CCE SAMPLE QUESTION PAPER

SECOND TERM (SA-II)

MATHEMATICS

(With Solutions)

CLASS X

Time Allowed : 3 Hours] [*Maximum Marks : 80*

General Instructions :

(*i*) *All questions are compulsory.*

(*ii*) *The question paper consists of 34 questions divided into four sections A, B, C and D. Section A comprises of 10 questions of 1 mark each, Section B comprises of 8 questions of 2 marks each, Section C comprises of 10 questions of 3 marks each and Section D comprises of 6 questions of 4 marks each.*

(*iii*) *Question numbers 1 to 10 in Section A are multiple choice questions where you are to select one correct option out of the given four.*

(*iv*) *There is no overall choice. However, internal choice has been provided in 1 question of two marks, 3 questions of three marks each and 2 questions of four marks each. You have to attempt only one of the alternatives in all such questions.*

(*v*) *Use of calculators is not permitted.*

Section 'A'

Question numbers 1 to 10 are of one mark each.

1. If the roots of the quadratic equation $x^2 + px + 12 = 0$ are in the ratio 1 : 3, then the value of p is

(*a*) ± 8 (*b*) ± 7

(*c*) ± 6 (*d*) ± 9

Solution. Choice (*a*) is correct.

Let the roots of the equation $x^2 + px + 12 = 0$ be α and 3α, then

$$\alpha + 3\alpha = -p \Rightarrow 4\alpha = -p \Rightarrow \alpha = \frac{-p}{4} \quad \ldots(1)$$

and $\alpha\cdot(3\alpha) = 12 \Rightarrow \alpha^2 = 4 \quad \ldots(2)$

From (1) and (2), we get

$$\left(-\frac{p}{4}\right)^2 = 4 \Rightarrow p^2 = 4 \times 16 = 64 \Rightarrow p = \pm 8$$

2. If $3p + 7$, 15, $8p + 12$ are three consecutive terms of an A.P., then p is equal to

(*a*) 1 (*b*) 2

(*c*) -1 (*d*) -2

Solution. Choice (*a*) is correct.

Since $3p + 7$, 15 and $8p + 12$ are three consecutive terms of an A.P., therefore,

$15 - (3p + 7) = d$ and $(8p + 12) - 15 = d$

$\Rightarrow$ $8 - 3p = d$ and $8p - 3 = d$

$\Rightarrow \quad 8 - 3p = 8p - 3$

$\Rightarrow \quad 11 = 11p$

$\Rightarrow \quad p = 1$

3. A bag contains 4 red and 6 black balls. A ball is taken out of the bag at random. The probability of getting a black ball is

(a) $\frac{3}{2}$ (b) $\frac{3}{5}$

(c) $\frac{5}{6}$ (d) $\frac{3}{4}$

Solution. Choice (b) is correct.

Total number of outcomes (*i.e.*, balls)

$= 4 \text{ red} + 6 \text{ black} = 10$

Out of 10 outcomes (*i.e.*, balls), the favourable outcomes (*i.e.*, a black ball) = 6, *i.e.*, the bag contains 6 black balls.

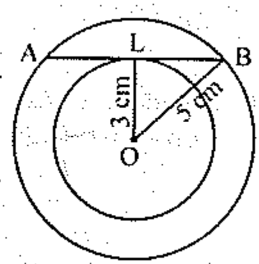
Thus, $P(\text{of getting a black ball}) = \frac{6}{10} = \frac{3}{5}$.

4. If radii of the two concentric circles are 3 cm and 5 cm, then the length of each chord of one circle which is tangent to other is

(a) 4 cm (b) 8 cm

(c) 10 cm (d) 12 cm

Solution. Choice (b) is correct.



Let O be the centre of two concentric circles and AB be a chord of the larger circle touching the smaller circle at L.

Join OL.

Since OL (= 3 cm) is the radius of the smaller circle and AB is a tangent to this circle at a point L.

$\therefore \quad OL \perp AB$

We know that the perpendicular drawn from the centre of a circle to any chord of the circle bisects the chord at L.

$\Rightarrow \quad AL = BL$

In right ΔOLB, we have

$LB^2 = OB^2 - OL^2$

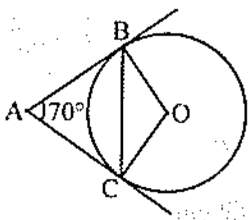
$\Rightarrow \quad LB^2 = (5)^2 - (3)^2$

$\Rightarrow \quad LB^2 = 25 - 9 = 16 = (4)^2$

$\Rightarrow \quad LB = 4 \text{ cm}$

Length of chord $AB = AL + LB = 4 + 4 = 8$ cm $\quad [\because AL = BL = 4 \text{ cm}]$

5. In figure, AB and AC are tangents to the circle with centre O such that $\angle BAC = 70°$, then $\angle OBC$ is equal to



(a) 25° (b) 35°

(c) 45° (d) 55°

Solution. Choice (b) is correct.

Since the angle between two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line segments joining the points of contact at the centre, *i.e.*,

$$\angle BAC + \angle BOC = 180°$$

But $\angle BAC = 70°$ (given)

$\therefore \quad 70° + \angle BOC = 180°$

$\Rightarrow \quad \angle BOC = 180° - 70° = 110° \quad \ldots(1)$

In ΔOBC, we have

$OB = OC$ [Each = radius]

$\Rightarrow \quad \angle OCB = \angle OBC \quad \ldots(2)$ [$\angle s$ opposite to equal sides of a Δ are equal]

In ΔOBC, we have

$\angle OBC + \angle OCB + \angle BOC = 180°$ [Sum of three $\angle s$ of a $\Delta = 180°$]

$\Rightarrow \quad 2\angle OBC + 110° = 180°$ [using (1) and (2)]

$\Rightarrow \quad 2\angle OBC = 180° - 110°$

$\Rightarrow \quad \angle OBC = 70° \div 2 = 35°.$

6. Two tangents making an angle of 120° with each other, are drawn to a circle of radius 9 cm, then the length of each tangent is equal to

(a) $\sqrt{3}$ cm (b) $6\sqrt{3}$ cm

(c) $3\sqrt{3}$ cm (d) $2\sqrt{3}$ cm

Solution. Choice (c) is correct.

In Δ's *PAO* and *PBO*, we have

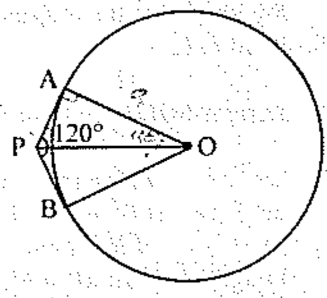
$PA = PB$ [Tangents to a circle from an external point *P*]

$OP = OP$ [Common]

$\angle OAP = \angle OBP$ [Each = 90°]

So, by RHS congruence rule, we have

$\Delta PAO \cong \Delta PBO$



$\Rightarrow \quad \angle OPA = \angle OPB = \frac{1}{2}\angle APB$

$\Rightarrow \quad \angle OPA = \frac{1}{2} \times 120° = 60° \quad$ [$\because \angle APB = 120°$ (given)]

In right-angled triangle *OAP*,

$\tan OPA = \frac{OA}{AP}$

$\Rightarrow \quad \tan 60° = \frac{9}{AP}$ [$OA = 9$ cm = radius (given)]

$\Rightarrow \quad \sqrt{3} = \frac{9}{AP}$

$\Rightarrow \quad AP = \frac{9}{\sqrt{3}} = \frac{3 \times 3}{\sqrt{3}} = 3\sqrt{3}\text{ cm}$

$\Rightarrow$ Length of the each tangent = **$3\sqrt{3}$ cm.**

7. To draw a pair of tangents to a circle which are inclined to each other at an angle of 110°, it is required to draw tangents at end points of those two radii of the circle, the angle between which should be

(*a*) 100° (*b*) 50°

(*c*) 70° (*d*) 80°

Solution. Choice (*c*) is correct.

Since tangent at a point to a circle is perpendicular to the radius through the point.

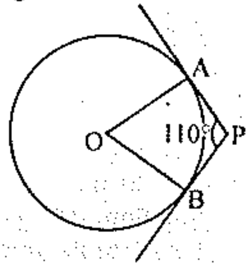
$\therefore \quad OA \perp AP \quad \text{and} \quad OB \perp BP$

$\Rightarrow \quad \angle OAP = 90° \quad \text{and} \quad \angle OBP = 90°$

It is given that a pair of tangents to a circle are inclined to each other at an angle of 110°.

$\therefore \quad \angle APB = 110°$

In quadrilateral *OAPB*, we have



$$\angle OAP + \angle APB + \angle AOB + \angle OBP = 360°$$

$\Rightarrow \quad 90° + 110° + \angle AOB + 90° = 360°$

$\Rightarrow \quad 290° + \angle AOB = 360°$

$\Rightarrow \quad \angle AOB = 360° - 290°$

$\Rightarrow \quad \angle AOB = 70°$

Thus the angle between the two radii of a circle is 70°.

8. A solid sphere of radius *r* is melted and cast into the shape of a solid cone of height *r*, then the radius of the base of cone is :

(*a*) r (*b*) $2r$

(*c*) $3r$ (*d*) $4r$

Solution. Choice (*b*) is correct.

Volume of a solid sphere of radius r = Volume of cone of base radius r_1 and height r

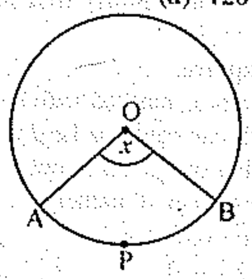
$$\Rightarrow \quad \frac{4}{3}\pi r^3 = \frac{1}{3}\pi r_1^2(r) \Rightarrow 4r^2 = r_1^2 \Rightarrow r_1 = 2r.$$

Thus, the radius of the base of the cone is **2*r***.

9. In figure, *O* is the centre of a circle. The area of sector *OAPB* is $\frac{5}{18}$ of the area of the circle. Find *x*.

(*a*) 70° (*b*) 80°

(*c*) 100° (*d*) 120°



Solution. Choice (*c*) is correct.

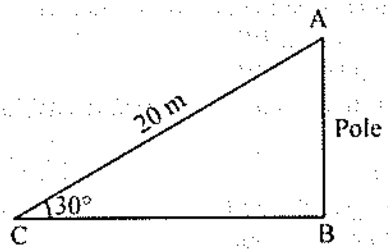
$$\text{Area of a sector } OAPB = \frac{5}{18} \times \text{Area of a circle}$$

$$\Rightarrow \quad \frac{x}{360°} \times \pi r^2 = \frac{5}{18}\pi r^2, \text{ where } r = OA = OB$$

$$\Rightarrow \quad x = \frac{5}{18} \times 360° = 100°$$

Thus, $x = 100°$.

10. A circus artist is climbing a 20 m long rope, which is tightly stretched and tied from the top of a vertical pole to the ground. If the angle made by the rope with the ground is 30°, then the height of the pole is :



(*a*) 5 m (*b*) 10 m

(*c*) 20 m (*d*) 15 m

Solution. Choice (*b*) is correct.

Let AB be the vertical pole and CA be the 20 m long rope such that its one end is tied from the top A of the vertical pole and the other end is tied to a point C on the ground.

In ΔABC, we have

$$\sin 30° = \frac{AB}{AC}$$

$$\Rightarrow \quad \frac{1}{2} = \frac{AB}{20} \qquad [\because AC = 20 \text{ m}]$$

$$\Rightarrow \quad AB = 10 \text{ m}$$

Hence, Height of the pole is **10 m.**

Section 'B'

Question numbers 11 to 18 carry 2 marks each.

11. Cards, marked with numbers 5 to 50 are placed in a box and mixed thoroughly. A card is drawn from the box at random. Find the probability that the number on the taken out card is

(*i*) a prime number less than 10.

(*ii*) a number which is a perfect square.

Solution. Total number of cards in a box, marked with numbers 5 to 50 are 46 (*i.e.*, 50 – 4 = 46).

∴ Total number of outcomes in which one card can be drawn are 46.

(*i*) Let A be the event that the number on the taken out card is "a prime number less than 10". There are 2 prime numbered cards less than 10 namely, 5, 7.

∴ Number of outcomes favourable to event $A = 2$.

Hence, required probability $P(A) = \frac{2}{46} = \frac{1}{23}$

(*ii*) Let B be the event that the number on the taken out card is "a number which is a perfect square".

There are 5 perfect square numbers from 5 to 50, namely

$$9(=3^2),\ 16(=4^2),\ 25(=5^2),\ 36(=6^2),\ 49(=7^2).$$

$\therefore$ Number of outcomes favourable to event $B = 5$

Hence, the required probability $P(B) = \dfrac{5}{46}$

12. For what value of k are the points (1, 1), (3, k) and (– 1, 4) collinear ?

Solution. Given three points (1, 1), (3, k) and (– 1, 4) will be collinear if the area of the triangle formed by them is zero.

$$\text{Area of triangle} = 0$$

$$\Rightarrow \frac{1}{2}[1(k-4)+3(4-1)+(-1)(1-k)] = 0 \qquad \left[\Delta = \frac{1}{2}[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)]\right]$$

$$\Rightarrow \frac{1}{2}[k-4+9-1+k] = 0$$

$$\Rightarrow \frac{1}{2}[2k+4] = 0$$

$$\Rightarrow k = -2$$

Hence, the given points are collinear if $k = -2$.

13. Find the area of the ΔABC with vertices $A(-5, 7)$, $B(-4, -5)$ and $C(4, 5)$.

Solution. The area of the triangle formed by the vertices $A(-5, 7)$, $B(-4, -5)$ and $C(4, 5)$ is given by

$\frac{1}{2}[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)]$, where $x_1 = -5$, $y_1 = 7$, $x_2 = -4$, $y_2 = -5$, $x_3 = 4$, $y_3 = 5$.

$$= \frac{1}{2}[(-5)(-5-5)+(-4)(5-7)+4(7+5)]$$

$$= \frac{1}{2}[50+8+48]$$

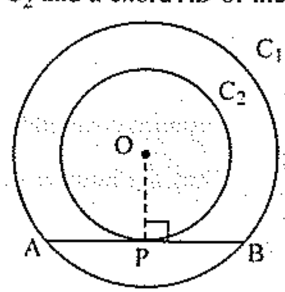
$$= \frac{1}{2}[106]$$

$$= 53$$

Thus, the area of the ΔABC = **53 square units.**

14. Prove that in two concentric circles, the chord of the larger circle, which touches the smaller circle, is bisected at the point of contact.

Solution. Let O be the common centre of two concentric circles C_1 and C_2 and a chord AB of the larger circle C_1 touching the smaller circle C_2 at the point P (see figure).



Join OP.

Since OP is the radius of the smaller circle and AB is a tangent to this circle at a point P

$\therefore \quad OP \perp AB$

We know that the perpendicular drawn from the centre of a circle to any chord of the circle, bisects the chord.

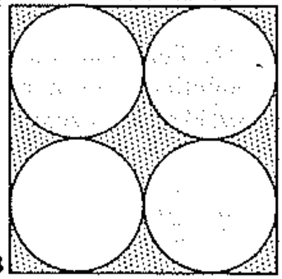
$\therefore \quad AP = BP$

Hence, AB is bisected at the point of contact.

15. Find the area of the shaded region in the figure, where *ABCD* is a square of side 14 cm.

Solution. Side of a square = 14 cm.

$\therefore$ Area of a square $ABCD = (\text{Side})^2 = (14)^2 = 196\ \text{cm}^2$ ...(1)



$$\text{Diameter of each circle} = \frac{\text{Side of a square}}{2} = \frac{14\ \text{cm}}{2} = 7\ \text{cm}$$

$\Rightarrow$ Radius (r) of each circle $= \frac{7}{2}$ cm

$\therefore$ Area of one circle $= \pi r^2$

$$= \pi\left(\frac{7}{2}\right)^2\ \text{cm}^2$$

$$= \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}\ \text{cm}^2$$

$$= \frac{154}{4}\ \text{cm}^2 = \frac{77}{2}\ \text{cm}^2$$

Thus, area of the four circles = 4 times the area of one circle

$$= 4 \times \frac{77}{2}\ \text{cm}^2$$

$$= 154\ \text{cm}^2 \quad ...(2)$$

Hence, area of the shaded region

= Area of a square – Area of the four circles

$= 196\ \text{cm}^2 - 154\ \text{cm}^2$ [using (1) and (2)]

$= \mathbf{42\ cm^2}$.

16. The diameter of the driving wheel of a bus is 140 cm. How many revolutions per minute must the wheel make in order to keep a speed of 66 km per hour ?

Solution. We have

Speed of the bus = 66 km/h

Distance covered by the wheel in 60 minutes = $66 \times 1000 \times 100$ cm

$\Rightarrow$ Distance covered by the wheel in 1 minute $= \frac{66 \times 1000 \times 100}{60} = 110000$ cm

Circumference of the wheel $= 2\pi r$

$= \pi(2r) = \pi(\text{diameter})$

$= \frac{22}{7} \times 140 = 440$ cm

Distance covered by the wheel in one revolution = 440 cm

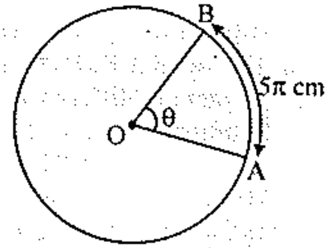
Thus, the number of revolutions in one minute $= \frac{\text{Distance covered by the wheel in one minute}}{\text{Distance covered by the wheel in one revolution}}$

$$= \frac{110000}{440} = \mathbf{250}$$

Or

An arc of a circle is of length 5π cm and the sector it bounds has an area of 20π cm^2. Find the radius of the circle.

Solution. We know that



Length of arc $AB = 5\pi$ cm (given)

$$\Rightarrow \quad \frac{\theta}{360°} \times 2\pi r = 5\pi$$

$$\Rightarrow \quad \frac{\theta}{180°} \times r = 5 \quad ...(1)$$

Area of a sector $OAB = 20\pi$ cm^2 (given)

$$\Rightarrow \quad \frac{\theta}{360°} \times \pi r^2 = 20\pi$$

$$\Rightarrow \quad \frac{\theta}{360°} \times r^2 = 20 \quad ...(1)$$

$$\Rightarrow \quad \left(\frac{\theta}{180°} \times r\right) \times \frac{r}{2} = 20$$

$$\Rightarrow \quad 5 \times \frac{r}{2} = 20 \quad \text{[using (1)]}$$

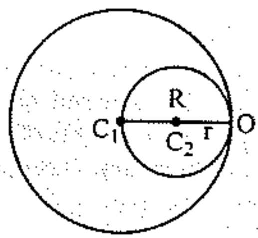
$$\Rightarrow \quad r = \frac{20 \times 2}{5} \text{ cm}$$

$$\Rightarrow \quad r = 8 \text{ cm}$$

17. Two circles touch internally. The sum of their areas is 116π cm^2 and distance between their centres is 6 cm. Find the radii of the circles.

Solution. Let R and r be the radii of the circles having centres C_1 and C_2 respectively, then

Sum of the areas of two circles $= 116\pi$ cm^2



$$\Rightarrow \quad \pi R^2 + \pi r^2 = 116\pi$$

$$\Rightarrow \quad \pi(R^2 + r^2) = 116\pi$$

$$\Rightarrow \quad R^2 + r^2 = 116 \quad ...(1)$$

Distance between the centres $= C_1C_2$

$$\Rightarrow \quad 6 \text{ cm (given)} = OC_1 - OC_2$$

$$\Rightarrow \quad 6 = R - r$$

$$\Rightarrow \quad R = 6 + r \quad ...(2)$$

From (1) and (2), we have

$$(6 + r)^2 + r^2 = 116$$

$$\Rightarrow \quad 36 + r^2 + 12r + r^2 = 116$$

$$\Rightarrow \quad 2r^2 + 12r - 80 = 0$$

$$\Rightarrow \quad r^2 + 6r - 40 = 0$$

$$\Rightarrow \quad r^2 + 10r - 4r - 40 = 0$$

$$\Rightarrow \quad (r^2 + 10r) - (4r + 40) = 0$$

$$\Rightarrow \quad r(r + 10) - 4(r + 10) = 0$$

$$\Rightarrow \quad (r + 10)(r - 4) = 0$$

$$\Rightarrow \quad r - 4 = 0 \text{ and } r + 10 = 0$$

$$\Rightarrow \quad r = 4 \text{ and } r = -10$$

$\Rightarrow$ $r = 4$ ($r = -10$ cannot be possible)

Putting $r = 4$ in (2), we get

$$R = 6 + 4 = 10$$

Hence, radii of the given circles are **10 cm** and **4 cm** respectively.

18. In an A.P., the first term is 22, nth term is -11 and sum to first n terms is 66. Find n and d, the common difference.

Solution. Let $a = 22$ be the first term and d be the common difference of an A.P.,

$22, 22 + d, 22 + 2d, \ldots\ldots$

Then, $t_n = n$th term of an A.P. $= a + (n-1)d$

$\Rightarrow$ $-11 = 22 + (n-1)d$

$\Rightarrow$ $(n-1)d = -11 - 22$

$\Rightarrow$ $(n-1)d = -33$ ...(1)

and $S_n =$ Sum of n terms of an A.P. $= \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow$ $66 = \frac{n}{2}[2 \times 22 + (n-1)d]$

$\Rightarrow$ $132 = n[44 + (n-1)d]$

$\Rightarrow$ $132 = n[44 - 33]$ [using (1)]

$\Rightarrow$ $132 = 11n$

$\Rightarrow$ $n = 132 \div 11 = 12$

Substituting $n = 12$ in (1), we get

$(12 - 1)d = -33$

$\Rightarrow$ $11d = -33$

$\Rightarrow$ $d = -3$

Thus, $\boldsymbol{n = 12}$ and $\boldsymbol{d = -3}$.

Section 'C'

Question numbers 19 to 28 carry 3 marks each.

19. The sum of the third and seventh terms of an A.P. is 6 and their product is 8. Find the sum of first ten terms of the A.P.

Solution. Let a and d be the first term and common difference of an A.P., $a, a + d, a + 2d, \ldots$

We have

$$t_3 + t_7 = 6 \text{ (given) and } t_3 t_7 = 8$$

Now, $t_3 + t_7 = 6$

$\Rightarrow$ $(a + 2d) + (a + 6d) = 6$

$\Rightarrow$ $2a + 8d = 6$

$\Rightarrow$ $a + 4d = 3$

$\Rightarrow$ $a = 3 - 4d$ ...(1)

and $t_3 . t_7 = 8$

$\Rightarrow$ $(a + 2d)(a + 6d) = 8$

$\Rightarrow$ $[(3 - 4d) + 2d][(3 - 4d) + 6d] = 8$ [using (1)]

$\Rightarrow$ $(3 - 2d)(3 + 2d) = 8$

$\Rightarrow$ $9 - 4d^2 = 8$

$\Rightarrow$ $4d^2 = 9 - 8$

$\Rightarrow \quad 4d^2 = 1$

$\Rightarrow \quad d^2 = \frac{1}{4} = \left(\frac{1}{2}\right)^2$

$\Rightarrow \quad d = \pm\frac{1}{2}$

When $d = \frac{1}{2}$, then $a = 3 - 4 \times \frac{1}{2} = 1$ [using (1)]

$$\therefore \quad S_{10} = \frac{10}{2}\left[2 \times 1 + (10-1) \times \frac{1}{2}\right]$$

$$= 5\left[2 + \frac{9}{2}\right] = 5\left[\frac{13}{2}\right]$$

$$= \frac{65}{2}$$

When $d = -\frac{1}{2}$, then $a = 3 - 4 \times \left(-\frac{1}{2}\right) = 5$ [using (1)]

$$\therefore \quad S_{10} = \frac{10}{2}\left[2 \times 5 + (10-1) \times \left(-\frac{1}{2}\right)\right]$$

$$= 5\left[10 - \frac{9}{2}\right] = 5\left[\frac{11}{2}\right]$$

$$= \frac{55}{2}.$$

Or

Find the sum of all three digits natural numbers, which are divisible by 7.

Solution. The smallest and the largest numbers of three digits, which are divisible by 7 are 105 and 994 respectively. So, the A.P. of three digit numbers which are divisible by 7 is

105, 112, 119, ... 994 ...(1)

Clearly, first term $(a) = 105$

and common difference $(d) = 112 - 105 = 7$

Let there be n terms in A.P. (1), then

nth term $(a_n) = a + (n-1)d$

$\Rightarrow \quad 994 = 105 + (n-1)7$

$\Rightarrow \quad 889 = (n-1)7$

$\Rightarrow \quad n - 1 = 889 \div 7$

$\Rightarrow \quad n - 1 = 127$

$\Rightarrow \quad n = 128$

$\therefore$ Required sum of n terms $= \frac{n}{2}$[First term + Last term]

$$= \frac{128}{2}[105 + 994]$$

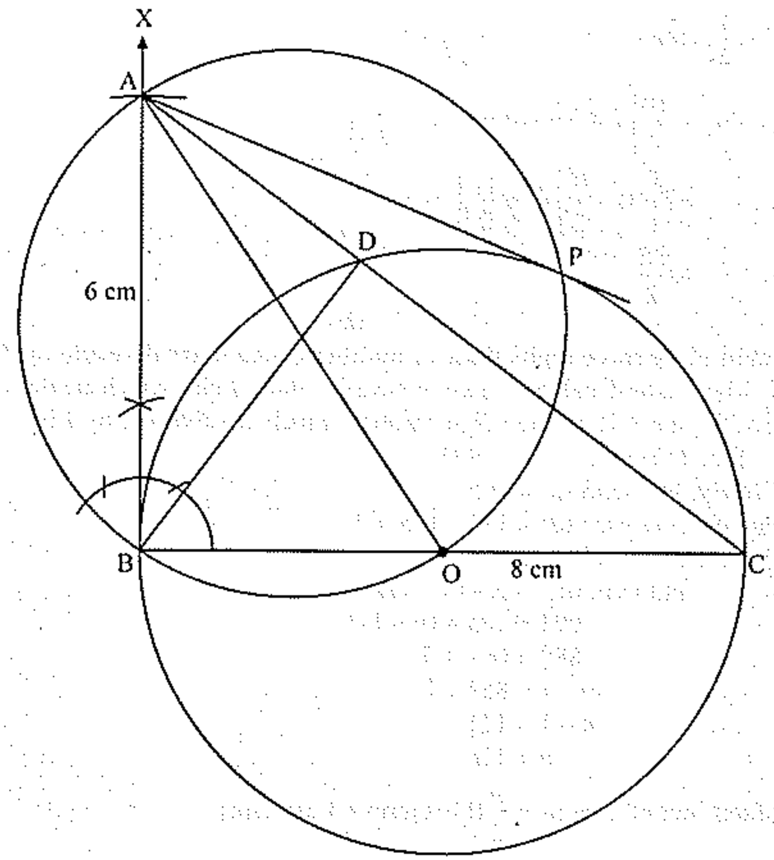
$$= 64(1099)$$

$$= \mathbf{70336}$$

20. Let ABC be a right triangle in which $AB = 6$ cm, $BC = 8$ cm and $\angle B = 90°$. BD is the perpendicular from B on AC. The circle through B, C and D is drawn. Construct the tangents from A to this circle.

Solution. Steps of Construction :

1. Draw a line segment $BC = 8$ cm.
2. At B construct $\angle CBX = 90°$.
3. With B as centre and radius = 6 cm, draw an arc intersecting the line BX at A.
4. Join AC to obtain ΔABC.
5. Draw perpendicular BD from B on AC.
6. Let O be the mid-point of BC. Draw a circle with centre O and radius $OB = OC$. This circle will pass through the point D.
7. Join AO. Draw a circle with AO as diameter. This circle cuts the circle drawn in step 6 at B and P.
8. Joint AP. AP and AB are desired tangents drawn from A to the circle passing through B, C and D.



21. Savita and Hamida are friends. What is the probability that both will have

(i) different birthdays ?

(ii) the same birthday ? (ignoring a leap year)

Solution. Savita's birthday can be anyone of the 365 days of the year.

Similarly, Hamida's birthday can be anyone of the 365 days of the year.

We assume that these 365 outcomes are equally likely.

(*i*) If Hamida's birthday is different from Savita's, the number of favourable outcomes for her birthday is

$$365 - 1 = 364.$$

So, P(Hamida's birthday is different from Savita's birthday)

$$= \frac{364}{365}$$

(*ii*) P(Savita and Hamida have the same birthday)

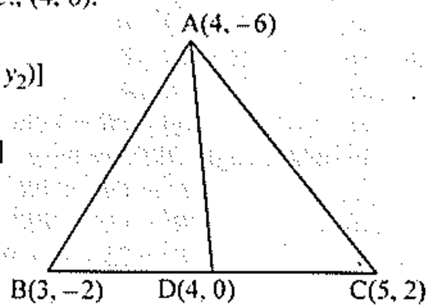
$$= 1 - P(\text{both have different birthdays})$$

$$= 1 - \frac{364}{365} \qquad [\text{using } P(\overline{E}) = 1 - P(E)]$$

$$= \frac{1}{365}.$$

22. You have studied in Class IX, that a median of a triangle divides it into two triangles of equal areas. Verify this result for ΔABC whose vertices are $A(4, -6)$, $B(3, -2)$ and $C(5, 2)$.

Solution. Given, $A(4, -6)$, $B(3, -2)$ and $C(5, 2)$ be the vertices of ΔABC. Let D be the mid-point of the side BC, then the coordinates of D are $\left(\frac{3+5}{2}, \frac{-2+2}{2}\right)$ *i.e.*, (4, 0).



$$\therefore \quad \text{Area of } \Delta ABC = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

$$= \frac{1}{2}[4(-2-2) + 3(2+6) + 5(-6+2)]$$

where $x_1 = 4$, $x_2 = 3$, $x_3 = 5$ and $y_1 = -6$, $y_2 = -2$, $y_3 = 2$

$$= \frac{1}{2}[-16 + 24 - 20]$$

$$= \frac{1}{2}[-12]$$

$$= -6$$

$$= 6 \text{ sq. units.} \qquad [\text{Taking numercial value as area cannot be} - \text{ve}]$$

Also, area of $\Delta ABD = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

$$= \frac{1}{2}[4(-2-0) + 3(0+6) + 4(-6+2)]$$

where $x_1 = 4$, $x_2 = 3$, $x_3 = 4$ and $y_1 = -6$, $y_2 = -2$, $y_3 = 0$

$$= \frac{1}{2}[-8 + 18 - 16]$$

$$= \frac{1}{2}[-6]$$

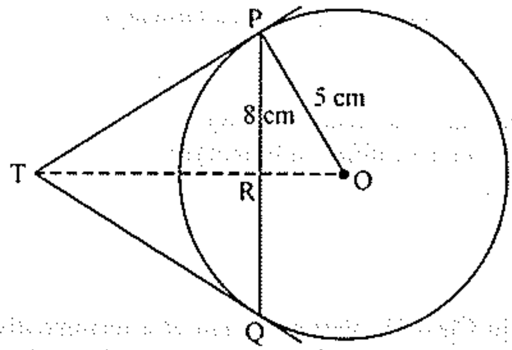
$$= -3$$

$$= 3 \text{ sq. units} \qquad [\text{Taking numercial value as area cannot be} - \text{ve}]$$

Thus, $\frac{\text{area of } \Delta ABC}{\text{area of } \Delta ABD} = \frac{6}{3} = \frac{2}{1}$

$\Rightarrow$ **area of ΔABC = 2 (area of ΔABD).**

23. PQ is a chord of length 8 cm of a circle of radius 5 cm. The tangents at P and Q intersect at a point T (see figure). Find the length of TP.



Solution. Since OT is perpendicular bisector of PQ, therefore

$PR = RQ$ ...(1)

But $PQ = 8$ cm (given)

$\Rightarrow PR + RQ = 8$

$\Rightarrow PR + PR = 8$ [using (1)]

$\Rightarrow PR = 4$

$\Rightarrow RQ = PR = 4$ cm ...(2) [using (1)]

In right triangle ORP, we have

$OP^2 = OR^2 + PR^2$

$\Rightarrow OR^2 = OP^2 - PR^2$

$\Rightarrow OR^2 = 25 - 16 = 9$

$\Rightarrow OR = 3$ cm ...(3)

Since TP is a tangent to circle with centre O and OP is its radius, therefore,

$OP \perp TP$

[$\because$ The tangent at any point of a circle is perpendicular to the radius through the point of contact]

$\therefore \angle OPT = 90°$

In right triangle OPT, we have

$OT^2 = PT^2 + OP^2$

$\Rightarrow (TR + OR)^2 = PT^2 + 25$

$\Rightarrow (TR + 3)^2 = PT^2 + 25$ ...(4)

In right triangle PRT, we have

$PT^2 = TR^2 + PR^2$

$\Rightarrow PT^2 = TR^2 + 16$ ...(5) [using (2)]

From (4) and (5), we have

$(TR + 3)^2 = (TR^2 + 16) + 25$

$\Rightarrow TR^2 + 9 + 6TR = TR^2 + 41$

$\Rightarrow 6TR = 32$

$$\Rightarrow \quad TR = \frac{16}{3} \qquad \ldots(6)$$

Now, from (5) and (6), we get

$$PT^2 = \left(\frac{16}{3}\right)^2 + 16 = \frac{256}{9} + 16$$

$$\Rightarrow \quad PT^2 = \frac{256 + 144}{9} = \frac{400}{9}$$

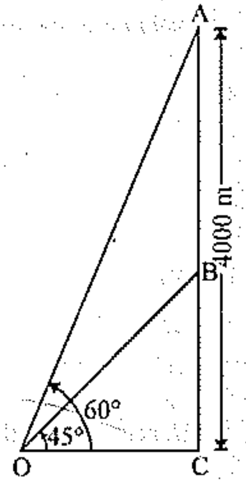
$$\Rightarrow \quad TP^2 = \left(\frac{20}{3}\right)^2$$

$$\Rightarrow \quad TP = \frac{20}{3} \text{ cm}$$

24. An aeroplane when flying at a height of 4000 m from the ground passes vertically above another aeroplane at an instant when the angles of elevation of the two aeroplanes from the same point on the ground are 60° and 45° respectively. Find the vertical distance between the two aeroplanes at that instant. [Take $\sqrt{3} = 1.73$]

Solution. Let A be the first aeroplane, vertically above another aeroplane B such that $AC = 4000$ m be the height of the first aeroplane from the ground.

Let O be a point on the ground such that the angle of elevation of the two aeroplanes A and B be $\angle AOC = 60°$ and $\angle BOC = 45°$.



In right ΔAOC, we have

$$\tan 60° = \frac{AC}{OC}$$

$$\Rightarrow \quad \sqrt{3} = \frac{AC}{OC} \qquad [\because \tan 60° = \sqrt{3}]$$

$$\Rightarrow \quad OC = \frac{4000}{\sqrt{3}} \qquad [\because AC = 4000 \text{ m}]$$

$$\Rightarrow \quad OC = \frac{4000\sqrt{3}}{\sqrt{3} \times \sqrt{3}} = \frac{4000\sqrt{3}}{3} \text{ m} \qquad \ldots(1)$$

Again, in right ΔBOC, we have

$$\tan 45° = \frac{BC}{OC}$$

$$\Rightarrow \quad 1 = \frac{BC}{OC} \qquad [\because \tan 45° = 1]$$

$$\Rightarrow \quad BC = OC$$

$$\Rightarrow \quad BC = \frac{4000\sqrt{3}}{3} \qquad [\text{using (1)}]$$

$$\Rightarrow \quad BC = \frac{4000 \times 1.73}{3} \qquad [\because \sqrt{3} = 1.73]$$

$$\Rightarrow \quad BC = \frac{6920}{3} \text{ m}$$

$$\Rightarrow \quad BC = 2306.7 \text{ m} \qquad \ldots(2)$$

Thus, $AB = AC - BC$

$\Rightarrow \quad AB = (4000 - 2306.7)$ m

$\Rightarrow \quad AB = 1693.3$ m

Hence, the vertical distance between the two aeroplanes is **1693.3 m.**

25. A solid iron rectangular block of dimensions 4.4 m, 2.6 m and 1 m is cast into a hollow cylindrical pipe of internal radius 30 cm and thickness 5 cm. Find the length of the pipe.

Solution. We have

$$\text{Volume of the block} = (4.4 \times 2.6 \times 1)\ \text{m}^3 = (440 \times 260 \times 100)\ \text{cm}^3$$

$$\text{Internal radius of the pipe } (r) = 30 \text{ cm}$$

$$\text{External radius of the pipe } (R) = r + \text{thickness of the pipe} = (30 + 5)\text{ cm} = 35 \text{ cm}$$

Let h be the length of the pipe, then

$$\begin{aligned}\text{Volume of the iron in cylindrical pipe} &= \text{External volume} - \text{Internal volume}\\ &= \pi R^2 h - \pi r^2 h\\ &= \pi h(R^2 - r^2)\\ &= \pi h(35^2 - 30^2)\\ &= \pi h(35 + 30)(35 - 30)\\ &= 325\pi h \text{ cm}^3\end{aligned}$$

It is given that the solid iron rectangular block is cast into a hollow cylindrical pipe. Therefore,

Volume of the iron in cylindrical pipe = Volume of the iron in rectangular block

$$\Rightarrow \quad 325\pi h = 440 \times 260 \times 100$$

$$\Rightarrow \quad h = \frac{440 \times 260 \times 100 \times 7}{325 \times 22} \text{ cm}$$

$$\Rightarrow \quad h = \frac{20 \times 260 \times 100 \times 7}{325} \text{ cm}$$

$$\Rightarrow \quad h = \frac{20 \times (4 \times 65) \times 100 \times 7}{(65 \times 5)} \text{ cm}$$

$$\Rightarrow \quad h = (800 \times 2 \times 7) \text{ cm}$$

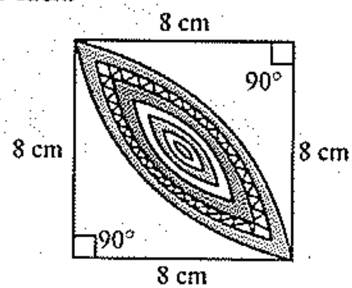
$$\Rightarrow \quad h = 11200 \text{ cm}$$

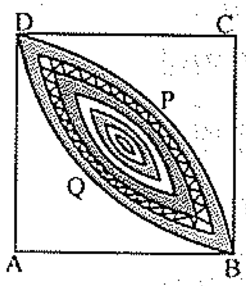
$$\Rightarrow \quad h = 112 \text{ m}$$

Hence, the length of the pipe is **112 m.**

Or

Calculate the area of the designed region in figure, common between the two quadrants of circles of radius 8 cm each.





Solution. From the figure, we have

Area of the shaded region

= (Area of the quadrant *ABPD* – Area of ΔABD)

+ (Area of the quadrant *CBQD* – Area of ΔCBD)

$$= \left[\frac{1}{4}\pi\times(AB)^2 - \frac{1}{2}\times AB\times AD\right] + \left[\frac{1}{4}\pi\times(BC)^2 - \frac{1}{2}BC\times CD\right]$$

$$= \frac{\pi}{4}[(AB)^2 + (BC)^2] - \frac{1}{2}\times AB\times AD - \frac{1}{2}\times BC\times CD$$

$$= \left\{\frac{\pi}{4}[(8)^2 + (8)^2] - \frac{1}{2}\times 8\times 8 - \frac{1}{2}\times 8\times 8\right\}\text{ cm}^2$$

$$= \left[\frac{\pi}{4}[64+64] - 8\times 8\right]\text{ cm}^2$$

$$= \left[\frac{\pi}{4}\times 128 - 64\right]\text{ cm}^2$$

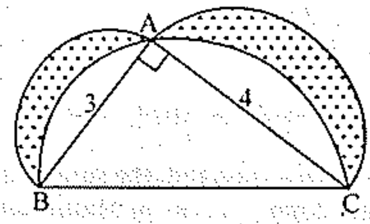
$$= \left(\frac{22}{7}\times 32 - 64\right)\text{ cm}^2$$

$$= 64\times\left(\frac{11}{7} - 1\right)\text{ cm}^2$$

$$= 64\times\left(\frac{4}{7}\right)\text{ cm}^2$$

$$= \mathbf{\frac{256}{7}}\textbf{ cm}^2$$

26. In figure, *ABC* is a right-angled triangle right-angled at *A*. Semicircles are drawn on *AB*, *AC* and *BC* as diameters. Find the area of the shaded region.



Solution. We have

In right ΔABC,

$$BC^2 = AB^2 + AC^2$$

$$= 9 + 16 = 25$$

$\Rightarrow$ $BC = 5$ cm

Required shaded area

= Area of a semi-circle with $AB = 3$ cm as a diameter + Area of a semi-circle with $AC = 4$ cm as diameter + Area of right-angled triangle right-angled at *A* – Area of a semi-circle with $BC = 5$ cm as diameter

$$= \frac{1}{2}\pi\times\left(\frac{AB}{2}\right)^2 + \frac{1}{2}\pi\times\left(\frac{AC}{2}\right)^2 + \frac{1}{2}AB\times AC - \frac{1}{2}\pi\times\left(\frac{BC}{2}\right)^2$$

$$= \left[\frac{\pi}{2}\times\left(\frac{3}{2}\right)^2 + \frac{\pi}{2}\times\left(\frac{4}{2}\right)^2 + \frac{1}{2}\times 3\times 4 - \frac{\pi}{2}\times\left(\frac{5}{2}\right)^2\right]$$

$$= \left\{\frac{\pi}{2}\left[\frac{9}{4} + \frac{16}{4} - \frac{25}{4}\right] + \frac{1}{2}\times 3\times 4\right\}\text{ sq. units}$$

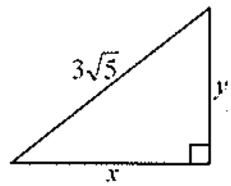
$$= \left\{\frac{\pi}{2}\left[\frac{25}{4} - \frac{25}{4}\right] + 6\right\} \text{ sq. units}$$

$$= \left[\frac{\pi}{2} \times (0) + 6\right] \text{sq. units}.$$

= 6 sq. units.

27. The hypotenuse of a right triangle is $3\sqrt{5}$ cm. If the smaller side is tripled and the larger side is doubled, the new hypotenuse will be 15 cm. Find the length of each side.

Solution. Let the smaller side and larger side of a right triangle be x cm and y cm respectively, then



$$x^2 + y^2 = (3\sqrt{5})^2 \qquad \text{[Pythagoras theorem]}$$

$$\Rightarrow \quad x^2 + y^2 = 45 \qquad \ldots(1)$$

It is given that : when the smaller side is tripled and the larger side be doubled, the new hypotenuse is 15 cm.

$$(3x)^2 + (2y)^2 = (15)^2$$

$$\Rightarrow \quad 9x^2 + 4y^2 = 225 \qquad \ldots(2)$$

Multiplying (1) by 4 and subtracting from (2), we get

$$(9x^2 + 4y^2) - (4x^2 + 4y^2) = 225 - 4 \times 45$$

$$\Rightarrow \quad 5x^2 = 225 - 180$$

$$\Rightarrow \quad 5x^2 = 45$$

$$\Rightarrow \quad x^2 = 9$$

$$\Rightarrow \quad x = \pm 3$$

$\Rightarrow \quad x = 3$, as the length of a side cannot be negative

Putting $x = 3$ in (1), we get

$$(3)^2 + y^2 = 45$$

$$\Rightarrow \quad y^2 = 45 - 9$$

$$\Rightarrow \quad y^2 = 36$$

$$\Rightarrow \quad y = \pm 6$$

$\Rightarrow \quad y = 6$, as the length of a side cannot be negative

Hence, the length of the smaller side is **3 cm** and the length of the larger side is **6 cm**.

Or

In a class test, the sum of the marks obtained by *P* in Mathematics and Science is 28. Had he got 3 more marks in Mathematics and 4 marks less in Science, the product of marks obtained in the two subjects would have been 180. Find the marks obtained in the two subjects separately.

Solution. Let the marks obtained by P in Mathematics be x and in Science y.

According to the first condition : "The sum of the marks obtained by P in Mathematics and Science is 28".

$$\therefore \quad x + y = 28 \Rightarrow y = 28 - x \qquad \ldots(1)$$

When he got 3 more marks in Mathematics, then marks in Mathematics = $x + 3$

When he got 4 marks less in Science, then marks in Science = $y - 4$.

According to the second condition : "Had he got 3 more marks in Mathematics and 4 marks less in Science, the product of his marks would have been 180".

$$\therefore \quad (x + 3)(y - 4) = 180$$

$$\Rightarrow \quad (x + 3)(28 - x - 4) = 180 \qquad \text{[using (1)]}$$

$\Rightarrow \quad (x + 3)(24 - x) = 180$

$\Rightarrow \quad 24x - x^2 + 72 - 3x = 180$

$\Rightarrow \quad x^2 - 21x + 108 = 0$

$\Rightarrow \quad x^2 - 12x - 9x + 108 = 0$

$\Rightarrow \quad x(x - 12) - 9(x - 12) = 0$

$\Rightarrow \quad (x - 12)(x - 9) = 0$

$\Rightarrow$ Either $x - 12 = 0$ or $x - 9 = 0$

$\Rightarrow$ Either $x = 12$ or $x = 9$

When $x = 12$, then from (1), $y = 28 - 12 = 16$

When $x = 9$, then from (1), $y = 28 - 9 = 19$

Thus, Marks in Mathematics = **12**, Marks in Science = **16**

or

Marks in Mathematics = **9**, Marks in Science = **19**.

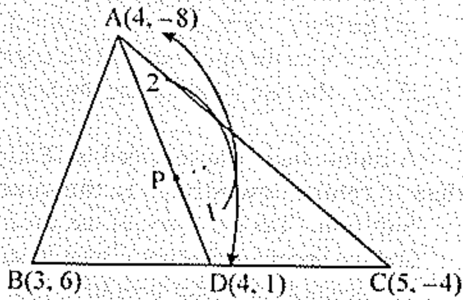
28. If $A(4, -8)$, $B(3, 6)$ and $C(5, -4)$ are the vertices of a ΔABC, D is the mid-point of BC and P is a point on AD joined such that $\frac{AP}{PD} = 2$, find the coordinates of P.

Solution. Let $A(4, -8)$, $B(3, 6)$ and $C(5, -4)$ are the vertices of ΔABC.

Also D is the mid-point of BC, then the coordinates of D are

$$\left(\frac{3+5}{2}, \frac{6-4}{2}\right)$$

i.e., (4, 1)



We have

$$\frac{AP}{PD} = \frac{2}{1}$$

$\Rightarrow \quad AP : PD = 2 : 1$

$\Rightarrow P$ divides A and D internally in the ratio 2 : 1

So, the coordinates of P are

$$\left(\frac{2 \times 4 + 1 \times 4}{2+1}, \frac{2 \times 1 + 1 \times (-8)}{2+1}\right)$$

i.e., $\left(\frac{8+4}{3}, \frac{2-8}{3}\right)$

i.e., $(4, -2)$

Thus, the coordinates of P are **(4, – 2)**.

Section 'D'

Question numbers 29 to 34 carry 4 marks each.

29. If a student had walked 1 km/h faster, he would have taken 15 minutes less to walk 3 km. Find the rate at which he was walking.

Solution. Let the rate at which a student walking = x km/h

Then the increased rate at which a student walking = $(x + 1)$ km/h

Walking distance = 3 km

Time taken by the student with a speed of x km/h $= \frac{3}{x}$ h

Time taken by the student with a speed of $(x + 1)$ km/h $= \frac{3}{(x+1)}$ h

According to the given condition, we have

$$\frac{3}{x} - \frac{3}{x+1} = \frac{15}{60}$$

$$\Rightarrow \quad \frac{3}{x} - \frac{3}{x+1} = \frac{1}{4}$$

$$\Rightarrow \quad \frac{3x+3-3x}{x(x+1)} = \frac{1}{4}$$

$$\Rightarrow \quad 12 = x^2 + x$$

$$\Rightarrow \quad x^2 + x - 12 = 0$$

$$\Rightarrow \quad x^2 + 4x - 3x - 12 = 0$$

$$\Rightarrow \quad x(x+4) - 3(x+4) = 0$$

$$\Rightarrow \quad (x+4)(x-3) = 0$$

$\Rightarrow$ Either $x + 4 = 0$ or $x - 3 = 0$

$\Rightarrow$ Either $x = -4$ or $x = 3$

$\Rightarrow x = 3$, as the speed cannot be negative.

Hence, the rate at which the student was walking = **3 km/h.**

Or

A motor boat whose speed is 18 km/h in still water takes 1 hour more to go 24 km upstream than to return downstream to the same spot. Find the speed of the stream.

Solution. Let the speed of the stream be x km/h

and the speed of the motor boat in still water = 18 km/h

$\therefore$ Speed of the motor boat upstream = $(18 - x)$ km/h

and speed of the motor boat downstream = $(18 + x)$ km/h

Thus, time taken in going 24 km upstream $= \frac{24}{18-x}$ hours

and time taken for returning 24 km downstream $= \frac{24}{18+x}$ hours

But the motor boat takes 1 hour more in going 24 km upstream and returning back 24 km downstream.

According to the given condition, we have

$$\frac{24}{18-x} - \frac{24}{18+x} = 1 \text{ h (given)}$$

$$\Rightarrow \quad 24 \times \left[\frac{1}{18-x} - \frac{1}{18+x}\right] = 1$$

$$\Rightarrow \quad 24 \times \left[\frac{18+x-18+x}{(18-x)(18+x)}\right] = 1$$

$\Rightarrow \quad 24 \times 2x = (18)^2 - x^2$

$\Rightarrow \quad 48x = 324 - x^2$

$\Rightarrow \quad x^2 + 48x - 324 = 0$

$\Rightarrow \quad x^2 + 54x - 6x - 324 = 0$

$\Rightarrow \quad (x^2 + 54x) - (6x + 324) = 0$

$\Rightarrow \quad x(x + 54) - 6(x + 54) = 0$

$\Rightarrow \quad (x + 54)(x - 6) = 0$

$\Rightarrow$ Either $x + 54 = 0$ or $x - 6 = 0$

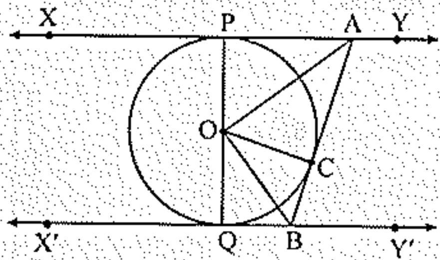
$\Rightarrow$ Either $x = -54$ or $x = 6$

Since x is the speed of the stream, it cannot be negative. So, we ignore the root $x = -54$.

Therefore, $x = 6$ gives the speed of the stream as **6 km/h**.

30. Prove that the intercept of a tangent between two parallel tangents to a circle subtends a right angle at the centre of the circle.

Solution. Let XY and $X'Y'$ be two parallel tangents to a circle with centre O. Let another tangent at C be intercepted between two parallel tangents XY and $X'Y'$ intersecting XY at A and $X'Y'$ at B.



Since $XY \parallel X'Y'$ and AB is the transversal, therefore

$\angle PAC + \angle CBQ = 180°$ ...(1)

[$\because$ Sum of the interior angles on the same side of a transversal is 180°]

In ΔPAO and ΔAOC, we have

$AP = AC$ [Tangents drawn from an external point are equal in length]

$AO = AO$ [Common]

$PO = OC$ [Radii of the same circle]

So, by SSS criterion of congruence, we have

$\Delta PAO \cong \Delta AOC$

$\therefore \quad \angle PAO = \angle CAO$ ...(2) [CPCT]

Similarly, we can prove

$\angle CBO = \angle QBO$ ...(3)

Now, (1) can be written as

$(\angle PAO + \angle CAO) + (\angle CBO + \angle QBO) = 180°$

$\Rightarrow (\angle CAO + \angle CAO) + (\angle CBO + \angle CBO) = 180°$ [using (2) and (3)]

$\Rightarrow \quad 2(\angle CAO) + 2(\angle CBO) = 180°$

$\Rightarrow \quad 2(\angle CAO + \angle CBO) = 180°$

$\Rightarrow \quad \angle CAO + \angle CBO = 90°$ ...(4)

In ΔAOB, we have

$\angle CAO + \angle CBO + \angle AOB = 180°$

$\Rightarrow \quad (\angle CAO + \angle CBO) + \angle AOB = 180°$

$\Rightarrow \quad 90° + \angle AOB = 180°$ [using (4)]

$\Rightarrow \quad \angle AOB = 180° - 90° = 90°$

Hence, $\angle AOB$ = **90°**, *i.e.*, the intercept AB of a tangent between two parallel tangents subtends a right angle at the centre O of the circle.

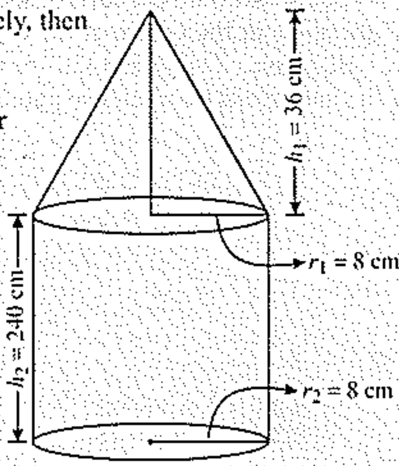
31. An iron pillar has lower part in the form of a right circular cylinder and the upper part in the form of a right circular cone. The radius of the base of each of the cone and a cylinder is 8 cm. The cylindrical part is 240 cm high and conical part is 36 cm high. Find the weight of the pillar if 1 cm³ of iron weighs 7.5 grams. $\left[\text{Take } \pi = \frac{22}{7}\right]$

Solution. Let r_1 and r_2 be the radii of the cone and cylinder respectively, then

$r_1 = r_2 = 8$ cm

Let h_1 and h_2 be the heights of the cone and cylinder respectively, then

$h_1 = 36$ cm and $h_2 = 240$ cm



Total volume of the iron pillar

$= \text{Volume of the cone} + \text{Volume of the cylinder}$

$= \frac{1}{3}\pi r_1^2 h_1 + \pi r_2^2 h_2$

$= \frac{\pi}{3} \times (8)^2 \times 36 + \pi \times (8)^2 \times 240$

$= \pi \times 64 \times 12 + \pi \times 64 \times 240$

$= 64\pi[12 + 240]$

$= 64 \times \frac{22}{7} \times 252$

$= 64 \times 22 \times 36 \text{ cm}^3$

$= 50688 \text{ cm}^3$

Hence, total weight of the iron pillar

$= 50688 \times 7.5$ g [$\because$ Weight of 1 cm³ = 7.5 grams (given)]

$= 50688 \times \frac{15}{2}$ g

$= 25344 \times 15$ g

$= 380160$ g

$= \frac{380160}{1000}$ kg

$=$ **380.16 kg.**

Or

A container (open at the top) made up of a metal sheet is in the form of a frustum of a cone of height 16 cm with radii of its lower and upper ends as 8 cm and 20 cm, respectively. Find

(*i*) the cost of milk when it is completely filled with milk at the rate of ₹ 15 per litre.

(*ii*) the cost of metal sheet used, if it costs ₹ 5 per 100 cm². **[Take π = 3.14]**

Solution. The container is a frustum of a cone of height 16 cm with radii of its upper and lower ends are 20 cm and 8 cm respectively.

$\therefore h = 16$ cm, $R = 20$ cm and $r = 8$ cm

Slant height of the frustum of cone is given by

$$l = \sqrt{h^2 + (R-r)^2}$$
$$= \sqrt{(16)^2 + (20-8)^2} \text{ cm}$$
$$= \sqrt{(16)^2 + (12)^2} \text{ cm}$$
$$= \sqrt{256 + 144} \text{ cm}$$
$$= \sqrt{400} \text{ cm} = 20 \text{ cm}$$

Now, the volume of the container, *i.e.*, the frustum of the cone

$$= \frac{1}{3} \times \pi h[R^2 + Rr + r^2]$$
$$= \frac{1}{3} \times 3.14 \times 16 \times [(20)^2 + (20)(8) + (8)^2] \text{ cm}^3$$
$$= \frac{1}{3} \times 3.14 \times 16 \times [400 + 160 + 64] \text{ cm}^3$$
$$= \frac{1}{3} \times 3.14 \times 16 \times [624] \text{ cm}^3$$
$$= 3.14 \times 16 \times 208 \text{ cm}^3$$
$$= 3.14 \times 3{,}328 \text{ cm}^3$$
$$= 10{,}449.92 \text{ c.c.}$$

$$= \frac{10{,}449.92}{1{,}000} \text{ litres} \qquad \left[\begin{array}{l} \because 1{,}000 \text{ c.c.} = 1 \text{ litre} \\ \Rightarrow \quad 1 \text{ c.c.} = \frac{1}{1000} \text{ litre} \end{array}\right]$$

$$= 10.44992 \text{ litres}$$
$$= 10.45 \text{ litres approx.}$$

Therefore, the quantity of milk = Volume of the container = 10.45 litres approx.

Thus, cost of the milk @ ₹ 15 per litre

= ₹ (10.45 × 15)

= **₹ 156.75**

Total surface area of the container *i.e.*, the frustum of the cone (excluding the upper end)

$$= [\pi l(R + r) + \pi r^2] \text{ cm}^2$$
$$= 3.14 \times [20 \times (20 + 8) + (8)^2] \text{ cm}^2$$
$$= 3.14 \times [20 \times 28 + 64] \text{ cm}^2$$
$$= 3.14 \times [560 + 64] \text{ cm}^2$$
$$= 3.14 \times 624 \text{ cm}^2$$
$$= 1{,}959.36 \text{ cm}^2$$

Thus, cost of the metal used @ ₹ 5 per 100 cm^2

$$= ₹\, \frac{1{,}959.36 \times 5}{100} \qquad \left[\begin{array}{l} \because \quad \text{Cost of } 100 \text{ cm}^2 = ₹\, 5 \\ \Rightarrow \text{Cost of } 1 \text{ cm}^2 \text{ of metal} = ₹\, \frac{5}{100} \end{array}\right]$$

$= ₹\ 19.5936 \times 5$
$= ₹\ 97.968$
$= ₹$ **98 approx.**

32. The sum of n, $2n$, $3n$ terms of an A.P. are S_1, S_2, S_3 respectively. Prove that :
$$S_3 = 3(S_2 - S_1).$$

Solution. Let a be the first term and d be the common difference of the given A.P., then

$$S_1 = \text{Sum of } n \text{ terms of the given A.P.} \Rightarrow S_1 = \frac{n}{2}[2a + (n-1)d] \quad \ldots(1)$$

$$S_2 = \text{Sum of } 2n \text{ terms of the given A.P.} \Rightarrow S_2 = \frac{2n}{2}[2a + (2n-1)d] \quad \ldots(2)$$

and $$S_3 = \text{Sum of } 3n \text{ terms of the given A.P.} \Rightarrow S_3 = \frac{3n}{2}[2a + (3n-1)d] \quad \ldots(3)$$

Now, R.H.S. $= 3(S_2 - S_1)$

$$= 3\left\{\left[\frac{2n}{2}\{2a + (2n-1)\,d\}\right] - \left[\frac{n}{2}\{2a + (n-1)d\}\right]\right\} \quad \text{[using (2)]}$$

$$= 3\left[2a\left(\frac{2n}{2} - \frac{n}{2}\right) + d\left\{n(2n-1) - \frac{n}{2}(n-1)\right\}\right] \quad \text{[using (2) and (1)]}$$

$$= 3\left[2a\left(\frac{n}{2}\right) + \frac{d}{2}(4n^2 - 2n - n^2 + n)\right]$$

$$= 3\left[2a\left(\frac{n}{2}\right) + \frac{nd}{2}(3n-1)\right]$$

$$= \frac{3n}{2}[2a + (3n-1)d]$$

$$= S_3 \quad \text{[using (3)]}$$

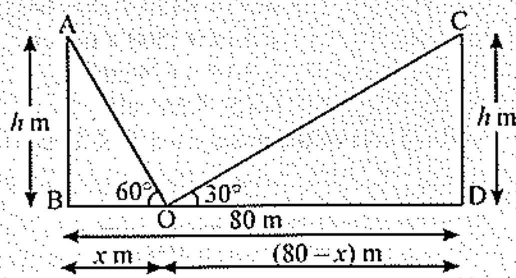
$$= \text{L.H.S.}$$

33. Two poles of equal heights are standing opposite each other on either side of the road, which is 80 m wide. From a point between them on the road, the angles of elevation of the top of the poles are 60° and 30°, respectively. Find the height of the poles and the distances of the point from the poles.

Solution. Let AB and CD be two poles of equal heights h metres.

Let O be a point on the road such that $\angle AOB = 60°$ and $\angle COD = 30°$. Let $OB = x$ metres, then $OD = (80 - x)$ m, where BD = width of the road = 80 metres. [See figure]

In right ΔAOB, we have



$$\tan 60° = \frac{AB}{OB}$$

$$\Rightarrow \quad \sqrt{3} = \frac{h}{x}$$

$$\Rightarrow \quad h = \sqrt{3}x \quad \ldots(1)$$

In right ΔCOD, we have

$$\tan 30° = \frac{CD}{OD}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{(80-x)}$$

$$\Rightarrow \quad h = \frac{80-x}{\sqrt{3}} \quad \ldots(2)$$

From (1) and (2), we get

$$\sqrt{3}x = \frac{80-x}{\sqrt{3}}$$

$$\Rightarrow \quad 3x = 80 - x$$

$$\Rightarrow \quad 4x = 80$$

$$\Rightarrow \quad x = 20 \text{ m}$$

Substituting $x = 20$ m in (1), we get

$$h = \sqrt{3} \times 20 \text{ m}$$

$$\Rightarrow \quad h = 1.732 \times 20 \text{ m} = 34.64 \text{ m}$$

Hence, the height of each pole is **34.64 m** and the distance of the point from pole AB is OB = **20 m** and from the pole CD is OD = **60 m.**

34. A shuttle cock used for playing badminton has the shape of a frustum of a cone mounted on a hemisphere as shown in the figure. The external diameters of the frustum are 5 cm and 2 cm, the height of the entire shuttle cock is 7 cm. Find its external surface area.

Solution. From the figure,

OA = Radius of the lower end of the frustum of a cone (r) = 1 cm

$O'A'$ = Radius of the upper end of the frustum of a cone (R) = 2.5 cm

OO' = Height of the frustum of a cone (h) = 6 cm

AA' = Slant height of the frustum of a cone (l)

$$= \sqrt{h^2 + (R-r)^2}$$

$$= \sqrt{(6)^2 + (2.5-1)^2}$$

$$= \sqrt{36 + 2.25} = \sqrt{38.25} = 6.19 \text{ cm}$$

$\therefore$ External surface area of a shuttle cock

= Curved surface area of the frustum of a cone + Surface area of a hemisphere

$= \pi(R+r)l + 2\pi r^2$

$= [\pi \times (2.5 + 1) \times 6.19 + 2\pi(1)^2] \text{ cm}^2$

$$= \left[\frac{22}{7} \times 3.5 \times 6.19 + 2 \times \frac{22}{7}\right] \text{cm}^2$$

$$= \left(\frac{77 \times 6.19 + 44}{7}\right) \text{cm}^2$$

$$= \left(\frac{476.63 + 44}{7}\right) \text{cm}^2$$

$$= \frac{520.63}{7} \text{ cm}^2$$

$= \mathbf{74.38 \ cm^2}$