CBSE
CCE SAMPLE QUESTION PAPER

SECOND TERM (SA-II)
MATHEMATICS
(With Solutions)
CLASS X

Time Allowed : 3 Hours] [*Maximum Marks* : 80

General Instructions :

(*i*) *All questions are compulsory.*

(*ii*) *The question paper consists of 34 questions divided into four sections A, B, C and D. Section A comprises of 10 questions of 1 mark each. Section B comprises of 8 questions of 2 marks each. Section C comprises of 10 questions of 3 marks each and Section D comprises of 6 questions of 4 marks each.*

(*iii*) *Question numbers 1 to 10 in Section A are multiple choice questions where you are to select one correct option out of the given four.*

(*iv*) *There is no overall choice. However, internal choice has been provided in 1 question of two marks, 3 questions of three marks each and 2 questions of four marks each. You have to attempt only one of the alternatives in all such questions.*

(*v*) *Use of calculators is not permitted.*

Section 'A'

Question numbers 1 to 10 are of one mark each.

1. Which of the following equations has the sum of its roots as 3 ?

(*a*) $x^2 + 3x - 5 = 0$ (*b*) $-x^2 + 3x + 3 = 0$

(*c*) $\sqrt{2}x^2 - \frac{3}{\sqrt{2}}x - 1 = 0$ (*d*) $3x^2 - 3x - 3 = 0$

Solution. Choice (*b*) is correct.

Given equation is $-x^2 + 3x + 3 = 0$ or $x^2 - 3x - 3 = 0$

Sum of the roots $= -\frac{b}{a} = -\frac{(-3)}{1} = \mathbf{3}$ [Here $a = 1$, $b = -3$, $c = -3$]

2. The sum of first five multiples of 3 is

(*a*) 45 (*b*) 65

(*c*) 75 (*d*) 90

Solution. Choice (*a*) is correct.

Sum of first five multiples of 3

$$= 3 + 6 + 9 + 12 + 15$$

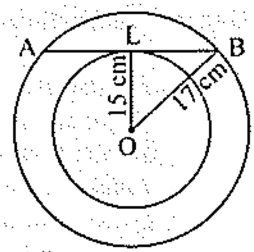
$$= \frac{5}{2}[3 + 15]$$

$= \frac{5}{2}[18]$ $\left[\text{Sum of } n \text{ terms of an A.P.} = \frac{n}{2}[a + l]. \text{ where } a \text{ is first term and } l \text{ is last term}\right]$

$= 45$

3. If radii of the two concentric circles are 15 cm and 17 cm, then the length of each chord of one circle which is tangent to other is

(*a*) 8 cm (*b*) 16 cm
(*c*) 30 cm (*d*) 17 cm



Solution. Choice (*b*) is correct.

Let O be the centre of two concentric circles and AB be a chord of the larger circle touching the smaller circle at L.

Join OL.

Since OL (= 15 cm) is the radius of the smaller circle and AB is a tangent to this circle at a point L.

$\therefore \quad OL \perp AB$

We know that the perpendicular drawn from the centre of a circle to any chord of the circle bisects the chord at L.

$\Rightarrow \quad AL = BL$

In right ΔOLB, we have

$LB^2 = OB^2 - OL^2$

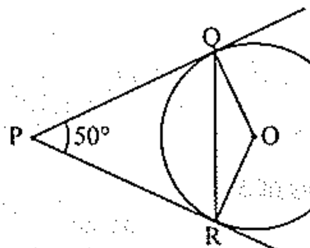
$\Rightarrow \quad LB^2 = (17)^2 - (15)^2$

$\Rightarrow \quad LB^2 = 289 - 225 = 64 = (8)^2$

$\Rightarrow \quad LB = 8$ cm

Length of chord $AB = AL + LB = 8 + 8 =$ **16 cm** $[\because AL = BL = 8 \text{ cm}]$

4. In figure, *PQ* and *PR* are tangents to the circle with centre *O* such that $\angle QPR = 50°$, then $\angle OQR$ is equal to



(*a*) 25° (*b*) 30°
(c) 40° (*d*) 50°

Solution. Choice (*a*) is correct.

Since the angle between two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line segments joining the points of contact at the centre, *i.e.*,

$\angle QPR + \angle QOR = 180°$

But $\angle QPR = 50°$ (given)

$\therefore \quad 50° + \angle QOR = 180°$

$\Rightarrow \quad \angle QOR = 180° - 50° = 130° \quad ...(1)$

In ΔOQR, we have

$$OQ = OR \quad \text{[Each = radius]}$$

$$\Rightarrow \angle ORQ = \angle OQR \quad ...(2) \text{ [}\angle s \text{ opposite to equal sides of a } \Delta \text{ are equal]}$$

In ΔOQR, we have

$$\angle OQR + \angle ORQ + \angle QOR = 180° \quad \text{[Sum of three } \angle s \text{ of a } \Delta = 180°\text{]}$$

$$\Rightarrow 2\angle OQR + 130° = 180° \quad \text{[using (1) and (2)]}$$

$$\Rightarrow 2\angle OQR = 180° - 130°$$

$$\Rightarrow \angle OQR = 50° \div 2 = 25°.$$

5. Two tangents making an angle of 120° with each other, are drawn to a circle of radius 6 cm, then the length of each tangent is equal to

(*a*) $\sqrt{3}$ cm (*b*) $6\sqrt{3}$ cm

(*c*) $\sqrt{2}$ cm (*d*) $2\sqrt{3}$ cm

Solution. Choice (*d*) is correct.

In Δ's *PAO* and *PBO*, we have

$$PA = PB \quad \text{[Tangents to a circle from an external point } P\text{]}$$

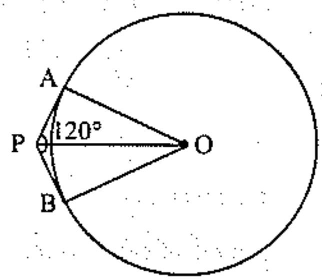
$$OP = OP \quad \text{[Common]}$$

$$\angle OAP = \angle OBP \quad \text{[Each = 90°]}$$

So, by RHS congruence rule, we have

$$\Delta PAO \cong \Delta PBO$$

$$\Rightarrow \angle OPA = \angle OPB = \frac{1}{2}\angle APB$$



$$\Rightarrow \angle OPA = \frac{1}{2} \times 120° = 60° \quad [\because \angle APB = 120° \text{ (given)}]$$

In right-angled triangle *OAP*,

$$\tan OPA = \frac{OA}{AP}$$

$$\Rightarrow \tan 60° = \frac{6}{AP} \quad [OA = 6 \text{ cm} = \text{radius (given)}]$$

$$\Rightarrow \sqrt{3} = \frac{6}{AP}$$

$$\Rightarrow AP = \frac{6}{\sqrt{3}} = \frac{3 \times 2}{\sqrt{3}} = 2\sqrt{3}\text{cm}$$

⇒ **Length of the each tangent = $2\sqrt{3}$ cm.**

6. To draw a pair of tangents to a circle which are inclined to each other at an angle of 100°, it is required to draw tangents at end points of those two radii of the circle, the angle between which should be

(*a*) 100° (*b*) 50°

(*c*) 80° (*d*) 200°

Solution. Choice (*c*) is correct.

Since tangent at a point to a circle is perpendicular to the radius through the point.

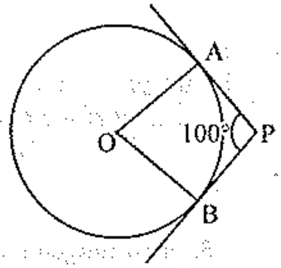
$$\therefore OA \perp AP \quad \text{and} \quad OB \perp BP$$

$$\Rightarrow \angle OAP = 90° \quad \text{and} \quad \angle OBP = 90°$$

It is given that a pair of tangents to a circle are inclined to each other at an angle of 100°.

$\therefore \quad \angle APB = 100°$

In quadrilateral $OAPB$, we have



$$\angle OAP + \angle APB + \angle AOB + \angle OBP = 360°$$

$$\Rightarrow \quad 90° + 100° + \angle AOB + 90° = 360°$$

$$\Rightarrow \quad 280° + \angle AOB = 360°$$

$$\Rightarrow \quad \angle AOB = 360° - 280°$$

$$\Rightarrow \quad \angle AOB = 80°$$

Thus, the angle between the two radii of a circle is **80°**.

7. The height of a cone is 60 cm. A small cone is cut off at the top by a plane parallel to the base and its volume is $\frac{1}{64}$th the volume of original cone. The height from the base at which the section is made is

(*a*) 15 cm (*b*) 30 cm

(*c*) 45 cm (*d*) 20 cm

Solution. Choice (*c*) is correct.

Let VAB be a cone of height 60 cm and base radius r cm.

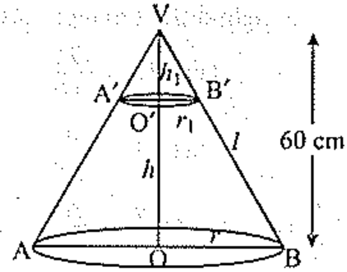
Suppose a small cone is cut off at the top by a plane parallel to the base at height 'h' cm from the base of the cone.

Clearly, $\Delta VOA \sim \Delta VO'A'$

$$\frac{VO}{VO'} = \frac{OA}{OA'}$$

$$\Rightarrow \quad \frac{60}{VO'} = \frac{r}{r_1} \Rightarrow \frac{h_1}{60} = \frac{r_1}{r} \quad \ldots(1) \; [\because VO' = h_1 \text{ and } VO = 60 \text{ cm}]$$

It is given that



$$\text{Volume of cone } VA'B' = \frac{1}{64} \text{ Volume of the original cone } VAB$$

$$\Rightarrow \quad \frac{1}{3}\pi r_1^2 h_1 = \frac{1}{64} \times \frac{1}{3}\pi r^2 \times 60 \quad [\because VO = 60 \text{ cm}]$$

$$\Rightarrow \quad r_1^2 h_1 = \frac{60}{64} r^2$$

$$\Rightarrow \quad \frac{r_1^2}{r^2} \times h_1 = \frac{60}{64} \quad \ldots(2)$$

$$\Rightarrow \quad \left(\frac{r_1}{r}\right)^2 \times h_1 = \frac{60}{64}$$

$$\Rightarrow \quad \left(\frac{h_1}{60}\right)^2 \times h_1 = \frac{60}{64}$$

$$\Rightarrow \quad h_1^3 = \frac{60 \times 60 \times 60}{64}$$

$\Rightarrow \quad h_1^3 = 15 \times 15 \times 15$

$\Rightarrow \quad h_1 = 15$ cm

$\therefore \quad h = 60 - h_1 = 60 - 15 = 45$ cm

Hence, the height from the base at which the section is made is **45 cm.**

8. If the circumference of a circle is equal to the perimeter of a square then the ratio of their areas is

(*a*) 22 : 7 (*b*) 14 : 11

(*c*) 7 : 22 (*d*) 7 : 11

Solution. Choice (*b*) is correct.

Let '*r*' be the radius of a circle and '*a*' be the side of a square. Then,

Circumference of a circle = Perimeter of a square [given]

$$\Rightarrow \quad 2\pi r = 4a$$

$$\Rightarrow \quad \frac{r}{a} = \frac{2}{\pi} \quad \text{...(1)}$$

Now, $$\frac{\text{Area of a circle}}{\text{Area of a square}} = \frac{\pi r^2}{a^2}$$

$$= \pi\left(\frac{r}{a}\right)^2$$

$$= \pi \times \left(\frac{2}{\pi}\right)^2 \quad \text{[using (1)]}$$

$$= \pi \times \frac{4}{\pi^2}$$

$$= \frac{4}{\pi}$$

$$= \frac{4 \times 7}{22}$$

$$= \frac{14}{11}$$

Hence, the ratio of their areas is **14 : 11.**

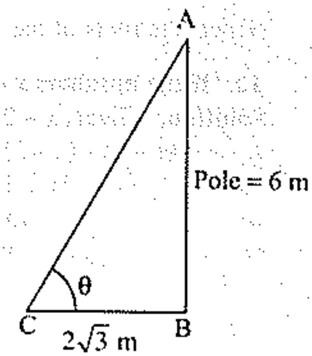
9. A pole 6 m high casts a shadow $2\sqrt{3}$ m long on the ground, then the sun's elevation is

(*a*) 60° (*b*) 45°

(*c*) 30° (*d*) 90°

Solution. Choice (*a*) is correct.

In figure, AB (= 6 m) is the pole and $BC = 2\sqrt{3}$ m is the shadow on the ground when the Sun's elevation is θ.



In right ΔABC, we have

$$\tan\theta = \frac{AB}{BC}$$

$$\Rightarrow \quad \tan\theta = \frac{6}{2\sqrt{3}}$$

$\Rightarrow \quad \tan\theta = \frac{3}{\sqrt{3}}$

$\Rightarrow \quad \tan\theta = \sqrt{3}$

$\Rightarrow \quad \tan\theta = \tan 60°$

$\Rightarrow \quad \theta = 60°.$

10. Which of the following cannot be the probability of an event ?

(*a*) $\frac{1}{5}$ (*b*) 0.3

(*c*) 4% (*d*) $\frac{5}{4}$

Solution. Choice (*d*) is correct.

The probability of an event lies between 0 and 1, *i.e.*,

$$0 \le P(A) \le 1$$

$\because \frac{5}{4} > 1 \Rightarrow \frac{5}{4}$ cannot be the probability of an event.

Section 'B'

Question numbers 11 to 18 carry 2 marks each.

11. Find the roots of the following quadratic equation :

$$\frac{2}{5}x^2 - x - \frac{3}{5} = 0$$

Solution. We have

$$\frac{2}{5}x^2 - x - \frac{3}{5} = 0$$

$\Rightarrow \quad 2x^2 - 5x - 3 = 0$

$\Rightarrow \quad 2x^2 - 6x + x - 3 = 0$

$\Rightarrow \quad 2x(x-3) + (x-3) = 0$

$\Rightarrow \quad (x-3)(2x+1) = 0$

$\Rightarrow$ Either $x - 3 = 0$ or $2x + 1 = 0$

$\Rightarrow$ Either $x = 3$ or $x = -\frac{1}{2}$

Hence, the roots of the given quadratic equation are **3** and $-\frac{1}{2}$.

12. If the numbers $x-2$, $4x-1$ and $5x+2$ are in A.P., find the value of x.

Solution. Given, $x-2$, $4x-1$ and $5x+2$ are in A.P.

$\therefore \quad (4x-1) - (x-2) = (5x+2) - (4x-1)$

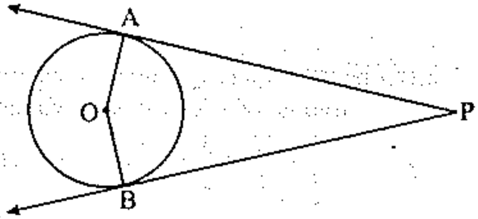
$\Rightarrow \quad 3x + 1 = x + 3$

$\Rightarrow \quad 2x = 2$

$\Rightarrow \quad x = 1$

13. **Two tangents PA and PB are drawn from an external point P to a circle with centre O. Prove that $AOBP$ is a cyclic quadrilateral.**

Solution. Since the tangent at a point to a circle is perpendicular to the radius through the point.



$\therefore$ $OA \perp AP$ and $OB \perp BP$

$\Rightarrow$ $\angle OAP = 90°$ and $\angle OBP = 90°$

$\therefore$ $\angle OAP + \angle OBP = 90° + 90° = 180°$ ...(1)

In a quadrilateral $OAPB$, we have

$\angle OAP + \angle APB + \angle OBP + \angle AOB = 360°$

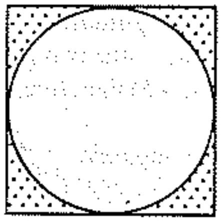
$\Rightarrow$ $(\angle APB + \angle AOB) + (\angle OAP + \angle OPB) = 360°$

$\Rightarrow$ $\angle APB + \angle AOB + 180° = 360°$ [using (1)]

$\Rightarrow$ $\angle APB + \angle AOB = 360° - 180° = 180°$ ...(2)

From (1) and (2), we conclude that the **quadrilateral $OABP$ is cyclic.**

14. **In figure, a circle of radius 7 cm is inscribed in a square. Find the area of the shaded region.** $\left[\text{Use } \pi = \frac{22}{7}\right]$



Solution.

Since a circle of radius 7 cm is inscribed in a square, therefore the diameter of a square is equal to the side of a square.

But the diameter of a circle = 2 × Radius of a circle

= 2 × 7 cm

= 14 cm

$\therefore$ Side of a square = 14 cm

Area of the shaded region = Area of a square – Area of a circle

$= (14 \times 14)\ \text{cm}^2 - \pi(7)^2\ \text{cm}^2$

$= 196\ \text{cm}^2 - \frac{22}{7} \times 7 \times 7\ \text{cm}^2$

$= 196\ \text{cm}^2 - 154\ \text{cm}^2$

$= 42\ \text{cm}^2$

15. **How many spherical lead shots each having diameter 3 cm can be made from a cuboidal lead solid of dimensions 9 cm × 11 cm × 12 cm ?**

Solution. We have

Volume of cuboidal lead solid of dimensions 9 cm × 11 cm × 12 cm

$= (9 \times 11 \times 12)\ \text{cm}^3$

Diameter of a spherical lead shot = 3 cm

$\therefore$ Radius of a spherical lead shot = 3 ÷ 2 = 1.5 cm

Volume of a spherical lead shot $= \frac{4}{3}\pi(1.5)^3$ cm^3

$$= \frac{4}{3} \times \frac{22}{7} \times \frac{3}{2} \times \frac{3}{2} \times \frac{3}{2} \text{ cm}^3$$

Let x be the required number of spherical lead shots, then

Volume of x lead shots = Volume of lead in cuboidal solid

$$\Rightarrow \quad x \times \frac{4}{3} \times \frac{22}{7} \times \frac{3}{2} \times \frac{3}{2} \times \frac{3}{2} = 9 \times 11 \times 12$$

$$\Rightarrow \quad x = \frac{9 \times 11 \times 12}{\frac{4}{3} \times \frac{22}{7} \times \frac{3}{2} \times \frac{3}{2} \times \frac{3}{2}}$$

$$\Rightarrow \quad x = \frac{9 \times 11 \times 12 \times 21 \times 8}{4 \times 22 \times 9 \times 3}$$

$$\Rightarrow \quad x = 21 \times 4 = 84$$

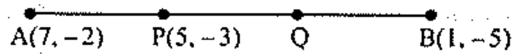
Hence, the number of spherical lead shots is **84.**

16. Point $P(5, -3)$ is one of the two points of trisection of the line segment joining the points $A(7, -2)$ and $B(1, -5)$ near to A. Find the coordinates of the other point of trisection.

Solution. Since the point $P(5, -3)$ is one of the two points of trisection (*viz.*, P and Q) of the line segment joining the points $A(7, -2)$ and $B(1, -5)$ therefore Q is the other point of trisection.

Thus, $AP = PQ = QB$

Clearly, Q is the mid-point of the line segment joining the points $P(5, -3)$ and $B(1, -5)$.



$\therefore$ Coordinates of Q are $\left(\frac{5+1}{2}, \frac{-3-5}{2}\right)$, *i.e.*, $(3, -4)$

Hence, the coordinates of the other point of trisection is **(3, – 4).**

17. Show that the point $P(-4, 2)$ lies on the line segment joining the points $A(-4, 6)$ and $B(-4, -6)$.

Solution. In order to show that the point $P(-4, 2)$ lies on the line segment joining the points $A(-4, 6)$ and $B(-4, -6)$, it is sufficient to show that the points $P(-4, 2)$, $A(-4, 6)$ and $B(-4, -6)$ are collinear.

If the point P, A and B are collinear, then the area of the ΔPAB must be zero.

$$\text{Area of } \Delta PAB = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

$$= \frac{1}{2}[(-4)(6+6) + (-4)(-6-2) + (-4)(2-6)]$$

$$= \frac{1}{2}[(-4)(12) + (-4)(-8) + (-4)(-4)]$$

$$= \frac{1}{2}[-48 + 32 + 16]$$

$$= \frac{1}{2}(0) = 0$$

Since the area of $\Delta PAB = 0$, therefore **the point $P(-4, 2)$ lies on the line segment joining the points $A(-4, 6)$ and $B(-4, -6)$.**

18. Two dice are thrown at the same time. Find the probability of getting different numbers on both dice.

Solution. When two dice are thrown at the same time, then the possible outcomes of the experiment are listed in the table.

	1	2	3	4	5	6
1	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
2	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
3	(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
4	(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
5	(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
6	(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

So, the number of possible outcomes $= 6 \times 6 = 36$

Let A be the event of getting the same number on both the dice, then the outcomes favourable to A are :

$$(1, 1)\ (2, 2)\ (3, 3)\ (4, 4)\ (5, 5)\ (6, 6)$$

$\therefore$ Favourable number of outcomes $= 6$

$\therefore$ P (different numbers on both dice) $= 1 - P$(same number on both dice)

$$= 1 - \frac{6}{36} = 1 - \frac{1}{6} = \frac{5}{6}$$

Or

A coin is tossed two times. Find the probability of getting atmost one head.

Solution. When a coin is tossed two times, we obtain any one of following as outcome

HH, HT, TH, TT

$\therefore$ Total number of outcomes $= 4$

Let A be the event of getting atmost one head, then the number of outcomes favourable to A are :

TT, HT, TH

$\therefore$ Favourable number of outcomes $= 3$

Hence, the required probability is $P(A) = \frac{3}{4}$.

Section 'C'

Question numbers 19 to 28 carry 3 marks each.

19. Find the roots of the equation $\frac{1}{2x-3} + \frac{1}{x-5} = 1, \quad x \neq \frac{3}{2}, 5.$

Solution. We have,

$$\frac{1}{2x-3} + \frac{1}{x-5} = 1, \quad x \neq \frac{3}{2}, 5$$

$$\Rightarrow \quad \frac{1}{2x-3} = 1 - \frac{1}{x-5}$$

$$\Rightarrow \quad \frac{1}{2x-3} = \frac{x-5-1}{x-5}$$

$$\Rightarrow \quad \frac{1}{2x-3} = \frac{x-6}{x-5}$$
$$\Rightarrow \quad x - 5 = (x-6)(2x-3)$$
$$\Rightarrow \quad x - 5 = 2x^2 - 12x - 3x + 18$$
$$\Rightarrow \quad 2x^2 - 16x + 23 = 0$$
$$\Rightarrow \quad x = \frac{16 \pm \sqrt{(-16)^2 - (4)(2)(23)}}{2(2)} \qquad \left[\because x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}\right]$$
$$\Rightarrow \quad x = \frac{16 \pm \sqrt{256-184}}{4}$$
$$\Rightarrow \quad x = \frac{16 \pm \sqrt{72}}{4}$$
$$\Rightarrow \quad x = \frac{16 \pm 6\sqrt{2}}{4}$$
$$\Rightarrow \quad x = \frac{16}{4} \pm \frac{6}{4}\sqrt{2}$$
$$\Rightarrow \quad x = 4 \pm \frac{3\sqrt{2}}{2}$$

Or

A natural number, when increased by 12, becomes equal to 160 times its reciprocal. Find the number.

Solution. Let x be the required natural number, then according to the given condition, we have

$$x + 12 = 160 \times \frac{1}{x}$$
$$\Rightarrow \quad x^2 + 12x - 160 = 0$$
$$\Rightarrow \quad x^2 + 20x - 8x - 160 = 0$$
$$\Rightarrow \quad (x^2 + 20x) - (8x + 160) = 0$$
$$\Rightarrow \quad x(x+20) - 8(x+20) = 0$$
$$\Rightarrow \quad (x+20)(x-8) = 0$$
$$\Rightarrow \text{Either} \quad x + 20 = 0 \quad \text{or} \quad x - 8 = 0$$
$$\Rightarrow \text{Either} \quad x = -20 \quad \text{or} \quad x = 8$$
$$\Rightarrow \quad x = 8 \qquad \text{[Natural number cannot be negative]}$$

Hence, the required natural number is **8**.

20. Find the sum of the integers between 100 and 200 that are divisible by 9.

Solution. The integers between 100 and 200 divisible by 9 are

108, 117, ..., 198

Clearly, it is an A.P. whose first term (a) = 108 and common difference (d) = 117 – 108 = 9.

Let the number of terms in the sequence 108, 117, ..., 198 be n, then,

$$t_n = 198$$
$$\Rightarrow \quad a + (n-1)d = 198$$

$\Rightarrow \quad 108 + (n-1)9 = 198$

$\Rightarrow \quad (n-1)9 = 198 - 108$

$\Rightarrow \quad (n-1)9 = 90$

$\Rightarrow \quad n - 1 = 90 \div 9$

$\Rightarrow \quad n - 1 = 10$

$\Rightarrow \quad n = 10 + 1 = 11$

∴ Required sum of the integers between 100 and 200 divisible by 9

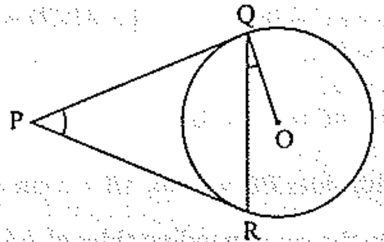
$$= \frac{11}{2}[108 + 198] \qquad \left[\because S_n = \frac{n}{2}(a+l)\right]$$

$$= \frac{11}{2} \times 306$$

$$= 11 \times 153$$

$$= 1683.$$

21. In figure, two tangents PQ and PR are drawn to a circle with centre O from an external point P. Prove that $\angle QPR = 2\angle OQR$.



Solution. Given : A circle with centre O, an external point P and two tangents PQ and PR to the circle, where Q and R are the points of contact (see figure).

To prove : $\angle QPR = 2\angle OQR$

Proof : Let $\angle QPR = \theta$

In ΔPQR, we have

$PQ = PR$ [Length of the tangents drawn from an external point to a circle are equal]

So, PQR is an isosceles triangle

$\therefore \quad \angle PQR = \angle PRQ \qquad ...(1)$

In ΔPQR, we have

$\angle PQR + \angle PRQ + \angle QPR = 180^\circ$ [∵ Sum of three ∠s of a Δ is 180°]

$\Rightarrow \quad 2\angle PQR + \theta = 180^\circ$ [using (1)]

$\Rightarrow \quad 2\angle PQR = 180^\circ - \theta$

$\Rightarrow \quad \angle PQR = \frac{1}{2}(180^\circ - \theta) = 90^\circ - \frac{1}{2}\theta \qquad ...(2)$

But $\quad \angle OQP = 90^\circ$ ...(3) [Angle between the tangent and radius of a circle is 90°]

Now, $\quad \angle OQR = \angle OQP - \angle PQR$

$$= 90^\circ - \left(90^\circ - \frac{1}{2}\theta\right) \qquad \text{[using (2) and (3)]}$$

$$= \frac{1}{2}\theta = \frac{1}{2}\angle QPR$$

$$\Rightarrow \quad \angle OQR = \frac{1}{2}\angle QPR$$

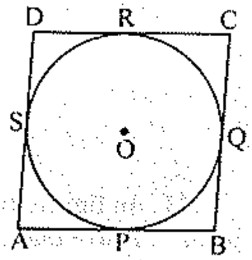
$$\Rightarrow \quad \angle QPR = 2\angle OQR$$

Or

Prove that the parallelogram circumscribing a circle is a rhombus.

Solution. Let $ABCD$ be a parallelogram such that all the sides of a parallelogram touch a circle (*i.e.*, a parallelogram circumscribing a circle) with centre O.

We know that tangents drawn from an external point to a circle are equal in length



$AP = AS$ ...(1) [Tangents from A]

$BP = BQ$ ...(2) [Tangents from B]

$CR = CQ$ ...(3) [Tangents from C]

and $DR = DS$ ...(4) [Tangents from D]

Adding the corresponding sides of (1), (2), (3) and (4), we get

$$AP + BP + CR + DR = AS + BQ + CQ + DS$$

$$\Rightarrow \quad (AP + BP) + (CR + DR) = (AS + DS) + (BQ + CQ)$$

$$\Rightarrow \quad AB + CD = AD + BC$$

$$\Rightarrow \quad AB + AB = BC + BC \qquad [\because ABCD \text{ is a llgm. } \therefore AB = CD \text{ and } BC = AD]$$

$$\Rightarrow \quad 2AB = 2BC$$

$$\Rightarrow \quad AB = BC$$

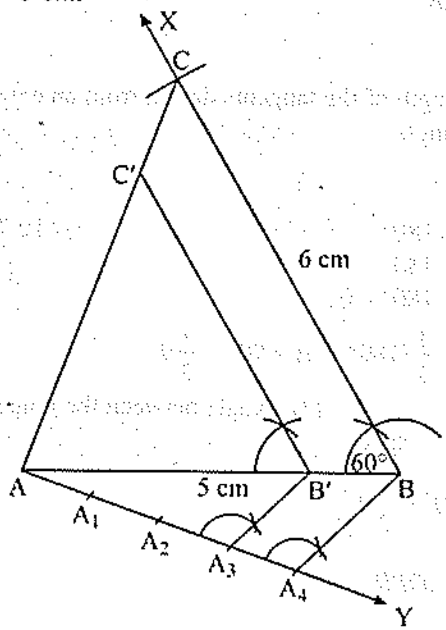
$$\Rightarrow \quad AB = BC = CD = AD$$

Thus, $ABCD$ is a **rhombus.**

22. Draw a triangle ABC with sides $BC = 6$ cm, $AB = 5$ cm and $\angle ABC = 60°$. Then construct a triangle whose sides are $\frac{3}{4}$ time the corresponding sides of ΔABC.

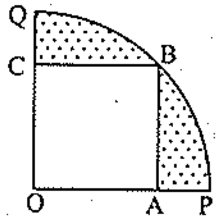
Solution. Steps of Construction :

1. Draw a line segment $AB = 5$ cm.



2. At B make $\angle ABX = 60°$.
3. With B as centre and radius equal to 6 cm draw an arc intersecting BX at C.
4. Join AC. Then ABC is the required triangle.
5. Draw any ray AY making an acute angle with AB on the opposite side of the vertex C.
6. Locate 4 points $\left(\text{the greater of 3 and 4 in } \frac{3}{4}\right)$ A_1, A_2, A_3 and A_4 on AY so that $AA_1 = A_1A_2 = A_2A_3 = A_3A_4$.
7. Join A_4B and draw a line through A_3 $\left(\text{the 3rd point, 3 being smaller of 3 and 4 in } \frac{3}{4}\right)$ parallel to A_4B intersecting AB at B'.
8. Draw a line through B' parallel to the line BC to intersect AC at C'.
   Then $AB'C'$ is the required triangle.

23. In figure, $OABC$ is a square inscribed in a quadrant $OPBQ$. If $OA = 20$ cm, find the area of shaded region. **[Use $\pi = 3.14$]**



Solution. Here, $OABC$ is a square inscribed in a quadrant $OPBQ$, where $OA = 20$ cm (given).

Diagonal of a square = OB

$$= \sqrt{OA^2 + AB^2} \quad \text{[Pythagoras theorem]}$$

$$= \sqrt{(20)^2 + (20)^2} \text{ cm} \quad [\because OA = AB = BC = OC = 20 \text{ cm}]$$

$$= \sqrt{400 + 400} \text{ cm}$$

$$= \sqrt{800} \text{ cm}$$

$$= 20\sqrt{2} \text{ cm}$$

Area of the shaded region

= Area of a quadrant of a circle of radius $OB = 20\sqrt{2}$ cm − Area of a square with side 20 cm

$$= \frac{90°}{360°}\pi r^2 - (\text{Side of square})^2, \text{ where } r = OB = 20\sqrt{2} \text{ cm}$$

$$= \left[\frac{1}{4} \times 3.14 \times (20\sqrt{2})^2 - (20)^2\right] \text{cm}^2 \quad [\because \pi = 3.14]$$

$$= \left[\frac{1}{4} \times 3.14 \times 800 - 400\right] \text{cm}^2$$

$$= [3.14 \times 200 - 400] \text{ cm}^2$$

$$= (628 - 400) \text{ cm}^2$$

$$= \mathbf{228 \text{ cm}^2}$$

24. A hemispherical depression is cut out from one face of a cubical wooden block such that the diameter 'l' of the hemisphere is equal to the edge of the cube. Determine the surface area of the remaining solid.

Solution. We have

Diameter of the hemisphere = l = Edge of a cube (given)

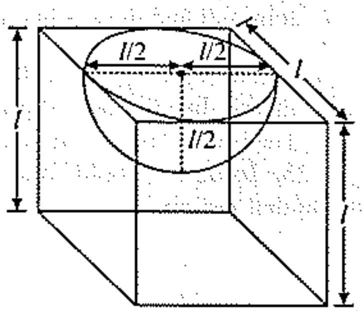
So, radius of the hemisphere = $l/2$

Surface area of the remaining solid

= Surface area of the cuboidal box whose each edge is of length 'l'

– Area of the top of the hemisphere part of radius $\frac{l}{2}$

+ Curved surface area of the hemispherical part



$$= \left[6(\text{edge of a cube})^2 - \pi\left(\frac{l}{2}\right)^2 + 2\pi\left(\frac{l}{2}\right)^2\right] \text{ sq. units}$$

$$= \left[6l^2 - \frac{\pi l^2}{4} + \frac{\pi l^2}{2}\right] \text{ sq. units}$$

$$= \left[\frac{24l^2 - \pi l^2 + 2\pi l^2}{4}\right] \text{ sq. units}$$

$$= \left[\frac{l^2}{4}(24 - \pi + 2\pi)\right] \text{ sq. units}$$

$$= \frac{l^2}{4}(24 + \pi) \text{ sq. units}$$

Or

A copper rod of diameter 1 cm and length 8 cm is drawn into a wire of length 18 m of uniform thickness. Find the thickness of the wire.

Solution. We have

Diameter of a copper rod = 1 cm

∴ Radius of a copper rod (r) = $\frac{1}{2}$ cm

Length of a copper rod (h) = 8 cm

∴ Volume of a copper rod = $\pi r^2 h$

$$= \pi\left(\frac{1}{2}\right)^2 \times 8 \text{ cm}^3$$

$$= 2\pi \text{ cm}^3 \quad \ldots(1)$$

Length of the wire (H) = 18 m = 1800 cm

Let the thickness of the wire be x cm, then

$$\text{Radius of the wire} = \frac{\text{Thickness}}{2} = \frac{x}{2} \text{ cm}$$

$$\text{Volume of a wire} = \pi \times \left(\frac{x}{2}\right)^2 \times 1800 \text{ cm}^3 \quad \ldots(2)$$

∴ Volume of a copper rod = Volume of a wire

$$\Rightarrow \quad 2\pi = \pi \times \frac{x^2}{4} \times 1800 \quad \text{[using (1) and (2)]}$$

$$\Rightarrow \quad 2 = x^2 \times 450$$

$$\Rightarrow \quad x^2 = \frac{2}{450} = \frac{1}{225}$$

$$\Rightarrow \quad x^2 = \frac{1}{(15)^2}$$

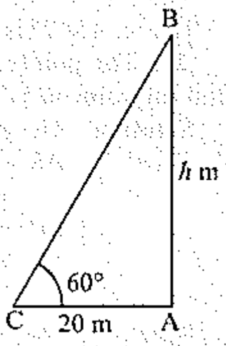
$$\Rightarrow \quad x = \frac{1}{15} \text{ cm}$$

Hence, the thickness of the wire = $\mathbf{\frac{1}{15}}$ **cm.**

25. A tower stands vertically on the ground. From a point on the ground which is 20 m away from the foot of the tower, the angle of elevation of the top of the tower is found to be 60°. Find the height of the tower.

Solution. Let AB be the tower of height 'h' m and let C be a point at a distance of 20 m away from the foot of the tower A.

It is given that the angle of elevation of the top of the tower B is 60° such that $\angle BCA = 60°$.



In right triangle BAC, we have

$$\tan 60° = \frac{BA}{CA}$$

$$\Rightarrow \quad \sqrt{3} = \frac{h}{20}$$

$$\Rightarrow \quad h = 20\sqrt{3} \text{ m}$$

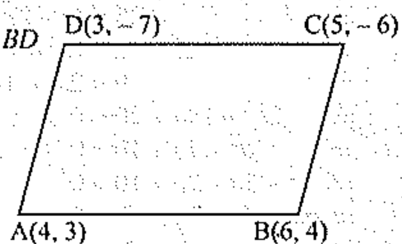
Hence, the height of the tower is $\mathbf{20\sqrt{3}}$ **m.**

26. Prove that the points $A(4, 3)$, $B(6, 4)$, $C(5, -6)$ and $D(3, -7)$ in that order are the vertices of a parallelogram.

Solution. In order to prove that the given four points in that order are the vertices of a parallelogram it is sufficient to show that

(*i*) opposite sides are equal and

(*ii*) mid-point of the diagonal AC = mid-point of the diagonal BD



The given points are $A(4, 3)$, $B(6, 4)$, $C(5, -6)$ and $D(3, -7)$.

$$\therefore \quad AB = \sqrt{(6-4)^2 + (4-3)^2}$$

$$= \sqrt{4+1}$$

$$= \sqrt{5}$$

$$DC = \sqrt{(5-3)^2 + (-6+7)^2}$$
$$= \sqrt{4+1}$$
$$= \sqrt{5}$$
$$BC = \sqrt{(5-6)^2 + (-6-4)^2}$$
$$= \sqrt{1+100}$$
$$= \sqrt{101}$$
$$AD = \sqrt{(3-4)^2 + (-7-3)^2}$$
$$= \sqrt{1+100}$$
$$= \sqrt{101}$$

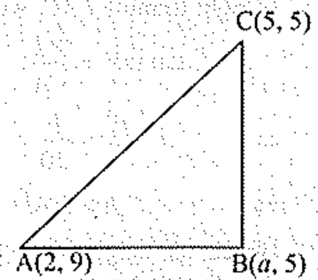
Also, coordinates of the mid-point of AC are $\left(\frac{4+5}{2}, \frac{3-6}{2}\right)$, *i.e.*, $\left(\frac{9}{2}, -\frac{3}{2}\right)$ and coordinates of the mid-point of BD are $\left(\frac{6+3}{2}, \frac{4-7}{2}\right)$ *i.e.*, $\left(\frac{9}{2}, -\frac{3}{2}\right)$.

Thus, $AB = DC$, $BC = AD$ and the diagonal AC and BD have the same point.

Hence, ***ABCD* is a parallelogram.**

27. The points $A(2, 9)$, $B(a, 5)$, $C(5, 5)$ are the vertices of a triangle *ABC* right angled at *B*. Find the value of '*a*' and hence the area of ΔABC.

Solution. The given points are $A(2, 9)$, $B(a, 5)$ and $C(5, 5)$.



Then, $AB^2 = (a-2)^2 + (5-9)^2$
$= (a-2)^2 + 16$
$BC^2 = (a-5)^2 + (5-5)^2$
$= (a-5)^2 + 0$
$= (a-5)^2$
and $AC^2 = (2-5)^2 + (9-5)^2$
$= 9 + 16$
$= \mathbf{25}$

It is given that the points $A(2, 9)$, $B(a, 5)$ and $C(5, 5)$ are the vertices of a triangle ABC right-angled at B.

$\therefore \quad AC^2 = AB^2 + BC^2$

$\Rightarrow \quad 25 = (a-2)^2 + 16 + (a-5)^2$

$\Rightarrow \quad 25 - 16 = (a^2 - 4a + 4) + (a^2 - 10a + 25)$

$\Rightarrow \quad 9 = 2a^2 - 14a + 29$

$\Rightarrow \quad 2a^2 - 14a + 20 = 0$

$\Rightarrow \quad a^2 - 7a + 10 = 0$

$\Rightarrow \quad a^2 - 5a - 2a + 10 = 0$

$\Rightarrow \quad a(a-5) - 2(a-5) = 0$

$\Rightarrow \quad (a-5)(a-2) = 0$

$\Rightarrow$ Either $a-5=0$ or $a-2=0$

$\Rightarrow$ Either $a=5$ or $a=2$

$\Rightarrow$ $a=2$, as a cannot be 5, *i.e.*, $a \neq 5$.

Further, area of $\Delta ABC = \frac{1}{2} AB \times BC$

$$= \frac{1}{2}\sqrt{(a-2)^2+16} \times \sqrt{(a-5)^2}$$

$$= \frac{1}{2}\sqrt{(2-2)^2+16} \times \sqrt{(2-5)^2}$$

$$= \frac{1}{2}\sqrt{0+16} \times \sqrt{9}$$

$$= \frac{1}{2} \times 4 \times 3 \text{ sq. units}$$

$$= \mathbf{6 \text{ sq. units}}$$

28. Cards with numbers 2 to 101 are placed in a box. A card is selected at random from the box. Find the probability that the card which is selected has a number which is a perfect square.

Solution. Total number of cards in a box, with numbers 2 to 101 are 100 (*i.e.*, $101-1=100$).

$\therefore$ Total number of outcomes in which one card can be selected are 100.

Let A be the event that the number on the selected card is "a number which is a perfect square."

There are 9 perfect square numbers from 2 to 101, namely

$4\ (=2^2)$, $9\ (=3^2)$, $16\ (=4^2)$, $25\ (=5^2)$, $36\ (=6^2)$, $49\ (=7^2)$, $64\ (=8^2)$, $81\ (=9^2)$, $100\ (=10^2)$

$\therefore$ Number of outcomes favourable to event $A = 9$

Hence, the required probability

$$= P(A) = \mathbf{\frac{9}{100}}.$$

Section 'D'

Question numbers 29 to 34 carry 4 marks each.

29. A train travels at a certain average speed for a distance of 63 km and then travels a distance of 72 km at an average speed of 6 km/h more than its original speed. If it takes 3 hours to complete the total journey, what is its original average speed ?

Solution. Let the original average speed of the train be x km/h, then time taken by the train to cover a journey of 63 km $= \frac{63}{x}$ h.

When the speed of the train is increased by 6 km/h, then the time taken by the train to cover a journey of 72 km with speed $(x+6)$ km/h $= \frac{72}{x+6}$ h.

It is given that the train takes 3 hours to complete the total journey.

$$\therefore \quad \frac{63}{x} + \frac{72}{x+6} = 3$$

$\Rightarrow \quad \dfrac{21}{x} + \dfrac{24}{x+6} = 1$ [Dividing both sides by 3]

$\Rightarrow \quad \dfrac{21(x+6)+24x}{x(x+6)} = 1$

$\Rightarrow \quad 21x + 126 + 24x = x^2 + 6x$

$\Rightarrow \quad 45x + 126 = x^2 + 6x$

$\Rightarrow \quad x^2 - 39x - 126 = 0$

$\Rightarrow \quad x^2 - 42x + 3x - 126 = 0$

$\Rightarrow \quad x(x-42) + 3(x-42) = 0$

$\Rightarrow \quad (x-42)(x+3) = 0$

$\Rightarrow$ Either $x - 42 = 0$ or $x + 3 = 0$

$\Rightarrow$ Either $x = 42$ or $x = -3$

$\Rightarrow \quad x = 42$, as x cannot be negative.

Hence, the original average speed of the train is **42 km/h.**

Or

Find two consecutive odd positive integers, sum of whose squares is 290.

Solution. Let the two consecutive odd positive integers be $2x - 1$ and $2x + 1$, then according to the given condition, we have

$$(2x-1)^2 + (2x+1)^2 = 290$$

$\Rightarrow (4x^2 - 4x + 1) + (4x^2 + 4x + 1) = 290$

$\Rightarrow \quad 8x^2 + 2 = 290$

$\Rightarrow \quad 4x^2 + 1 = 145$

$\Rightarrow \quad 4x^2 = 144$

$\Rightarrow \quad x^2 = 36$

$\Rightarrow \quad x = \pm 6.$

When $x = 6$, then the two consecutive odd positive integers are :

$$2 \times 6 - 1 \text{ and } 2 \times 6 + 1, \textit{ i.e.}, 11 \text{ and } 13$$

Reject $x = -6$ as this gives two consecutive odd negative integers.

Hence, the two consecutive odd positive integers are **11 and 13.**

30. A sum of ₹ 1400 is to be used to give seven cash prizes to students of a school for their overall academic performance. If each prize is ₹ 40 less than the preceding price, find the value of each of the prizes.

Solution. Number of cash prizes (n) = 7 (given)

Total sum of 7 prizes = ₹ 1400

Let the first prize given to a student of a school for his overall academic performance be ₹ a.

Then, according to the given condition each prize is ₹ 40 less than the preceding price, *i.e.*,

$$a - 40, a - 80, a - 120.....,$$

Thus, the prizes form a sequence as

$$a, a - 40, a - 80, a - 120, a - 160, a - 200, a - 240$$

First term = a, Common different (d) = $a - 40 - a = -40$

$\therefore$ Sum of 7 terms of an A.P. = 1400 (given)

$\Rightarrow \quad \dfrac{7}{2}[2a + (7-1)(-40)] = 1400$

$\Rightarrow \quad \frac{7}{2}[2a + 6(-40)] = 1400$

$\Rightarrow \quad 7(a - 120) = 1400$

$\Rightarrow \quad a - 120 = 1400 \div 7$

$\Rightarrow \quad a - 120 = 200$

$\Rightarrow \quad a = 200 + 120$

$\Rightarrow \quad a = 320$

Hence, the value of each prize in ₹ are **320, 280, 240, 200, 160, 120** and **80.**

31. Prove that the lengths of the tangents drawn from an external point to a circle are equal.

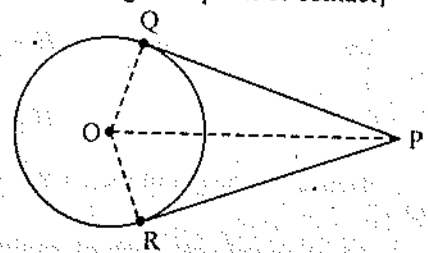
Solution. Given : PQ and PR are two tangents, from an external point P to a circle.

To prove : $PQ = PR$.

Construction : Join OP, OQ and OR.

Proof : $\angle OQP$ and $\angle ORP$ are right angles, because these are angles between the radii and tangents. [The tangent at any point of a circle is perpendicular to the radius through the point of contact]

Now in right triangles OQP and ORP, we have



$OQ = OR$ [Radii of the same circle]

$\angle OQP = \angle ORP$ [$OQ \perp PQ$ and $OR \perp PR$]

$OP = OP$ [Common]

By RHS theorem of congruence,

$\Delta OQP \cong \Delta QRP$

$\Rightarrow \quad PQ = PR$ [CPCT]

32. A well of diameter 3 m and 14 m deep is dug. The earth, taken out of it has been evenly spread all around it in the shape of a circular ring of width 4 m to form an embankment. Find the height of the embankment.

Solution. Here, Diameter of a well = 3 m

$\therefore$ Radius of the well $(r) = \frac{3}{2}$ m

Depth of the well $(h) = 14$ m

Volume of earth taken out from the well $= \pi r^2 h$

$$= \left[\pi \times \left(\frac{3}{2}\right)^2 \times 14\right] \text{m}^3 \quad \ldots(1)$$

Inner radius of the embankment $(r_1) = \frac{3}{2}$ m

Width of the circular ring to form an embankment = 4 m

Outer radius of the embankment $(r_2) = \left(\frac{3}{2} + 4\right) \text{m} = \frac{11}{2}$ m

Let H m be the height of the embankment.

Volume of the embankment $= \pi(r_2^2 - r_1^2) \times H$

$$= \pi\left[\left(\frac{11}{2}\right)^2 - \left(\frac{3}{2}\right)^2\right] \times H \text{ m}^3$$

$$= \pi\left[\left(\frac{11}{2}+\frac{3}{2}\right)\left(\frac{11}{2}-\frac{3}{2}\right)\right] \times H \text{ m}^3$$

$$= \pi\,[7 \times 4] \times H \text{ m}^3$$

$$= 28\pi H \text{ m}^3 \qquad \ldots(2)$$

$\therefore$ $$\pi \times \left(\frac{3}{2}\right)^2 \times 14 = 28\pi H$$

$\Rightarrow$ $$H = \frac{\left(\frac{3}{2}\right)^2 \times 14}{28}$$

$\Rightarrow$ $$H = \frac{9}{4} \times \frac{1}{2}$$

$$H = \frac{9}{8} = 1.125 \text{ m}$$

Hence, the height of the embankment is **1.125 m.**

Or

21 glass spheres each of radius 2 cm are packed in a cuboidal box of internal dimensions 16 cm × 8 cm × 8 cm and then the box is filled with water. Find the volume of water filled in the box.

Solution. Radius of a glass sphere $(r) = 2$ cm

$\therefore$ $$\text{Volume of a glass sphere} = \frac{4}{3}\pi r^3$$

$$= \frac{4}{3} \times \frac{22}{7} \times (2)^3 \text{ cm}^3$$

$$= \frac{88}{21} \times 8 \text{ cm}^3$$

$$= \frac{704}{21} \text{ cm}^3$$

$\Rightarrow$ $$\text{Volume of 21 glass spheres} = 21 \times \frac{704}{21} \text{ cm}^3$$

$$= 704 \text{ cm}^3$$

Volume of the cuboidal box of internal dimensions 16 cm × 8 cm × 8 cm

$$= (16 \times 8 \times 8) \text{ cm}^3$$

$$= 1024 \text{ cm}^3$$

∴ Volume of water filled in the cuboidal box

$= \text{Volume of a cuboidal box of internal dimensions } 16\text{ cm} \times 8\text{ cm} \times 8\text{ cm}$
$- \text{Volume of 21 glass spheres}$
$= 1024\text{ cm}^3 - 704\text{ cm}^3$
$= \mathbf{320\text{ cm}^3}$

33. The slant height of the frustum of a cone is 4 cm and the circumferences of its circular ends are 18 cm and 6 cm. Find curved surface area of the frustum.

Solution. Let 'r_1' and 'r_2' be the radii of the circular ends of the frustum, 'l' be the slant height and 'h' be the height of the frustum.

Then, $l = 4$ cm

Circumference of the bigger circular end of a frustum is $2\pi r_1$.

$\therefore \quad 2\pi r_1 = 18$ (given)

$\Rightarrow \quad r_1 = \dfrac{9}{\pi}$ cm

Circumference of the smaller circular end of a frustum is $2\pi r_2$.

$\therefore \quad 2\pi r_2 = 6$

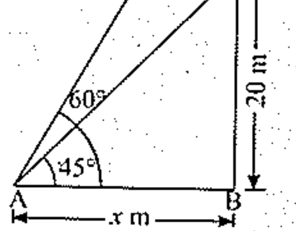
$\Rightarrow \quad r_2 = \dfrac{3}{\pi}$ cm

∴ Curved surface area of the frustum

$$= \pi(r_1 + r_2) \times l$$
$$= \pi \times \left(\frac{9}{\pi} + \frac{3}{\pi}\right) \times 4\text{ cm}^2$$
$$= \pi \times \frac{12}{\pi} \times 4\text{ cm}^2$$
$$= \mathbf{48\text{ cm}^2}.$$

34. From a point on the ground, the angles of elevation of the bottom and top of a transmission tower fixed at the top of a 20 m high building are 45° and 60° respectively. Find height of the tower.

Solution. Let BC be the building of height 20 metres and CD be the tower of height h metres. Let A be the point on the ground at a distance of x m from the foot of the building.



In ΔACB, we have

$$\tan 45° = \frac{BC}{AB}$$

$\Rightarrow \quad 1 = \dfrac{20}{x}$

$\Rightarrow \quad x = 20$ metres ...(1)

In ΔADB, we have

$$\tan 60° = \frac{BD}{AB}$$

$\Rightarrow \quad \sqrt{3} = \dfrac{h + 20}{20}$ [using (1)]

$\Rightarrow \quad h + 20 = 20\sqrt{3}$

$\Rightarrow \quad h = 20\sqrt{3} - 20$

$\Rightarrow \quad h = 20(\sqrt{3} - 1)$ metres

$\Rightarrow \quad h = 20[1.732 - 1]$ metres

$\Rightarrow \quad h = 20 \times 0.732$ metres

$\Rightarrow \quad h = 14.64$ metres

Hence, the height of the tower is **14.64 metres.**