \quad 2a = 14$

$\Rightarrow \quad a = 7$

Thus, the sum of n terms of an A.P. is S_n, *i.e.*,

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$\Rightarrow \quad S_n = \frac{n}{2}[2 \times 7 + (n - 1) \times 2]$$

$$\Rightarrow \quad S_n = n[7 + (n - 1)]$$

$$\Rightarrow \quad S_n = n[6 + n]$$

$$\Rightarrow \quad \mathbf{S_n = n^2 + 6n.}$$

Or

Find the sum of the first 30 positive integers divisible by 6.

Solution. The first 30 positive integers divisible by 6 are

6, 12, 18, ..., 180.

Here, $a = 6$, $d = 12 - 6 = 6$ and $n = 30$

$\therefore$ Sum of the first 30 positive integers divisible by 6 is S_{30}, *i.e.*,

$$S_{30} = \frac{30}{2}[2a + (30 - 1)d]$$

$$\Rightarrow \quad S_{30} = 15[2 \times 6 + 29 \times 6]$$

$$\Rightarrow \quad S_{30} = 15[12 + 174]$$

$$\Rightarrow \quad S_{30} = 15 \times 186$$

$$\Rightarrow \quad \mathbf{S_{30} = 2790}$$

Note : Also, $S_{30} = \frac{30}{2}[6 + 180] = 15[6 + 180] = 15 \times 186 = \mathbf{2790}.$ $\quad \left[\because S_n = \frac{n}{2}(a + l)\right]$

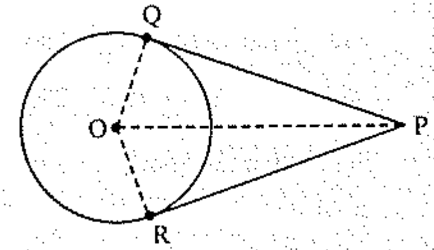
31. Prove that the lengths of tangents drawn from an external point to a circle are equal.

Solution. Given : PQ and PR are two tangents, from an external point P to a circle.

To prove : $PQ = PR$.

Construction : Join OP, OQ and OR.

Proof : $\angle OQP$ and $\angle ORP$ are right angles, because these are angles between the radii and tangents. [The tangent at any point of a circle is perpendicular to the radius through the point of contact]



Now in right triangles OQP and ORP, we have

$OQ = OR$ [Radii of the same circle]

$\angle OQP = \angle ORP$ [$OQ \perp PQ$ and $OR \perp PR$]

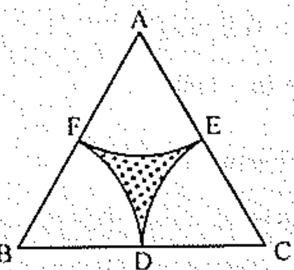
$OP = OP$ [Common]

By RHS theorem of congruence,

$\Delta OQP \cong \Delta QRP$

$\Rightarrow$ $PQ = PR$ [CPCT]

32. In figure, arcs are drawn by taking vertices A, B and C of equilateral triangle ABC of side 14 cm as centres to intersect the sides BC, CA and AB at their respective mid-point D, E and F. Find the area of the shaded region. $\left[\text{Use } \pi = \frac{22}{7} \text{ and } \sqrt{3} = 1.73\right]$



Solution. Clearly, arcs are drawn by taking vertices A, B and C of an equilateral triangle ABC of sides 14 cm as centres to intersect the sides BC, CA and AB at their respective mid-points D, E and F.

Therefore, the radius of each sector of an angle $60° = \frac{14}{2}$ cm $= 7$ cm.

Let **A** be the area of three sectors each of angle 60° and a radius of each sector be 7 cm, then,

A = 3 × Area of one sector of angle of 60° and the radius of sector is 7 cm.

$$= 3 \times \left[\frac{60°}{360°} \times \pi \times (7)^2\right] \text{cm}^2 \qquad \left[\because \text{Area of sector} = \frac{\theta}{360°} \times \pi r^2\right]$$

$$= \frac{3}{6} \times \pi \times 49 \text{ cm}^2$$

$$= \frac{3}{6} \times \frac{22}{7} \times 49 \text{ cm}^2$$

$$= 77 \text{ cm}^2$$

Now, required area of shaded region

$$= \text{Area of an equilateral } \Delta ABC - \mathbf{A}$$

$$= \frac{\sqrt{3}}{4}(\text{side})^2 - \mathbf{A}$$

$$= \left[\frac{\sqrt{3}}{4} \times (14)^2 - 77\right] \text{cm}^2$$

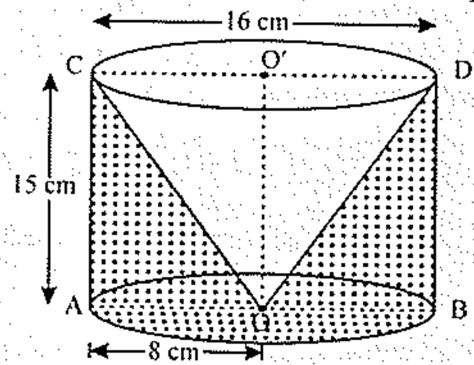
$$= \left[\frac{\sqrt{3}}{4} \times 196 - 77\right] \text{cm}^2$$

$= [1.73 \times 49 - 77]\ cm^2$
$= [84.77 - 77]\ cm^2$
$= \mathbf{7.77\ cm^2}$.

33. From a solid cylinder whose height is 15 cm and diameter 16 cm, a conical cavity of the same height and same diameter is hollowed out. Find the total surface area of the remaining solid. [Take π = 3.14]

Solution. We have

Slant height of the cone is OC, *i.e.*,



$$OC = \sqrt{(OO')^2 + (O'C)^2}$$
$$= \sqrt{(15)^2 + (8)^2}\ cm$$
$$= \sqrt{225 + 64}\ cm$$
$$= \sqrt{289}\ cm$$
$$= 17\ cm$$

Total surface area of the remaining solid

= Curved surface area of the cylinder + Area of the base of the cylinder + Curved surface area of the cone.

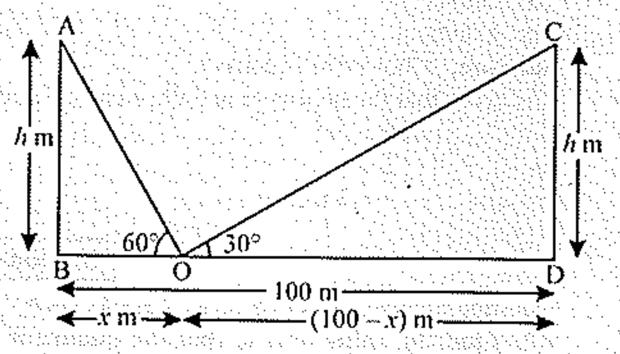
$= 2\pi rh + \pi r^2 + \pi rl$
$= \pi r[2h + r + l]$
$= 3.14 \times 8[2 \times 15 + 8 + 17]\ cm^2$ [$\because r = 8$ cm, $h = 15$ cm, $l = 17$ cm]
$= 3.14 \times 8[30 + 8 + 17]\ cm^2$
$= 3.14 \times 8[55]\ cm^2$
$= 3.14 \times 440\ cm^2$
$= 31.4 \times 44\ cm^2$
$= \mathbf{1381.6\ cm^2}$

34. Two poles of equal heights are standing opposite to each other on either side of the road, which is 100 m wide. From a point between them on the road, the angles of elevation of the top of the poles are 60° and 30°, respectively. Find the height of the poles.

Solution. Let AB and CD be two poles of equal height h metres. Let O be a point on the road such that $\angle AOB = 60°$ and $\angle COD = 30°$.

Let $OB = x$ m, then $OD = (100 - x)$ m, where BD = width of the road = 100 m

In right ΔAOB, we have



$$\tan 60° = \frac{AB}{OB}$$
$$\Rightarrow \quad \sqrt{3} = \frac{h}{x}$$
$$\Rightarrow \quad x = \frac{h}{\sqrt{3}} \quad ...(1)$$

In right ΔCOD, we have

$$\tan 30° = \frac{CD}{OD}$$

$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{100-x}$

$\Rightarrow \quad \sqrt{3}h = 100 - x$ ...(2)

From (1) and (2), we get

$\sqrt{3}h = 100 - \frac{h}{\sqrt{3}}$

$\Rightarrow \quad 3h = 100\sqrt{3} - h$

$\Rightarrow \quad 4h = 100\sqrt{3}$

$\Rightarrow \quad h = 25\sqrt{3}$

$\Rightarrow \quad h = 25 \times 1.732$ m

$\Rightarrow \quad h = 43.3$ m

Hence, the height of each pole is **43.3 m.**

SET II

Section 'A'

Question numbers 1 to 10 are of one mark each.

9. In an A.P., if $a = -10$, $n = 6$ and $a_n = 10$, then the value of d is

(*a*) 0 (*b*) 4

(*c*) -4 (*d*) $\frac{10}{3}$

Solution. Choice (*b*) is correct.

Given : $a = -10$, $n = 6$ and $a_n = 10$

$\therefore \quad a_n = 10$

$\Rightarrow \quad a + (n-1)d = 10$

$\Rightarrow \quad -10 + (6-1)d = 10$ [$\because n = 6, a = -10$]

$\Rightarrow \quad 5d = 20$

$\Rightarrow \quad \mathbf{d = 4}$

10. If the perimeter and the area of a circle are numerically equal, then the radius of the circle is

(*a*) 2 units (*b*) π units

(*c*) 4 units (*d*) 7 units

Solution. Choice (*a*) is correct.

It is given that the perimeter and the area of a circle are numerically equal.

$\therefore \quad 2\pi r = \pi r^2$

$\Rightarrow \quad 2 = r$

$\Rightarrow \quad r =$ **2 units.**

Section 'B'

Question numbers 11 to 18 carry 2 marks each.

17. Find the value of k so that the quadratic equation $kx(3x - 10) + 25 = 0$, has two equal roots.

Solution. The given quadratic equation is

$$kx(3x - 10) + 25 = 0$$

$\Rightarrow$ $$3kx^2 - 10kx + 25 = 0$$

Here $a = 3k$, $b = -10k$, $c = 25$

For equal roots :

$$D = b^2 - 4ac = 0$$

$\Rightarrow$ $$(-10k)^2 - 4(3k)(25) = 0$$

$\Rightarrow$ $$100k^2 - 300k = 0$$

$\Rightarrow$ $$100k(k-3) = 0$$

$\Rightarrow$ Either $k = 0$ or $k = 3$

Hence, the value of k is **3** and reject $k = 0$.

18. A coin is tossed two times. Find the probability of getting not more than one head.

Solution. When a coin is tossed two times, the total possible outcomes are four *viz.*, {HH, HT, TH and TT} which are equally likely.

The outcome favourable to the event E, "getting not more than one head" are {HT, TH, TT}

So, the number of outcomes favourable to event E is 3.

$\therefore$ Probability of getting not more than one head

$$= P(E) = \mathbf{\frac{3}{4}}$$

Section 'C'

Question numbers 19 to 28 carry 3 marks each.

26. If $P(2, 4)$ is equidistant from $Q(7, 0)$ and $R(x, 9)$, find the values of x. Also find the distance PQ.

Solution. Here, $P(2, 4)$, $Q(7, 0)$ and $R(x, 9)$ are the given points.

It is given that $P(2, 4)$ is equidistant from the points $Q(7, 0)$ and $R(x, 9)$.

$\therefore$ $$PQ = PR$$

$\Rightarrow$ $$PQ^2 = PR^2$$

$\Rightarrow$ $$(7-2)^2 + (0-4)^2 = (x-2)^2 + (9-4)^2$$

$\Rightarrow$ $$25 + 16 = (x-2)^2 + 25$$

$\Rightarrow$ $$(x-2)^2 = 16$$

$\Rightarrow$ $$x - 2 = \pm 4$$

$\Rightarrow$ $x - 2 = 4$ or $x - 2 = -4$

$\Rightarrow$ Either $x = 6$ or $x = -2$

Thus, the values of x are **6** or **– 2.**

Distance PQ between the points $P(2, 4)$ and $Q(7, 0)$

$$= \sqrt{(7-2)^2 + (0-4)^2}$$

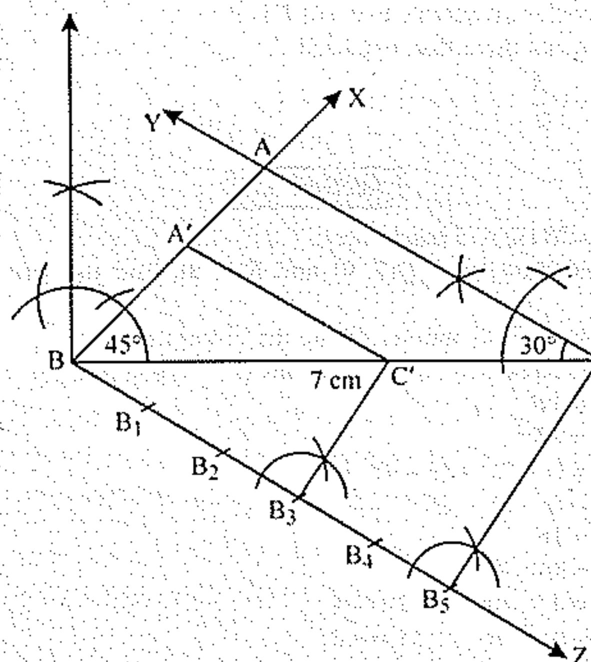
$$= \sqrt{25 + 16}$$

$$= \sqrt{41} \text{ units}$$

28. Draw a triangle ABC with side $BC = 7$ cm, $\angle B = 45°$ and $\angle A = 105°$. Then construct a triangle whose sides are $\frac{3}{5}$ times the corresponding sides of ΔABC.

Solution. Steps of construction :

1. Draw a line segment $BC = 7$ cm.
2. At B construct $\angle CBX = 45°$.
3. At C construct $\angle BCY = 180° - (45° + 105°) = 30°$.
4. Suppose BX and CY intersect at A. Then, ABC is the given triangle.
5. Construct an acute angle CBZ at B on opposite side of vertex A of ΔABC.
6. Locate 5 points $\left(\text{the greater of 3 and 5 in } \frac{3}{5}\right)$, B_1, B_2, B_3, B_4, B_5 such that $BB_1 = B_1B_2 = B_2B_3 = B_3B_4 = B_4B_5$.
7. Join B_5C and draw a line through B_3 $\left(\text{the 3rd point, 3 smaller of 3 and 5 in } \frac{3}{5}\right)$ parallel to B_5C to intersecting BC at C'.
8. Draw a line through C', parallel to the line CA to intersect BA at A'. Then $A'BC'$ is required triangle.



Section 'D'

Question numbers 29 to 34 carry 4 marks each.

32. From a point on the ground, the angles of elevation of the bottom and top of a transmission tower fixed at the top of a 10 m high building are 30° and 60° respectively. Find the height of the tower.

Solution. Let BC be the building of height 10 metres and CD be the tower of height h metres. Let A be the point on the ground at a distance of x m from the foot of the building.

In ΔABC, we have

$$\tan 30° = \frac{BC}{AB}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{10}{x}$$

$$\Rightarrow \quad x = 10\sqrt{3} \text{ metres}$$

In ΔADB, we have

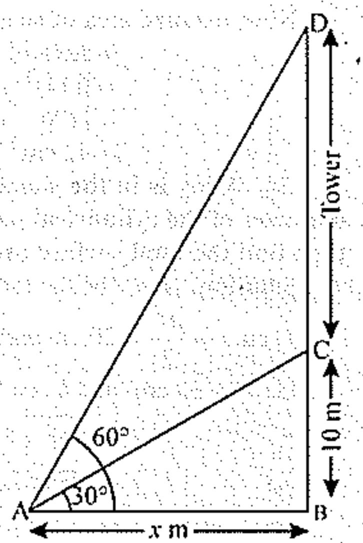
$$\tan 60° = \frac{BD}{AB}$$

$$\Rightarrow \quad \sqrt{3} = \frac{h+10}{10\sqrt{3}} \qquad [\because AB = x = 10\sqrt{3} \text{ m}]$$

$$\Rightarrow \quad h + 10 = 30$$

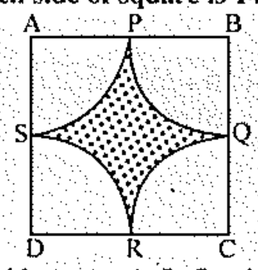
$$\Rightarrow \quad h = (30 - 10) \text{ m}$$

$$\Rightarrow \quad h = 20 \text{ m}$$



Hence the height of the tower is **20 metres.**

33. Find the area of the shaded region in figure, where arcs drawn with centres *A*, *B*, *C* and *D* intersect in pairs at mid-points *P*, *Q*, *R* and *S* of the sides *AB*, *BC*, *CD* and *DA* respectively of a square *ABCD*, where the length of each side of square is 14 cm.



Solution. Clearly arcs are drawn with centres *A*, *B*, *C* and *D* intersect in pairs at mid-points *P*, *Q*, *R* and *S* of the sides *AB*, *BC*, *CD* and *DA* respectively of a square *ABCD*, where the length of each side of square is 14 cm.

Therefore, the radius of each sector of an angle $90° = \frac{14}{2}$ cm = 7 cm.

Let **A** be the area of four quadrants each of angle 90° and the radius of each quadrant is 7 cm, then

A = 4 × Area of one quadrant of angle of 90° and the radius of quadrant is 7 cm.

$$= 4 \times \left[\frac{90°}{360°} \times \pi \times (7)^2\right] \text{cm}^2 \qquad \left[\text{Area of a quadrant} = \frac{\theta}{360°} \times \pi r^2\right]$$

$$= 4 \times \frac{1}{4}\pi \times 49 \text{ cm}^2$$

$$= 49\pi \text{ cm}^2$$

$$= 49 \times \frac{22}{7} \text{ cm}^2$$

$$= 154 \text{ cm}^2$$

Now, required area of shaded region

$= \text{Area of a square } ABCD \text{ with side 14 cm} - \mathbf{A}$
$= [(14)^2 - 154]\text{cm}^2$
$= [196 - 154]\text{cm}^2$
$= \mathbf{42\ cm^2}$

34. A toy is in the shape of a solid cylinder surmounted by a conical top. If the height and diameter of the cylindrical part are 21 cm and 40 cm respectively, and the height of cone is 15 cm, then find the total surface area of the toy. **[$\pi = 3.14$, be taken]**

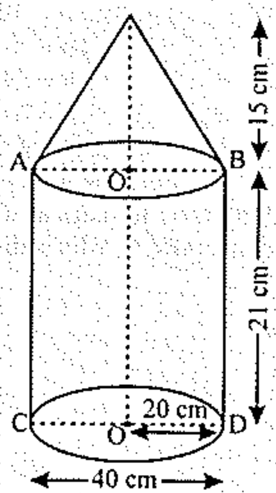
Solution. Let r be the radius of a solid cylinder and h be its height.

Then, $r = \frac{40}{2} = 20$ cm and $h = 21$ cm

Let r (= 20 cm) and h_1 (= 15 cm) be the radius and height of the cone and l_1 be its slant height.

$\therefore \quad l_1 = \sqrt{h_1^2 + r^2} = \sqrt{(15)^2 + (20)^2}$ cm

$\Rightarrow \quad l_1 = \sqrt{225 + 400}$ cm $= \sqrt{625}$ cm $= 25$ cm



Total surface area of the toy

= Curved surface area of the cylinder
+ Area of the base of the cylinder
+ Curved surface area of the cone

$= 2\pi rh + \pi r^2 + \pi r l_1$
$= \pi r[2h + r + l_1]$
$= 3.14 \times 20[2 \times 21 + 20 + 25]\ \text{cm}^2$
$= 31.4 \times 2[42 + 20 + 25]\ \text{cm}^2$
$= 31.4 \times 2 \times 87\ \text{cm}^2$
$= 62.8 \times 87\ \text{cm}^2$
$= \mathbf{5463.6\ cm^2}$.

SET III

Section 'A'

Question numbers 1 to 10 are of one mark each.

9. In an A.P., if $a = 15$, $d = -3$ and $a_n = 0$, then the value of n is

(*a*) 5 (*b*) 6
(*c*) 19 (*d*) 4

Solution. Choice (*b*) is correct.

Given : $a = 15$, $d = -3$, and $a_n = 0$

$\therefore \quad a_n = 0$
$\Rightarrow \quad a + (n-1)d = 0$
$\Rightarrow \quad 15 + (n-1)(-3) = 0$
$\Rightarrow \quad 15 - 3n + 3 = 0$
$\Rightarrow \quad 18 - 3n = 0$
$\Rightarrow \quad 3n = 18$
$\Rightarrow \quad \mathbf{n = 6}$

10. The radii of two circles are 8 cm and 6 cm respectively. The diameter of the circle having area equal to the sum of the areas of the two circles (in cm) is

(*a*) 10 (*b*) 14

(*c*) 20 (*d*) 28

Solution. Choice (*c*) is correct.

Let D be the diameter of the required circle, then

Area of a circle of diameter D cm

= Area of a circle of radius 8 cm + Area of a circle of radius 6 cm

[According to the given condition of the question]

$$\Rightarrow \quad \pi\left(\frac{D}{2}\right)^2 = \pi(8)^2 + \pi(6)^2$$

$$\Rightarrow \quad \left(\frac{D}{2}\right)^2 = 64 + 36$$

$$\Rightarrow \quad \frac{D^2}{4} = 100$$

$$\Rightarrow \quad D^2 = 400$$

$$\Rightarrow \quad D = 20 \text{ cm}$$

Hence, the diameter of the required circle is **20 cm.**

Section 'B'

Question numbers 11 to 18 carry 2 marks each.

11. A coin is tossed two times. Find the probability of getting both heads or both tails.

Solution. When a coin is tossed two times, the total possible outcomes are four, *viz.*, {HH, HT, TH and TT} which are equally likely.

The outcome favourable to the event E "getting both heads or both tails" are : {HH, TT}

So the number of outcomes favourable to event E is 2.

∴ Probability of getting both heads or both tails

$$= P(E) = \frac{2}{4} = \frac{1}{2}.$$

12. Find the value of m so that the quadratic equation $mx(5x - 6) + 9 = 0$ has two equal roots.

Solution. The given quadratic equation is

$$mx(5x - 6) + 9 = 0$$

$$\Rightarrow \quad 5mx^2 - 6mx + 9 = 0$$

Here, $a = 5m$, $b = -6m$, $c = 9$

For equal roots :

$$D = b^2 - 4ac = 0$$

$$\Rightarrow \quad (-6m)^2 - 4(5m)(9) = 0$$

$$\Rightarrow \quad 36m^2 - 180m = 0$$

$$\Rightarrow \quad 36m(m - 5) = 0$$

$$\Rightarrow \quad m = 0 \quad \text{or} \quad m = 5$$

Hence, the value of m is **5** and reject $m = 0$.

Section 'C'

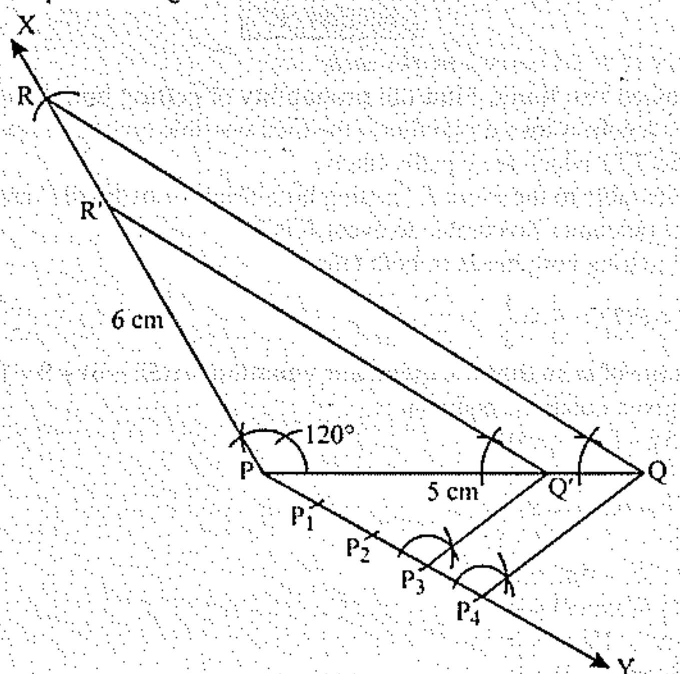
Question numbers 19 to 28 carry 3 marks each.

27. Draw a triangle PQR such that $PQ = 5$ cm, $\angle P = 120°$ and $PR = 6$ cm. Construct another triangle whose sides are $\frac{3}{4}$ times the corresponding sides of ΔPQR.

Solution. Steps of construction :

1. Draw a line segment $PQ = 5$ cm.
2. At P construct $\angle QPX = 120°$.
3. With P as centre and radius $PR = 6$ cm, draw an arc intersecting the line PX at R.
4. Join QR to obtain ΔPQR
5. Draw any other ray PY making an acute angle with PQ on the opposite side to the vertex R.
6. Locate 4 points $\left(\text{the greater of 3 and 4 in } \frac{3}{4}\right)$ P_1, P_2, P_3 and P_4 on PY so that $PP_1 = P_1P_2 = P_2P_3 = P_3P_4$.
7. Join P_4 $\left(\text{the fourth point being the greater of 3 and 4 in } \frac{3}{4}\right)$ to Q and draw a line through P_3 parallel to P_4Q intersecting the line PQ at Q'.
8. Draw a line through Q' parallel to QR intersecting PR at R'.

Thus, $PQ'R'$ is the required triangle.



28. Find the point on y-axis which is equidistant from the points $(-5, -2)$ and $(3, 2)$.

Solution. Since the required point, say P, is on the y-axis, therefore its abscissa will be zero.

Let the ordinate of the point P be y.

$\therefore$ Coordinates of the given point P are $P(0, y)$.

Let the two given points be $A(-5, -2)$ and $B(3, 2)$.

It is given that the point P on the y-axis is equidistant from the points $A(-5, -2)$ and $B(3, 2)$.

$$\therefore \quad AP = BP$$
$$\Rightarrow \quad AP^2 = BP^2$$
$$\Rightarrow \quad (0+5)^2 + (y+2)^2 = (0-3)^2 + (y-2)^2$$
$$\Rightarrow \quad 25 + y^2 + 4y + 4 = 9 + y^2 - 4y + 4$$
$$\Rightarrow \quad 29 + 4y = 13 - 4y$$
$$\Rightarrow \quad 8y = 13 - 29$$
$$\Rightarrow \quad 8y = -16$$
$$\Rightarrow \quad y = -2$$

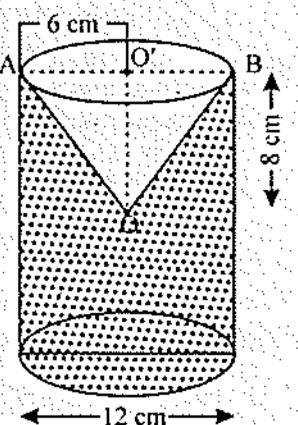
Thus, the required point P is $\mathbf{(0, -2)}$.

Section 'D'

Question numbers 29 to 34 carry 4 marks each.

29. From a solid cylinder of height 20 cm diameter 12 cm, a conical cavity of height 8 cm and radius 6 cm is hollowed out. Find the total surface area of the remaining solid.

Solution. We have



Slant height of the cone is OA, *i.e.*,

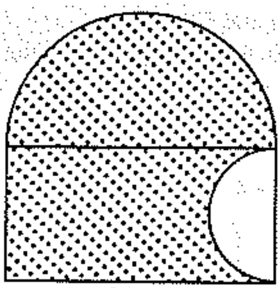
$$OA = \sqrt{(OO')^2 + (O'A)^2}$$
$$= \sqrt{(8)^2 + (6)^2} \text{ cm}$$
$$= \sqrt{64 + 36} \text{ cm}$$
$$= \sqrt{100} \text{ cm}$$
$$= 10 \text{ cm.}$$

Total surface area of the remaining solid

= Curved surface area of the cylinder + Area of the base of the cylinder + Curved surface area of the cone

$$= 2\pi rh + \pi r^2 + \pi rl$$
$$= \pi r[2h + r + l]$$
$$= \frac{22}{7} \times 6[2 \times 20 + 6 + 10] \text{ cm}^2 \qquad [\because h = 20 \text{ cm}, l = OA = 10 \text{ cm}]$$
$$= \frac{132}{7}[40 + 6 + 10] \text{ cm}^2$$
$$= \frac{132}{7} \times 56 \text{ cm}^2$$
$$= 132 \times 8 \text{ cm}^2$$
$$= \mathbf{1056 \text{ cm}^2}.$$

30. **The length and breadth of a rectangular piece of paper are 28 cm and 14 cm respectively. A semi-circular portion is cut off from the breadth's side and a semicircular portion is added on length's side, as shown in figure. Find the area of the shaded region.** $\left[\text{Use } \pi = \frac{22}{7}\right]$



Solution. We have

$AD = 28$ cm *i.e.*, length of a rectangular piece of paper

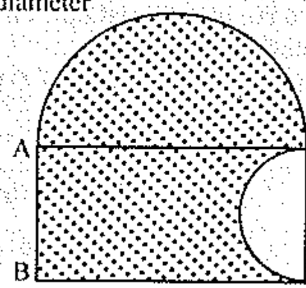
$DC = 14$ cm *i.e.*, breadth of a rectangular piece of paper.

$\mathbf{A_1}$ = Area of a rectangle of length $AD = 28$ cm and breadth $DC = 14$ cm.

$= (28 \times 14)\text{ cm}^2$

$= 392\text{ cm}^2$ ...(1)

$\mathbf{A_2}$ = Area of a semi-circular portion of the width DC as diameter



$= \frac{1}{2}\pi\left(\frac{14}{2}\right)^2\text{ cm}^2$

$= \frac{\pi}{2} \times 49\text{ cm}^2$

$= \frac{1}{2} \times \frac{22}{7} \times 49\text{ cm}^2$

$= (11 \times 7)\text{ cm}^2$

$= 77\text{ cm}^2$ ...(2)

$\mathbf{A_3}$ = Area of a semi-circular portion of the width AD as diameter

$= \frac{1}{2}\pi\left(\frac{28}{2}\right)^2\text{ cm}^2$

$= \frac{1}{2} \times \frac{22}{7} \times (14)^2\text{ cm}^2$

$= \frac{11}{7} \times 14 \times 14\text{ cm}^2$

$= 11 \times 2 \times 14$

$= 308\text{ cm}^2$ ...(3)

Now, required area of the shaded region

$= \mathbf{A_1 - A_2 + A_3}$

$= [392 - 77 + 308]\text{ cm}^2$

$= [700 - 77]\text{ cm}^2$

$= \mathbf{623\text{ cm}^2}$