CCE SAMPLE QUESTION PAPER

SECOND TERM (SA-II)

MATHEMATICS

(With Solutions)

CLASS X

Time Allowed : 3 Hours] *[Maximum Marks : 80*

General Instructions :

(i) *All questions are compulsory.*

(ii) *The question paper consists of 34 questions divided into four sections A, B, C and D. Section A comprises of 10 questions of 1 mark each, Section B comprises of 8 questions of 2 marks each, Section C comprises of 10 questions of 3 marks each and Section D comprises of 6 questions of 4 marks each.*

(iii) *Question numbers 1 to 10 in Section A are multiple choice questions where you are to select one correct option out of the given four.*

(iv) *There is no overall choice. However, internal choice has been provided in 1 question of two marks, 3 questions of three marks each and 2 questions of four marks each. You have to attempt only one of the alternatives in all such questions.*

(v) *Use of calculators is not permitted.*

Section 'A'

Question numbers 1 to 10 are of one mark each.

1. If α and β are the roots of the equation $x^2 + 3x + 4 = 0$, then the equation whose roots are $\alpha + 1$ and $\beta + 1$ is

(a) $x^2 + x + 1 = 0$ (b) $x^2 + x + 2 = 0$

(c) $x^2 + 3x + 1 = 0$ (d) $x^2 + 3x + 4 = 0$

Solution. Choice (b) is correct.

Since α and β are the roots of the equation $x^2 + 3x + 4 = 0$, therefore,

$$\alpha + \beta = -\frac{(3)}{1} = -3$$

$$\alpha\beta = \frac{4}{1} = 4$$

Let S and P denote respectively the sum and product of the roots $(\alpha + 1)$ and $(\beta + 1)$ of the required equation, then,

$$S = (\alpha + 1) + (\beta + 1) = \alpha + \beta + 2$$

$\Rightarrow$ $S = -3 + 2 = -1$

and $P = (\alpha + 1)(\beta + 1) = \alpha\beta + (\alpha + \beta) + 1$

$\Rightarrow$ $P = 4 + (-3) + 1 = 2$

Hence, the required equation is

$$x^2 - Sx + P = 0$$

$\Rightarrow \quad x^2 - (-1)x + 2 = 0$

$\Rightarrow \quad x^2 + x + 2 = 0$

2. The 10th term of the sequence $\sqrt{2}, \sqrt{8}, \sqrt{18}, \ldots\ldots$ is

(*a*) $\sqrt{162}$ (*b*) $\sqrt{200}$

(*c*) $\sqrt{242}$ (*d*) $\sqrt{288}$

Solution. Choice (*b*) is correct.

Here, $a_1 = \sqrt{2}$, $a_2 = \sqrt{8} = 2\sqrt{2}$, $a_3 = \sqrt{18} = \sqrt{9 \times 2} = 3\sqrt{2}, \ldots\ldots$

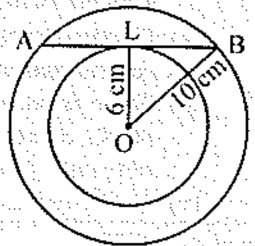
$\therefore \quad a_{10} = 10\sqrt{2} = \sqrt{100 \times 2} = \sqrt{200}$

3. If radii of the two concentric circles are 6 cm and 10 cm, then the length of each chord of one circle which is tangent to other is

(*a*) 8 cm (*b*) 16 cm

(*c*) 20 cm (*d*) 10 cm

Solution. Choice (*b*) is correct.



Let O be the centre of two concentric circles and AB be a chord of the larger circle touching the smaller circle at L.

Join OL.

Since OL (= 6 cm) is the radius of the smaller circle and AB is a tangent to this circle at a point L.

$\therefore \quad OL \perp AB$

We know that the perpendicular drawn from the centre of a circle to any chord of the circle bisects the chord at L.

$\Rightarrow \quad AL = BL$

In right ΔOLB, we have

$LB^2 = OB^2 - OL^2$

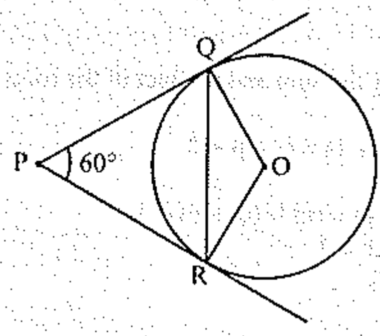
$\Rightarrow \quad LB^2 = (10)^2 - (6)^2$

$\Rightarrow \quad LB^2 = 100 - 36 = 64 = (8)^2$

$\Rightarrow \quad LB = 8$ cm

Length of chord $AB = AL + LB = 8 + 8 =$ **16 cm** $\quad [\because AL = BL = 8 \text{ cm}]$

4. In figure, PQ and PR are tangents to the circle with centre O such that $\angle QPR = 60°$, then $\angle OQR$ is equal to



(*a*) 20° (*b*) 30°

(*c*) 40° (*d*) 50°

Solution. Choice (*b*) is correct.

Since the angle between two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line segments joining the points of contact at the centre, *i.e.*,

$$\angle QPR + \angle QOR = 180°$$

But $\angle QPR = 60°$ (given)

$\therefore \quad 60° + \angle QOR = 180°$

$\Rightarrow \quad \angle QOR = 180° - 60° = 120° \quad ...(1)$

In ΔOQR, we have

$OQ = OR$ [Each = radius]

$\Rightarrow \quad \angle ORQ = \angle OQR \quad ...(2)$ [$\angle s$ opposite to equal sides of a Δ are equal]

In ΔOQR, we have

$\angle OQR + \angle ORQ + \angle QOR = 180°$ [Sum of three $\angle s$ of a $\Delta = 180°$]

$\Rightarrow \quad 2\angle OQR + 120° = 180°$ [using (1) and (2)]

$\Rightarrow \quad 2\angle OQR = 180° - 120°$

$\Rightarrow \quad \angle OQR = 60° \div 2 = $ **30°.**

5. Two tangents making an angle of 60° with each other, are drawn to a circle of radius 3 cm, then the length of each tangent is equal to

(*a*) $4\sqrt{3}$ cm (*b*) $6\sqrt{3}$ cm

(*c*) $3\sqrt{3}$ cm (*d*) $2\sqrt{3}$ cm

Solution. Choice (*c*) is correct.

In Δ's *PAO* and *PBO*, we have

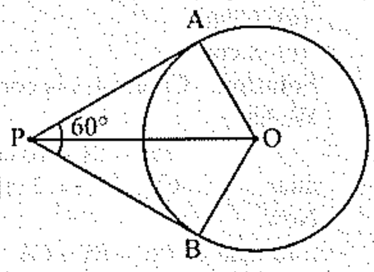
$PA = PB$ [Tangents to a circle from an external point *P*]

$OP = OP$ [Common]

$\angle OAP = \angle OBP$ [Each = 90°]

So, by RHS congruence rule, we have

$\Delta PAO \cong \Delta PBO$



$\Rightarrow \quad \angle OPA = \angle OPB = \frac{1}{2}\angle APB$

$\Rightarrow \quad \angle OPA = \frac{1}{2} \times 60° = 30°$ [$\because \angle APB = 60°$ (given)]

In right-angled triangle *OAP*,

$$\tan OPA = \frac{OA}{AP}$$

$\Rightarrow \quad \tan 30° = \frac{3}{AP}$ [$OA = 6$ cm = radius (given)]

$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{3}{AP}$

$\Rightarrow \quad AP = 3\sqrt{3}$ cm

$\Rightarrow$ Length of the each tangent = $\mathbf{3\sqrt{3}}$ **cm.**

6. Cards each marked with one of the numbers 4, 5, 6, ..., 20 are placed in a box and mixed thoroughly. One card is drawn at random from the box. The probability of getting an even prime number is

(*a*) 0 (*b*) $\frac{1}{2}$

(*c*) $\frac{3}{4}$ (*d*) $\frac{2}{5}$

Solution. Choice (*a*) is correct.

Out of 17 (= 20 − 3) cards, one card can be drawn in 17 ways.

∴ Total number of possible outcomes = 17

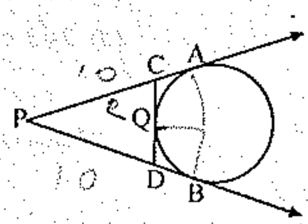
There are 6 prime numbers, namely, 5, 7, 11, 13, 17, 19

Here, there is no even prime number.

∴ Favourable number of outcomes out of 17 = 0

Hence, P(Getting an even prime number) = $\frac{0}{17} = \mathbf{0}$.

7. In the figure given below, *PA* and *PB* are tangents to the circle drawn from an external point *P*. *CQD* is another tangent touching the circle at *Q*. If *PB* = 10 cm, and *CQ* = 2 cm, then the length of *PC* is



(*a*) 6 cm (*b*) 7 cm

(*c*) 8 cm (*d*) 5 cm

Solution. Choice (*c*) is correct.

We have

$PB = PA = 10$ cm

and $CQ = CA = 2$ cm [∵ Tangent from external point to a circle are equal in length]

Now,

Length of $PC = PA - CA$

$= 10 - 2$

$= \mathbf{8}$ **cm.**

8. A cylinder and a cone are of same base radius and of same height. The ratio of the volume of cylinder to that of the cone is :

(*a*) 2 : 1 (*b*) 3 : 1

(*c*) 1 : 2 (*d*) 1 : 3

Solution. Choice (*b*) is correct.

It is given that : a cylinder and a cone are of same base radius (r) and of same height (h).

$$\therefore \quad \frac{V_1}{V_2} = \frac{\text{Volume of the cylinder}}{\text{Volume of the cone}} = \frac{\pi r^2 h}{\frac{1}{3}\pi r^2 h} = \frac{3}{1}$$

Thus, the ratio of the volume of cylinder to that of the cone is **3 : 1.**

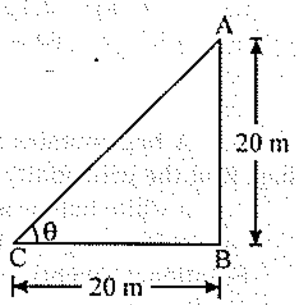
9. The angle of elevation of the top of a 20 m high tower at a point 20 m away from the base of the tower is

(*a*) 30° (*b*) 45°

(*c*) 60° (*d*) 75°

Solution. Choice (*b*) is correct.

In figure, AB is the tower and C be a point on the ground 20 m away from the base of the tower such that $BC = 20$ m.



In ΔABC, we have

$$\tan\theta = \frac{AB}{BC} = \frac{20\text{ m}}{20\text{ m}}$$

$$\Rightarrow \quad \tan\theta = 1 \Rightarrow \theta = 45°$$

Hence the angle of elevation is **45°.**

10. The difference between the circumference and diameter of a circle is 30 cm. The area of the circle is

(*a*) 115 cm^2 (*b*) 135 cm^2

(*c*) 154 cm^2 (*d*) 217 cm^2

Solution. Choice (*c*) is correct.

Let r be the radius of a circle, then according to given information, we have

$$\text{Circumference} - \text{Diameter} = 30 \text{ cm}$$

$$\Rightarrow \quad 2\pi r - 2r = 30$$

$$\Rightarrow \quad 2r(\pi - 1) = 30$$

$$\Rightarrow \quad 2r\left(\frac{22}{7} - 1\right) = 30$$

$$\Rightarrow \quad 2r \times \frac{15}{7} = 30$$

$$\Rightarrow \quad r = \frac{7 \times 30}{2 \times 15}$$

$$\Rightarrow \quad r = 7 \text{ cm}$$

Thus, area of a circle of radius 7 cm $= \frac{22}{7} \times (7)^2$

$= (22 \times 7)\text{ cm}^2 = \mathbf{154\ cm^2}$.

Section 'B'

Question numbers 11 to 18 carry 2 marks each.

11. If 2 is a root of the equation $x^2 + kx + 12 = 0$ and the equation $x^2 + kx + q = 0$ has equal roots, find the value of q.

Solution. Since 2 is a root of the equation $x^2 + kx + 12 = 0$, therefore

$$(2)^2 + k(2) + 12 = 0$$

$$\Rightarrow \quad 4 + 2k + 12 = 0$$

$$\Rightarrow \quad 2k + 16 = 0$$

$$\Rightarrow \quad k = -16 \div 2$$

$$\Rightarrow \quad k = -8$$

Putting $k = -8$ in the equation $x^2 + kx + q = 0$, we get

$$x^2 - 8x + q = 0 \quad \ldots(1)$$

The equation (1) will have equal roots, if Discriminant = 0

$\Rightarrow \quad (-8)^2 - 4(1)(q) = 0$

$\Rightarrow \quad 64 - 4q = 0$

$\Rightarrow \quad q = 64 \div 4$

$\Rightarrow \quad q = 16$

12. A bag contains 5 red, 8 green and 7 white balls. One ball is drawn at random from the bag, find the probability of getting

(*i*) a white ball or a green ball

(*ii*) neither a green ball nor a red ball.

Solution. Number of red balls in the bag = 5

Number of green balls in the bag = 8

Number of white balls in the bag = 7

Then, total number of balls in the bag = 5 + 8 + 7 = 20

(*i*) Number of white or green balls in the bag

= Number of white balls in the bag + Number of green balls in the bag

= 7 + 8 = 15

∴ Probability of getting a white ball or a green ball

$$= \frac{15}{20} = \mathbf{\frac{3}{4}}$$

(*ii*) Number of green or red balls in the bag

= Number of green balls in the bag + Number of red balls in the bag

= 8 + 5 = 13

Number of neither green nor red balls

= Total number of balls – Number of green or red balls

= 20 – 13 = 7

∴ Probability of getting neither a green ball nor a red ball

$$= \mathbf{\frac{7}{20}}.$$

Or

One card is drawn from a well-shuffled deck of 52 playing cards. Find the probability of getting

(*i*) a non-face card

(*ii*) a black king or a red queen.

Solution. (*i*) Number of face cards in a deck of 52 playing cards

= 4 kings + 4 queens + 4 jacks

= 12

Number of non-face cards in a deck of 52 playing cards

= Total number of cards – Number of face cards

= 52 – 12

= 40

∴ Probability of getting a non-face card

$$= \frac{40}{52} = \mathbf{\frac{10}{13}}$$

(*ii*) As there are 2 black kings and 2 red queens in a deck of 52 playing cards.

∴ Number of a black king or a red queen

$= 2$ black kings + 2 red queens

$= 4$

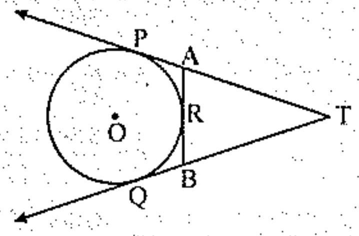
∴ Probability of getting a black king or a red queen

$$= \frac{4}{52} = \frac{1}{13}.$$

13. In figure, *TP* and *TQ* are tangents from *T* to the circle with centre *O* and *R* is any point on the circle. If *AB* is a tangent to the circle at *R*, prove that :

$$TA + AR = TB + BR.$$

Solution. Since tangents from an external point to a circle are equal in length.



∴ $TP = TQ$ ...(1) [Tangents from T]

$AP = AR$ ...(2) [Tangents from A]

$BQ = BR$ ...(3) [Tangents from B]

Now, $TP = TQ$ [using (1)]

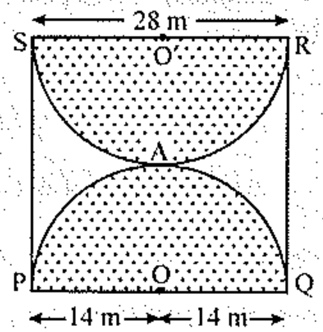
⇒ $TA + AP = TB + BQ$

⇒ $TA + AR = TB + BR$ [using (2) and (3)]

14. *PQRS* is a square land of side 28 m. Two semicircular grass covered portions are to be made on two of its opposite sides as shown in the figure. How much area will be left uncovered ?

$$\left[\textbf{Take } \pi = \frac{22}{7}\right]$$

Solution. We have



Area left uncovered

= Area of the square *PQRS* – Area of two semicircular grass

$$= [28 \times 28]\ \text{m}^2 - \left[2\left(\frac{1}{2} \times \frac{22}{7} \times 14^2\right)\right] \text{m}^2$$

$$= 784\ \text{m}^2 - (22 \times 2 \times 14)\ \text{m}^2$$

$$= 784\ \text{m}^2 - 616\ \text{m}^2$$

$$= \mathbf{168\ m^2}.$$

15. Find a point on the *y*-axis which is equidistant from the points *A*(6, 5) and *B*(–4, 3).

Solution. Since the required point, say *P*, is on the *y*-axis, therefore its abscissa will be zero.

Let the ordinate of the point *P* be *y*.

∴ Coordinates of the point *P* are *P*(0, *y*).

The two given points are *A*(6, 5) and *B*(–4, 3).

It is given that the point *P* on the *y*-axis is equidistant from the points *A*(6, 5) and *B*(–4, 3).

∴ $AP = BP$

⇒ $AP^2 = BP^2$

⇒ $(0 - 6)^2 + (y - 5)^2 = (0 + 4)^2 + (y - 3)^2$

⇒ $36 + y^2 - 10y + 25 = 16 + y^2 - 6y + 9$

⇒ $-10y + 6y + 36 = 0$

$\Rightarrow \quad -4y + 36 = 0$

$\Rightarrow \quad -4y = -36$

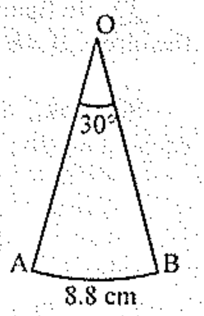
$\Rightarrow \quad y = 9$

Thus, the required point **P is (0, 9).**

16. A pendulum swings through an angle of 30° and describe an arc 8.8 cm in length. Find the length of the pendulum. **[Use $\pi = 22/7$]**

Solution. We know that the length of a sector of an angle θ in a circle of radius r is given by :

$$l = \frac{\theta}{360°} \times 2\pi r$$



$$\Rightarrow \text{(given) } 8.8 = \frac{30°}{360°} \times 2 \times \frac{22}{7} \times r, \text{ where } l = 8.8 \text{ cm}, \theta = 30°$$

$$\Rightarrow \quad 8.8 = \frac{1}{12} \times 2 \times \frac{22}{7} \times r$$

$$\Rightarrow \quad r = \frac{8.8 \times 12 \times 7}{2 \times 22} \text{ cm}$$

$$\Rightarrow \quad r = \frac{0.4 \times 12 \times 7}{2} \text{ cm}$$

$$\Rightarrow \quad r = 16.8 \text{ cm}$$

Hence, the length of the pendulum (= radius of a sector) is **16.8 cm.**

17. Find the value of k if the points (k, 3), (6, – 2) and (– 3, 4) are collinear.

Solution. Given points (k, 3), (6, – 2) and (– 3, 4) will be collinear if the area of the triangle formed by them is zero.

$\therefore$ Area of triangle = 0

$$\Rightarrow \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] = 0 \text{ where } x_1 = k, y_1 = 3, x_2 = 6, y_2 = -2, x_3 = -3, y_3 = 4$$

$$\Rightarrow \frac{1}{2}[k(-2-4) + 6(4-3) + (-3)(3+2)] = 0$$

$$\Rightarrow \quad \frac{1}{2}[-6k + 6 - 15] = 0$$

$$\Rightarrow \quad -6k - 9 = 0$$

$$\Rightarrow \quad -6k = 9$$

$$\Rightarrow \quad k = \frac{-9}{6}$$

$$\Rightarrow \quad k = -\frac{3}{2}$$

Hence, the value of k is $-\frac{3}{2}$.

18. Two A.P.'s have the same common difference. The first term of one of these is 3, and that of the other is 8. What is the difference between their 10th terms ?

Solution. Let the first term and common difference of one A.P. be 3 and d respectively. Let a_{10} denote the 10th term of this A.P. Then

$$a_{10} = 3 + (10 - 1)d \qquad [\because a = 3, n = 10]$$

$$\Rightarrow \quad a_{10} = 3 + 9d \qquad ...(1)$$

Again, let the first term and common difference of the other A.P. be 8 and d respectively. Let t_{10} denote the 10th term of this A.P. Then

$$t_{10} = 8 + (10 - 1)d \qquad [\because t = 8, n = 10]$$

$$\Rightarrow \quad t_{10} = 8 + 9d \qquad ...(2)$$

Now, $t_{10} - a_{10} = (8 + 9d) - (3 + 9d)$ [using (1) and (2)]

$$= 8 - 3 = 5$$

Hence, the difference between 10th terms of two A.P.'s is **5.**

Section 'C'

Question numbers 19 to 28 carry 3 marks each.

19. There are three consecutive positive integers such that the sum of the square of the first and the product of the other two is 154. What are the integers ?

Solution. Let the three consecutive positive integers be

$x, x + 1$ and $x + 2$, respectively.

According to the given condition :

The sum of the square of the first integer and the product of the 2nd and 3rd integers is 154.

$$\therefore \quad x^2 + (x + 1)(x + 2) = 154$$

$$\Rightarrow \quad x^2 + (x^2 + 3x + 2) = 154$$

$$\Rightarrow \quad 2x^2 + 3x + 2 = 154$$

$$\Rightarrow \quad 2x^2 + 3x - 152 = 0$$

$$\Rightarrow \quad 2x^2 + 19x - 16x - 152 = 0$$

$$\Rightarrow \quad (2x^2 - 16x) + (19x - 152) = 0$$

$$\Rightarrow \quad 2x(x - 8) + 19(x - 8) = 0$$

$$\Rightarrow \quad (x - 8)(2x + 19) = 0$$

$\Rightarrow$ Either $x - 8 = 0$ or $2x + 19 = 0$

$\Rightarrow$ Either $x = 8$ or $x = -\frac{19}{2}$

$\Rightarrow$ $x = 8$. Reject $x = -\frac{19}{2}$, because it is not a positive integer.

Hence, the three consecutive positive integers are 8, $8 + 1 = 9$ and $8 + 2 = 10$ respectively, *i.e.*, **8, 9,** and **10** respectively.

20. Determine an A.P. whose 3rd term is 16 and when 5th term is subtracted from 7th term, we get 12.

Solution. Let the first term and common difference of A.P. be a and d, respectively.

Let a_3, a_5 and a_7 denote the 3rd term, 5th term and 7th term of an A.P., then

$$a_3 = a + (3 - 1)d \qquad [\because n = 3]$$

$$\Rightarrow \quad 16 = a + 2d \qquad ...(1) [\because a_3 = 16 \text{ (given)}]$$

It is given that the difference of 5th term from the 7th term is 12

$$\therefore \quad a_7 - a_5 = 12 \text{ (given)}$$

$$\Rightarrow [a + (7 - 1)d] - [a + (5 - 1)d] = 12$$

$$\Rightarrow \quad (a + 6d) - (a + 4d) = 12$$

$$\Rightarrow \quad 2d = 12$$

$$\Rightarrow \quad d = 6 \quad \ldots(2)$$

From (1) and (2), we get

$$a + 2 \times 6 = 16$$

$$\Rightarrow \quad a = 16 - 12 = 4$$

Thus, the A.P. is **4, 10, 16, 22, 28, 34, 40, ...**

Or

Find the sum of all the three digit numbers which leave the remainder 3 when divided by 5.

Solution. The smallest and the largest numbers of three digits which leave the remainder 3 when divided by 5 are 103, and 998, respectively.

So, the sequence of three digit numbers which are divisible by 5 and leaving the remainder 3 are

103, 108, 113, ..., 998

Clearly, it is an A.P. with first term $a = 103$ and common difference $d = 5$.

Let there be n terms in this sequence, then

$$a_n = 998$$

$$\Rightarrow \quad a + (n-1)d = 998$$

$$\Rightarrow \quad 103 + (n-1) \times 5 = 998$$

$$\Rightarrow \quad (n-1) \times 5 = 895$$

$$\Rightarrow \quad n - 1 = 179$$

$$\Rightarrow \quad n = 179 + 1 = 180$$

Now, required sum $= \frac{n}{2}[a + l]$

$$= \frac{180}{2}[103 + 998]$$

$$= 90[1101]$$

$$= \mathbf{99090.}$$

21. The king, queen and jack of clubs are removed from a deck of 52 playing cards and the remaining cards are shuffled. A card is drawn from the remaining cards. Find the probability of getting a card of (*i*) heart (*ii*) queen (*iii*) club.

Solution. Total number of playing cards in deck are 52. When the king, queen and jack of clubs are removed from a deck of 52 playing cards, then the remaining cards are 49 (*i.e.*, 52 – a king of club – a queen of club – a jack of club = 52 – 1 – 1 – 1 = 52 – 3).

Total number of outcomes in which one card is selected from the remaining 49 cards are 49.

(*i*) Out of 49 outcomes, 13 outcomes (as there are 13 hearts) are favourable of getting a heart. Hence,

$$P(\text{of getting a heart}) = \mathbf{\frac{13}{49}}.$$

(*ii*) Out of 49 outcomes, 3 outcomes (as one queen of club is removed) are favourable of getting a queen. Hence,

$$P(\text{of getting a queen}) = \mathbf{\frac{3}{49}}.$$

(*iii*) Out of 49 outcomes, 10 outcomes (as a king of clubs, queen of clubs and jack of clubs are removed) are favourable of getting a club. Hence,

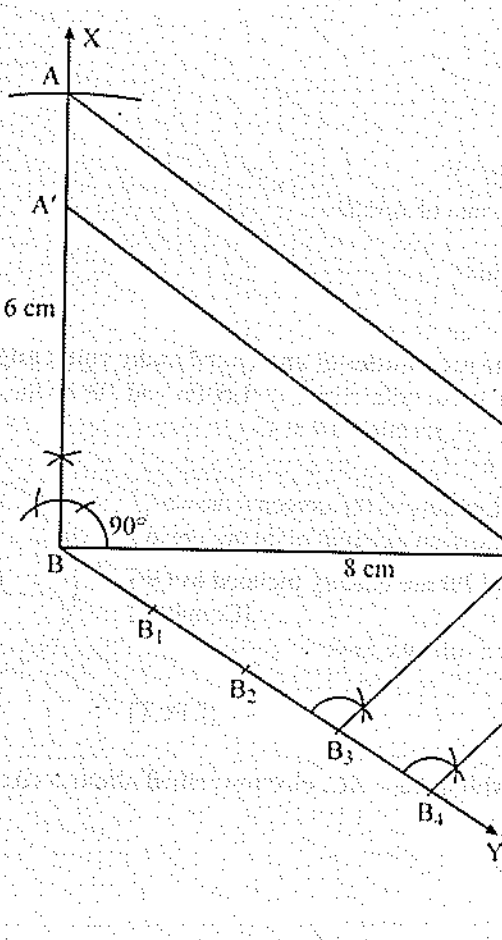
$$P(\text{of getting a club}) = \mathbf{\frac{10}{49}}.$$

22. Draw a right triangle in which sides (other than hypotenuse) are of lengths 8 cm and 6 cm. Then construct another triangle whose sides are $\frac{3}{4}$ times the corresponding sides of the first triangle.

Solution. Steps of Construction :

1. Draw a line segment $BC = 8$ cm.
2. At B construct $\angle CBX = 90°$.
3. With B as centre and radius = 6 cm, draw an arc intersecting the line BX at A.
4. Join AC to obtain the required ΔABC.
5. Draw any ray BY making an acute angle with BC on the opposite side of the vertex A.
6. Locate 4 points $\left(\text{the greater of 3 and 4 in } \frac{3}{4}\right)$ B_1, B_2, B_3 and B_4 on BY so that $BB_1 = B_1B_2 = B_2B_3 = B_3B_4$.
7. Join B_4C and draw a line through B_3 $\left(\text{the 3rd point, 3 being the smaller of 3 and 4 in } \frac{3}{4}\right)$ parallel to B_4C intersecting BC at C'.
8. Draw a line through C' parallel to line CA to intersect BA and A' (see figure).

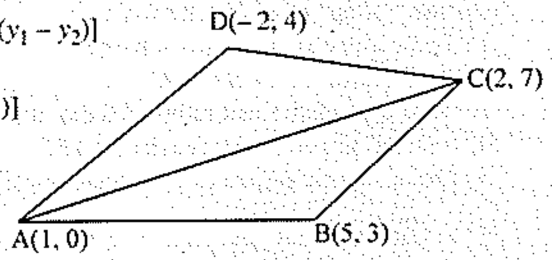
Then $A'BC'$ is the required triangle.



23. Find the area of the quadrilateral *ABCD* whose vertices are *A*(1, 0), *B*(5, 3), *C*(2, 7) and *D*(– 2, 4).

Solution. By joining *A* to *C*, we will get two triangles *ABC* and *ACD*.

Now, the area of ΔABC



$$= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

$$= \frac{1}{2}[1(3 - 7) + 5(7 - 0) + 2(0 - 3)]$$

$$= \frac{1}{2}[-4 + 35 - 6]$$

$$= \frac{25}{2} \text{ sq. units}$$

Also, the area of ΔACD

$$= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

$$= \frac{1}{2}[1(7 - 4) + 2(4 - 0) + (-2)(0 - 7)]$$

$$= \frac{1}{2}[3 + 8 + 14]$$

$$= \frac{25}{2} \text{ sq. units}$$

So, the area of the quadrilateral *ABCD*

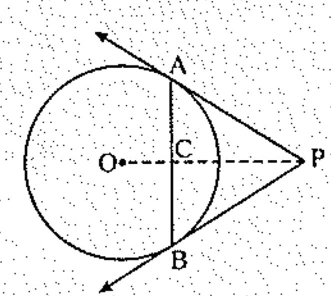
$$= \text{area of } \Delta ABC + \text{area of } \Delta ACD$$

$$= \left(\frac{25}{2} + \frac{25}{2}\right) \text{ sq. units}$$

= 25 sq. units.

24. Prove that the tangents at the extermities of any chord make equal angles with the chord.

Solution. Let *AB* be a chord of a circle with centre *O*. Let *PA* and *PB* be the tangents at *A* and *B* respectively, meeting at a point *P*. Join *OP*, meeting *AB* at *C*.



In triangles *PCA* and *PCB*, we have

$PA = PB$ [$\because$ Lengths of the tangents drawn from an external point are equal]

$\angle APC = \angle BPC$ [$\because$ *PA* and *PB* are equally inclined to *OP*]

and $PC = PC$ [Common]

So, by SAS criterion of congruence rule, we have

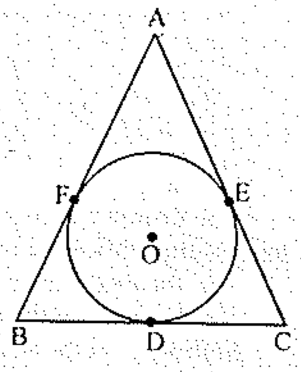
$\Delta PCA \cong \Delta PCB$

$\Rightarrow \angle PAC = \angle PBC$ [CPCT]

Or

***ABC* is an isosceles triangle, in which *AB* = *AC*, circumscribed about a circle. Show that *BC* is bisected at the point of contact.**

Solution. ABC is an isosceles triangle, in which $AB = AC$, circumscribed about a circle with centre O. Since tangents drawn from an external point to a circle are equal in length.



$\therefore \quad AF = AE$ ...(1) [Tangents from A]

$BF = BD$ ...(2) [Tangents from B]

$CD = CE$ ...(3) [Tangents from C]

Adding (1), (2) and (3), we get

$AF + BF + CD = AE + BD + CE$

$\Rightarrow \quad AB + CD = AC + BD$

But $\quad AB = AC$ [given]

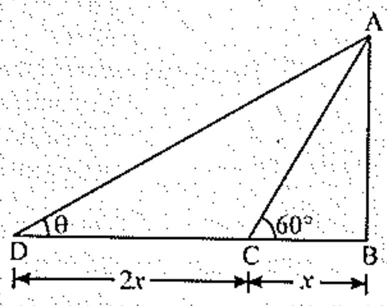
$\Rightarrow \quad CD = BD$

$\Rightarrow BC$ is bisected at the point of contact D.

25. The shadow of a flag-staff is three times as long as the shadow of the flag-staff when the sun rays meet the ground at an angle of 60°. Find the angle between the rays and the ground at the time of longer shadow.

Solution. Let AB be the flag-staff and $BC = x$ be the length of its shadow when the sun rays meet the ground at angle of 60°.

In ΔABC, we have



$$\tan 60° = \frac{AB}{BC}$$

$$\Rightarrow \quad \sqrt{3} = \frac{AB}{x}$$

$$\Rightarrow \quad AB = \sqrt{3}x \quad ...(1)$$

Let θ be the angle between the sun rays and the ground when the length of the shadow of the flag-staff is $BD = 3x$.

In ΔABD, we have

$$\tan \theta = \frac{AB}{BD}$$

$$\Rightarrow \quad \tan \theta = \frac{\sqrt{3}x}{3x} \quad \text{[using (1)]}$$

$$\Rightarrow \quad \tan \theta = \frac{\sqrt{3}}{3}$$

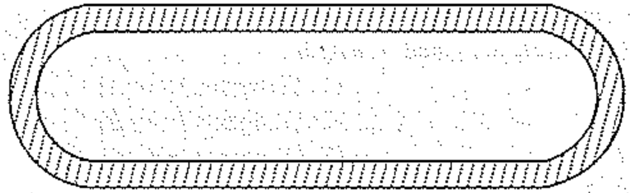
$$\Rightarrow \quad \tan \theta = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad \tan \theta = \tan 30°$$

$$\Rightarrow \quad \theta = 30°$$

Thus, the angle between the sun rays and the ground is **30°** at the time of longer shadow.

26. Figure depicts a racing track whose left and right ends are semicircular.

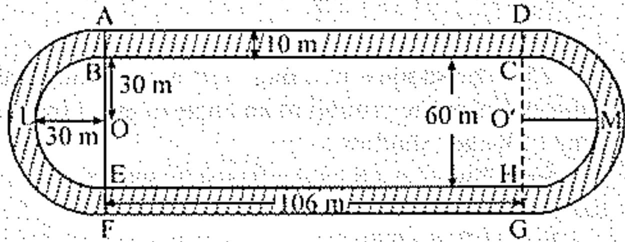


The distance between the two inner parallel line segments is 60 m and they are each 106 m long. If the track is 10 m wide, find :

(*i*) the distance around the track along its inner edge

(*ii*) the area of the track.

Solution. In the figure,



$OB = O'C$
$= 30$ m
$AB = CD$
$= 10$ m
$OA = O'D$
$= (30 + 10)$ m
$= 40$ m.

(*i*) The distance around the track along its inner edge

= Circumference of the semicircle with radius $OB = OL = 30$ m
+ Distance BC + Circumference of the semicircle with radius $O'C = O'M = 30$ m
+ Distance EH

$$= \left[\frac{1}{2}(2\pi \times 30) + 106 + \frac{1}{2}(2\pi \times 30) + 106\right] \text{ m}$$

$$= [2\pi \times 30 + 212] \text{ m}$$

$$= \left[\frac{60 \times 22}{7} + 212\right] \text{ m}$$

$$= \left[\frac{1320 + 1484}{7}\right] \text{ m}$$

$$= \mathbf{\frac{2804}{7} \text{ m.}}$$

(*ii*) Area of the track

= Area of rectangle $ABCD$ + Area of rectangle $EFGH$
+ 2 [Area of semicircle with radius 40 m – Area of semicircle with radius 30 m]

$$= [106 \times 10 + 106 \times 10] \text{ m}^2 + 2\left[\frac{1}{2} \times \pi \times (40)^2 - \frac{1}{2} \times \pi \times (30)^2\right] \text{m}^2$$

$$= [1060 + 1060 + \pi \times (40^2 - 30^2)] \text{ m}^2$$

$$= \left[2120 + \frac{22}{7} \times (1600 - 900)\right] \text{m}^2$$

$$= \left[2120 + \frac{22}{7} \times 700\right] \text{m}^2$$

$$= [2120 + 2200] \text{ m}^2$$

$$= \mathbf{4320 \text{ m}^2.}$$

Or

How many spherical lead shots each 4.2 cm in diameter can be obtained from a rectangular solid of lead with dimensions 66 cm, 42 cm, 21 cm. **[Use $\pi = 22/7$]**

Solution. We have

Volume of a rectangular solid of lead

$$= (66 \times 42 \times 21) \text{ cm}^3 \quad ...(1)$$

$$\text{Radius of a lead shot} = \frac{4.2}{2} \text{ cm} = 2.1 \text{ cm}$$

$$\text{Volume of spherical lead shot} = \frac{4}{3}\pi r^3$$

$$= \left[\frac{4}{3} \times \frac{22}{7} \times (2.1)^3\right] \text{cm}^3 \quad ...(2)$$

$\therefore$ Required number of lead shots

$$= \frac{\text{Volume of a rectangular solid of lead}}{\text{Volume of a spherical lead shot}}$$

$$= \frac{66 \times 42 \times 21}{\frac{4}{3} \times \frac{22}{7} \times (2.1)^3}$$

$$= \frac{3 \times 42 \times 21 \times 3 \times 7}{4 \times (2.1)^3}$$

$$= \frac{3}{4} \times \left(\frac{42}{2.1}\right) \times \left(\frac{21}{2.1}\right) \times \left(\frac{21}{2.1}\right)$$

$$= \frac{3}{4} \times 20 \times 10 \times 10$$

$$= 1500$$

Hence, the number of spherical lead shots is **1500.**

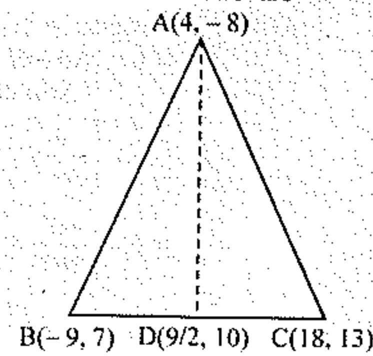
27. If $A(4, -8)$, $B(-9, 7)$ and $C(18, 13)$ are the vertices of a triangle ABC, find the length of the median through A and coordinates of centroid of the triangle.

Solution. The coordinates of the vertices of the triangle ABC are $A(4, -8)$, $B(-9, 7)$ and $C(18, 13)$ respectively.

Let D be the mid-point of the side BC of the triangle ABC. Then the coordinates of D are

$$D\left(\frac{-9+18}{2}, \frac{7+13}{2}\right)$$

or $$D\left(\frac{9}{2}, \frac{20}{2}\right) \text{ or } D\left(\frac{9}{2}, 10\right)$$



Thus the length of the median AD through A is

$$AD = \sqrt{\left(\frac{9}{2} - 4\right)^2 + (10 + 8)^2}$$

$$= \sqrt{\left(\frac{9-8}{2}\right)^2 + (18)^2}$$

$$= \sqrt{\frac{1}{4} + 324}$$

$$= \sqrt{\frac{1+1296}{4}}$$

$$= \frac{\sqrt{1297}}{2}$$

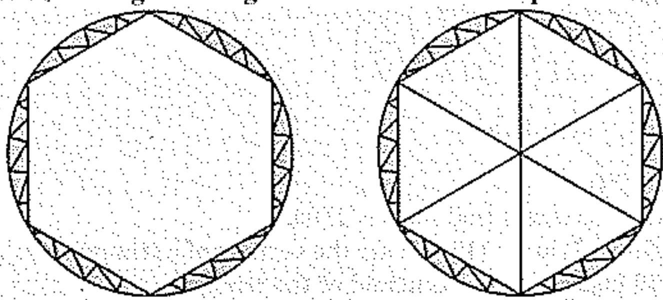
The coordinates of the centroid G of the triangle ABC are

$$G\left(\frac{4+(-9)+18}{3}, \frac{(-8)+7+13}{3}\right)$$

or $G\left(\frac{13}{3}, \frac{12}{3}\right)$ or $G\left(\frac{13}{3}, 4\right)$

Hence, the length of the median through A is $\frac{\sqrt{1297}}{2}$ and the coordinates of centroid of the triangle ABC are $\left(\frac{13}{3}, 4\right)$.

28. A round table cover has six equal designs as shown in figure (1). If the radius of the cover is 28 cm, find the cost of making the designs at the rate of ₹ 0.35 per cm². [Use $\sqrt{3} = 1.7$]



(1) (2)

Solution. We observe in figure (2) that the designs form six segments of a circle of radius 28 cm and each angle 60°.

∴ Area of six designs

= 6 × [Area of a sector of circle with radius 28 cm at an angle of 60° − Area of an equilateral Δ with side 28 cm]

$$= 6 \times \left[\frac{\theta}{360^\circ} \times \pi \times (28)^2 - \frac{\sqrt{3}}{4} \times (28)^2\right] \text{cm}^2$$

$$= 6 \times \left[\frac{60^\circ}{360^\circ} \times \pi \times (28)^2 - \frac{\sqrt{3}}{4} \times (28)^2\right] \text{cm}^2 \quad [\because \theta = 60^\circ]$$

$$= 6 \times \left[\frac{1}{6} \times \frac{22}{7} \times 28 \times 28 - \frac{\sqrt{3}}{4} \times 28 \times 28\right] \text{cm}^2$$

$$= 6 \times \left[\frac{1}{6} \times 22 \times 4 \times 28 - \sqrt{3} \times 7 \times 28\right] \text{cm}^2$$
$$= [22 \times 4 \times 28 - \sqrt{3} \times (6 \times 7 \times 28)] \text{ cm}^2$$
$$= [88 \times 28 - \sqrt{3} \times (1176)] \text{ cm}^2$$
$$= [2464 - 1176 \times (1.7)] \text{ cm}^2 \qquad [\text{given } \sqrt{3} = 1.7]$$
$$= [2464 - 1999.2] \text{ cm}^2$$
$$= 464.8 \text{ cm}^2$$

Hence, cost of making the designs at the rate of ₹ 0.35 per cm^2

$$= ₹\ (464.8 \times 0.35)$$
$$= ₹\ \mathbf{162.68.}$$

Section 'D'

Question numbers 29 to 34 carry 4 marks each.

29. Some students arranged a picnic. The budget for food was ₹ 240. Because four students of the group failed to go, the cost of food to each student got increased by ₹ 5. How many students went for the picnic ?

Solution. Let the original number of students be x.

Total cost of food for x students = ₹ 240

∴ The cost of food for each student = ₹ $\frac{240}{x}$

It is given that 4 students of the group failed to go.

∴ The number of students go for a picnic = $(x - 4)$

Now, the cost of food for each student = ₹ $\frac{240}{x-4}$

It is given that the four students of a group failed to go, the cost of food to each student got increased by ₹ 5.

$$\therefore \quad \frac{240}{x-4} - \frac{240}{x} = 5 \text{ (given)}$$
$$\Rightarrow \quad \frac{240x - 240x + 960}{x(x-4)} = 5$$
$$\Rightarrow \quad 960 = 5(x^2 - 4x)$$
$$\Rightarrow \quad 192 = x^2 - 4x$$
$$\Rightarrow \quad x^2 - 4x - 192 = 0$$
$$\Rightarrow \quad x^2 - 16x + 12x - 192 = 0$$
$$\Rightarrow \quad x(x - 16) + 12(x - 16) = 0$$
$$\Rightarrow \quad (x - 16)(x + 12) = 0$$
$$\Rightarrow \quad \text{Either } x = 16 \text{ or } x = -12$$

$\Rightarrow x = 16$, as x cannot be negative.

Hence, the number of students go for the picnic = $x - 4$, *i.e.*, $16 - 4 =$ **12.**

Or

A plane left 30 minutes late than its scheduled time and in order to reach the destination 1500 km away in time, it had to increase the speed by 250 km/h from the usual speed. Find its usual speed.

Solution. Let the usual speed of the plane be x km/h.

Then, time taken to cover 1500 km with usual speed $= \frac{1500}{x}$ h

When the speed of a plane is increased by 250 km/h, then the time taken to cover 1500 km with the speed of $(x+250)$ km/h $= \frac{1500}{x+250}$ h

$$\therefore \quad \frac{1500}{x} - \frac{1500}{x+250} = \frac{1}{2} \text{ hour (or 30 minutes)}$$

$$\Rightarrow \quad \frac{1500x + 1500 \times 250 - 1500x}{x(x+250)} = \frac{1}{2}$$

$$\Rightarrow \quad \frac{1500 \times 250}{x^2 + 250x} = \frac{1}{2}$$

$$\Rightarrow \quad 3000 \times 250 = x^2 + 250x$$

$$\Rightarrow \quad x^2 + 250x - 750000 = 0$$

$$\Rightarrow \quad x^2 + 1000x - 750x - 750000 = 0$$

$$\Rightarrow \quad x(x+1000) - 750(x+1000) = 0$$

$$\Rightarrow \quad (x+1000)(x-750) = 0$$

$$\Rightarrow \quad \text{Either } x = 750 \text{ or } x = -1000$$

$\Rightarrow x = 750$, as x cannot be negative.

Hence, the usual speed of the plane is **750 km/h.**

30. Prove that the lengths of tangents drawn from an external point to a circle are equal.

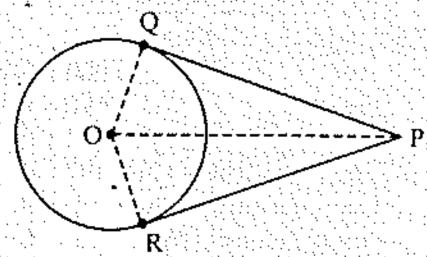
Solution. Given : PQ and PR are two tangents, from an external point P to a circle.

To prove : $PQ = PR$.

Construction : Join OP, OQ and OR.

Proof : $\angle OQP$ and $\angle ORP$ are right angles, because these are angles between the radii and tangents.

[The tangent at any point of a circle is perpendicular to the radius through the point of contact]



Now in right triangles OQP and ORP, we have

$OQ = OR$ [Radii of the same circle]

$\angle OQP = \angle ORP$ [$OQ \perp PQ$ and $OR \perp PR$]

$OP = OP$ [Common]

By RHS theorem of congruence,

$\Delta OQP \cong \Delta ORP$

$\Rightarrow$ $PQ = PR$ [CPCT]

31. 200 logs are stacked in the following manner : 20 logs in the bottom row, 19 in the next row, 18 in the row next to it and so on (see figure). In how many rows are the 200 logs placed and how many logs are in the top row ?

Solution. Suppose 200 logs are stacked in 'n' rows.
There are **20 logs in the first row,**
19 logs in the second row,
18 logs in the third row, and so on.
So, numbers of logs in various rows form an A.P. whose first term $a(= 20)$ and common difference d $(19 - 20) = -1$
$\therefore$ Sum of n terms (rows) of an A.P. with $a = 20$, $d = -1$ is equal to 200 (given)

$$\Rightarrow \quad \frac{n}{2}[2 \times 20 + (n-1) \times (-1)] = 200 \qquad \left[\because S_n = \frac{n}{2}[2a + (n-1)d]\right]$$
$$\Rightarrow \quad 40n - n(n-1) = 400$$
$$\Rightarrow \quad 40n - n^2 + n = 400$$
$$\Rightarrow \quad n^2 - 41n + 400 = 0$$
$$\Rightarrow \quad n^2 - 25n - 16n + 400 = 0$$
$$\Rightarrow \quad n(n-25) - 16(n-25) = 0$$
$$\Rightarrow \quad (n-25)(n-16) = 0$$
$$\Rightarrow \quad n - 25 = 0 \quad \text{or} \quad n - 16 = 0$$
$$\Rightarrow \quad n = 25 \quad \text{or} \quad n = 16$$

When n = 25, then number of logs in the 25th row
= 25th term of an A.P. with first term $a = 20$ and common difference $d = -1$
$= a + (25 - 1)d$
$= 20 + 24(-1)$
$= -4$, not possible

When n = 16, then number of logs in the 16th row
= 16th term of an A.P. with first term $a = 20$ and common difference $d = -1$
$= a + (16 - 1)d$
$= 20 + 15(-1)$
$= 20 - 15$
$= 5$

Hence, there are **16 rows** in which 200 logs are placed and **5 logs** are in the top row.

32. A vessel is a hollow cylinder fitted with a hemispherical bottom of the same base. The depth of the cylinder is $4\frac{2}{3}$ m and the diameter of hemisphere is 3.5 m. Calculate the volume and the internal surface area of the solid.

Solution. Depth (*i.e.*, height) of the cylinder $(h) = 4\frac{2}{3}$ m $= \frac{14}{3}$ m.
Diameter of the cylinder = Diameter of the hemispherical bottom = 3.5 m.
Radius of the cylinder (r) = Radius of the hemispherical bottom (r).
$= (3.5 \div 2)$ m $= 1.75$ m
Volume of the solid = Volume of the cylinder + Volume of the hemispherical portion

$$= \left(\pi r^2 h + \frac{2}{3}\pi r^3\right)$$

$$= \pi r^2 \left(h + \frac{2}{3} r\right)$$

$$= \frac{22}{7} \times (1.75)^2 \left[\frac{14}{3} + \frac{2}{3} \times 1.75\right] \text{m}^3$$

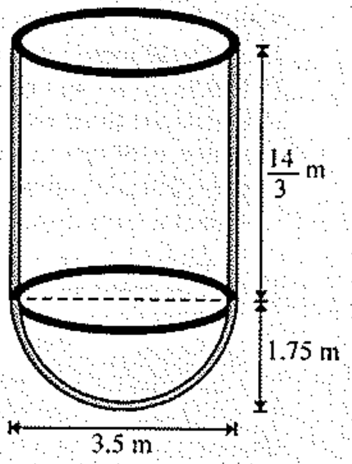
$$= \frac{22}{7} \times \left(\frac{7}{4}\right)^2 \left[\frac{14}{3} + \frac{2}{3} \times \frac{3.5}{2}\right] \text{m}^3$$

$$= \frac{22}{7} \times \left(\frac{7}{4}\right)^2 \left[\frac{14 + 3.5}{3}\right] \text{m}^3$$

$$= \frac{22}{7} \times \frac{7}{4} \times \frac{7}{4}\left(\frac{17.5}{3}\right) \text{m}^3$$

$$= \frac{11 \times 7}{2 \times 4} \times \frac{17.5}{3} \text{m}^3$$

$$= \frac{1347.5}{24} \text{m}^3 = \mathbf{56.15\ m^3}$$



Internal surface area of the solid

= Curved surface area of the cylinder + Curved surface of the hemispherical portion

$= (2\pi rh + 2\pi r^2)$

$= 2\pi r(h + r)$

$$= 2 \times \frac{22}{7} \times 1.75 \times \left(\frac{14}{3} + 1.75\right) \text{m}^2$$

$$= 2 \times \frac{22}{7} \times \frac{7}{4} \times \left(\frac{14}{3} + \frac{7}{4}\right) \text{m}^2$$

$$= 11 \times \left(\frac{56 + 21}{12}\right) \text{m}^2$$

$$= 11 \times \frac{77}{12} \text{m}^2$$

$$= \frac{847}{12} \text{m}^2$$

$$= \mathbf{70\frac{7}{12}\ m^2}$$

Or

A solid toy in the form of a hemisphere surmounted by a right circular cone. The height of the cone is 4 cm and the diameter of the base is 6 cm.

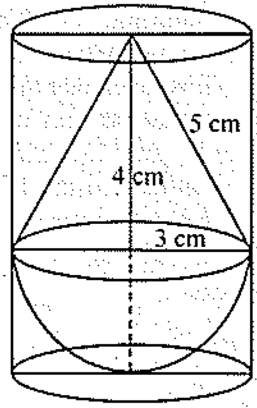
Determine :

(*i*) the volume of the toy

(*ii*) surface area of the toy

(*iii*) the difference of the volumes of the cylinder and the toy, when a right circular cylinder circumscribes the toy.

Solution. Height of the cone = 4 cm
Diameter of the base of the cone = 6 cm
Radius of the base of the cone (r) = 3 cm
Radius of the hemisphere (r) = 3 cm
Slant height of the cone (l)



$$l = \sqrt{h^2 + r^2}$$

$$\Rightarrow \quad l = \sqrt{(4)^2 + (3)^2}$$

$$\Rightarrow \quad l = \sqrt{16 + 9}$$

$$\Rightarrow \quad l = \sqrt{25} = 5 \text{ cm}$$

Height of the cylinder (H) = Height of the cone (h) + Radius of the hemisphere (r)

$= (4 + 3) = 7$ cm

Radius of the base of the cylinder (r) = 3 cm

(i) Volume of the toy

= Volume of the hemispherical part + Volume of the conical part

$= \frac{2}{3}\pi r^3 + \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi r^2[2r + h]$

$= \frac{1}{3} \times 3.14 \times (3)^2 \times [2 \times 3 + 4] \text{ cm}^3$

$= 3.14 \times 3 \times 10 \text{ cm}^3$

$= 31.4 \times 3 \text{ cm}^3$

$= \mathbf{94.2 \text{ cm}^3}$ ...(1)

(ii) Surface area of the toy

= Surface area of conical part + Surface area of a hemispherical part

$= (\pi rl + 2\pi r^2)$

$= \pi r(l + 2r)$

$= 3.14 \times 3 \times [5 + 2 \times 3] \text{ cm}^2$

$= 3.14 \times 3 \times (11) \text{ cm}^2$

$= \mathbf{103.62 \text{ cm}^2}$

(iii) Difference of the volumes of the cylinder and the toy

= Volume of the right circular cylinder – Volume of the toy

$= [\pi r^2 h - 94.2] \text{ cm}^3$ [using (1)]

$= [3.14 \times (3)^2 \times 7 - 94.2] \text{ cm}^3$

$= [21.98 \times 9 - 94.2] \text{ cm}^3$

$= (197.82 - 94.2) \text{ cm}^3$

$= \mathbf{103.62 \text{ cm}^3}$

33. The angle of elevation of a jet fighter from a point *A* on the ground is 60°. After a flight of 15 seconds, the angle of elevation changes to 30°. If the jet is flying at a speed of 720 km/hour, find the constant height at which the jet is flying. **[Use $\sqrt{3} = 1.732$]**

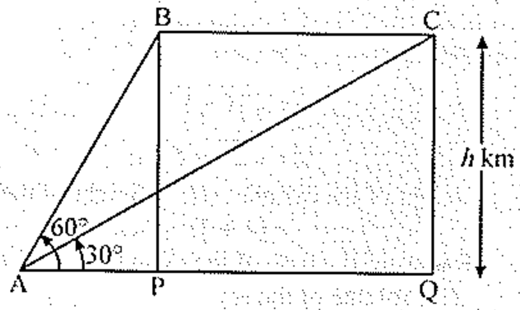
Solution. Let B and C be the two positions of a jet fighter as observed from a point A on the ground. Let APQ be the horizontal line through A.

It is given that the angles of elevation of a jet fighter in two positions B and C as observed from a point A are 60° and 30°, respectively.

$\therefore \quad \angle BAP = 60°$ and $\angle CAQ = 30°$

Let the constant height of a jet fighter be h km, *i.e.*, $BP = CQ = h$ km.

It is also given that a jet is flying at a speed of 720 km/hour.



In right-angled ΔAPB, we have

$$\tan 60° = \frac{BP}{AP}$$

$$\Rightarrow \quad \sqrt{3} = \frac{h}{AP} \quad [\because QC = PB = h \text{ km}]$$

$$\Rightarrow \quad AP = \frac{h}{\sqrt{3}} \quad \ldots(1)$$

In right-angled ΔAQC, we have

$$\tan 30° = \frac{CQ}{AQ}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{AQ} \quad [\because CQ = h]$$

$$\Rightarrow \quad AQ = \sqrt{3}\, h \quad \ldots(2)$$

Now, $\quad BC = PQ = AQ - AP$

$$\Rightarrow \quad BC = \left(\sqrt{3}h - \frac{h}{\sqrt{3}}\right) \text{ km} \quad [\text{using (1) and (2)}]$$

$$\Rightarrow \quad BC = \left(\frac{3h - h}{\sqrt{3}}\right) \text{ km}$$

$$\Rightarrow \quad BC = \frac{2h}{\sqrt{3}} \text{ km} \quad \ldots(3)$$

As the jet fighter travels BC km in 15 seconds.

$\therefore$ Time taken in hours, the jet fighter travels BC km $= \dfrac{15}{60 \times 60}$ hour

$\therefore \quad$ Distance = Speed × Time taken

$$\Rightarrow \quad BC = \left[720 \times \left(\frac{15}{60 \times 60}\right)\right] \text{ km}$$

$$\Rightarrow \quad \frac{2h}{\sqrt{3}} = \frac{12 \times 15}{60} \text{ km}$$

$$\Rightarrow \quad h = \frac{\sqrt{3}}{2} \times 3 \text{ km}$$

$$\Rightarrow \quad h = \frac{1.732 \times 3}{2} \text{ km}$$

$\Rightarrow \quad h = 0.866 \times 3$ km

$\Rightarrow \quad h = 2.598$ km

Thus, the constant height at which the jet is flying = **2.598 km** or **2598 m.**

34. Selvi's house has an overhead tank in the shape of a cylinder. This is filled by pumping water from a sump (an underground tank) which is in the shape of a cuboid. The sump has dimensions 1.57 m × 1.44 m × 95 cm. The overhead tank has its radius 60 cm and height 95 cm. Find the height of the water left in the sump after the overhead tank has been completely filled with water from the sump which had been full. Compare the capacity of the tank with that of the sump. **[Use $\pi = 3.14$]**

Solution. Radius of an overhead tank (R) = 60 cm

Height of an overhead tank (H) = 95 cm

The volume of water in the overhead tank equals the volume of the water removed from the sump.

$\therefore$ Volume of water in the overhead tank in the shape of a cylinder

$$= \pi R^2 H$$
$$= [3.14 \times 60 \times 60 \times 95] \text{ cm}^3$$
$$= [3.14 \times 0.6 \times 0.6 \times 0.95] \text{ m}^3$$

The volume of water in the sump when full

$$= 1.57 \text{ m} \times 1.44 \text{ m} \times 0.95 \text{ m}^3 \qquad \text{[given]}$$

The volume of water left in the sump after filling the tank

$$= (1.57 \times 1.44 \times 0.95) \text{ m}^3 - (3.14 \times 0.6 \times 0.6 \times 0.95) \text{ m}^3$$
$$= 1.57 \times 0.95[1.44 - 2 \times 0.6 \times 0.6] \text{ m}^3$$
$$= 1.57 \times 0.95[1.44 - 0.72] \text{ m}^3$$
$$= (1.57 \times 0.95 \times 0.72) \text{ m}^3 \qquad \text{...(1)}$$

If h be the height of the water left in the sump

$$= l \times b \times h$$
$$= (1.57 \times 1.44 \times h) \text{ m}^3 \text{ where } l = 1.57, b = 1.44 \qquad \text{...(2)}$$

From (1) and (2), we have

The height of the water left in the sump

$$= \frac{\text{volume of water left in the sump}}{l \times b}$$
$$= \left(\frac{1.57 \times 0.95 \times 0.72}{1.57 \times 1.44}\right) \text{m} = \frac{0.95}{2} \text{ m}$$
$$= 0.475 \text{ m}$$
$$= 0.475 \times 100 \text{ cm}$$
$$= \mathbf{47.5 \text{ cm.}}$$

Also, $\dfrac{\text{Capacity of tank}}{\text{Capacity of sump}} = \dfrac{3.14 \times 0.6 \times 0.6 \times 0.95}{1.57 \times 1.44 \times 0.95}$

$$= \frac{1}{2}$$

$\Rightarrow \quad$ Capacity of tank $= \frac{1}{2} \times$ (Capacity of the sump)

Thus, the capacity of the tank is **half** the capacity of the sump.