CCE SAMPLE QUESTION PAPER

SECOND TERM (SA-II)

MATHEMATICS

(With Solutions)

CLASS X

Time Allowed : 3 Hours] [*Maximum Marks* : 80

General Instructions :

(*i*) *All questions are compulsory.*

(*ii*) *The question paper consists of 34 questions divided into four sections A, B, C and D. Section A comprises of 10 questions of 1 mark each, Section B comprises of 8 questions of 2 marks each, Section C comprises of 10 questions of 3 marks each and Section D comprises of 6 questions of 4 marks each.*

(*iii*) *Question numbers 1 to 10 in Section A are multiple choice questions where you are to select one correct option out of the given four.*

(*iv*) *There is no overall choice. However, internal choice has been provided in 1 question of two marks, 3 questions of three marks each and 2 questions of four marks each. You have to attempt only one of the alternatives in all such questions.*

(*v*) *Use of calculators is not permitted.*

Section 'A'

Question numbers 1 to 10 are of one mark each.

1. If α, β are roots of the equation

$$(a+1)x^2 + (2a+3) + (3a+4) = 0$$

If $\alpha\beta = 2$, then $(\alpha + \beta)$ is equal to

(*a*) 0 (*b*) – 1

(*c*) 2 (*d*) – 2

Solution. Choice (*b*) is correct.

Since α and β are the roots of the given equation, therefore

$$\alpha + \beta = -\left(\frac{2a+3}{a+1}\right) \quad ...(1)$$

and $$\alpha\beta = \frac{3a+4}{a+1}$$

But $\alpha\beta = 2$ (given)

$$\therefore \quad \frac{3a+4}{a+1} = 2$$

$$\Rightarrow \quad 3a + 4 = 2a + 2$$

$$\Rightarrow \quad a = -2$$

Putting $a = -2$ in (1), we get

$$\alpha + \beta = -\left(\frac{-4+3}{-2+1}\right)$$

$$\Rightarrow \quad \alpha + \beta = -1$$

2. The value of k for which the difference between the roots of the equation $x^2 + kx + 8 = 0$ is 2, are

(*a*) ± 2 (*b*) ± 4

(*c*) ± 6 (*d*) ± 8

Solution. Choice (*c*) is correct.

Let α and β be the roots of the equation $x^2 + kx + 8 = 0$, then

$$\alpha + \beta = -k \quad \ldots(1)$$

and $$\alpha\beta = 8 \quad \ldots(2)$$

It is given that the difference between the roots of the given equation is 2.

Let $$\alpha - \beta = \pm 2 \quad \ldots(3)$$

We know that

$$(\alpha + \beta)^2 - (\alpha - \beta)^2 = 4\,\alpha\beta$$

$$\Rightarrow \quad (-k)^2 - (\pm 2)^2 = 4(8) \quad \text{[using (1), (2) and (3)]}$$

$$\Rightarrow \quad k^2 - 4 = 32$$

$$\Rightarrow \quad k^2 = 36$$

$$\Rightarrow \quad k = \pm 6$$

3. If the sum of first m terms of an A.P. is $2m^2 + 3m$, then the second term is

(*a*) 3 (*b*) 5

(*c*) 7 (*d*) 9

Solution. Choice (*d*) is correct.

Let S_m denote the sum of m terms of an A.P., then

$$S_m = 2m^2 + 3m$$

Putting $m = 1$ in (1), $S_1 = 2(1)^2 + 3(1) = 2 + 3 = 5 = a_1$ $[\because S_1 = a_1]$

Putting $m = 2$ in (1), $S_2 = 2(2)^2 + 3(2) = 8 + 6 = 14 = a_1 + a_2$ $[\because S_2 = a_1 + a_2]$

$$\therefore \quad (a_1 + a_2) - a_1 = S_2 - S_1 = 14 - 5$$

$$\Rightarrow \quad a_2 = 9$$

$\Rightarrow$ Second term of an A.P. is **9**.

4. The 6th term from the end of an A.P. 17, 14, 11, ..., – 40 is equal to

(*a*) – 21 (*b*) – 25

(*c*) – 22 (*d*) – 24

Solution. Choice (*b*) is correct.

Here, l = last term = – 40 and common difference $(d) = -3$.

$\therefore$ nth term from the end of a given A.P. $= l - (n-1)d$

$$= -40 - (n-1)(-3)$$

$$= -40 + 3n - 3$$

$$= -43 + 3n$$

Hence, 6th term from the end of a given A.P.

$$= -43 + 3(6)$$

$$= -43 + 18$$

$$= \mathbf{-25}$$

5. A girl calculates that the probability of her winning the first prize in a lottery is 0.08. If 6000 tickets are sold, how many tickets has she bought ?

(*a*) 40 (*b*) 60

(*c*) 240 (*d*) 480

Solution. Choice (*d*) is correct.

Probability of an event E is calculated by the formula as

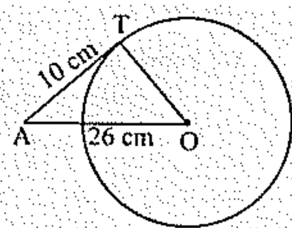
$$P(E) = \frac{\text{Number of outcomes favourable to an event } E}{\text{Total number of outcomes}}$$

$$\Rightarrow \quad 0.08 = \frac{\text{Number of outcomes favourable to an event } E}{6000}$$

$\Rightarrow$ Number of outcomes favourable to an event $E = 0.08 \times 6000 = 480$

$\Rightarrow$ She has bought **480 tickets** to win the first prize.

6. If the length of tangent from a point A at a distance of 26 cm from the centre of the circle is 10 cm, than the radius of the circle is



(*a*) 22 cm (*b*) 24 cm

(*c*) 21 cm (*d*) 23 cm

Solution. Choice (*b*) is correct.

Since the tangent to a circle is perpendicular to the radius through the point of contact.

$\therefore \quad \angle OTA = 90°$

In right ΔOTA, we have

$$OA^2 = OT^2 + AT^2 \Rightarrow (26)^2 = OT^2 + (10)^2$$

$$\Rightarrow \quad OT^2 = 676 - 100 = 576 = (24)^2$$

$$\Rightarrow \quad OT = 24$$

Hence, the radius of the circle is **24 cm.**

7. The distance between the points (a cos θ, a sin θ) and (– a sin θ, a cos θ) is

(*a*) $\sqrt{a}$ (*b*) a

(*c*) $2\sqrt{a}$ (*d*) $\sqrt{2}a$

Solution. Choice (*d*) is correct.

Distance between the given points (a cos θ, a sin θ) (– a sin θ, a cos θ)

$$= \sqrt{(a\cos\theta + a\sin\theta)^2 + (a\sin\theta - a\cos\theta)^2}$$

$$= \sqrt{a^2(\cos^2\theta + \sin^2\theta + 2\sin\theta\cos\theta) + a^2(\sin^2\theta + \cos^2\theta - 2\sin\cos\theta)}$$

$$= \sqrt{a^2(1 + 2\sin\theta\cos\theta) + a^2(1 - 2\sin\theta\cos\theta)}$$

$$= \sqrt{a^2 + a^2} = \sqrt{2a^2} = \sqrt{2}a.$$

8. If the surface areas of two spheres is in the ratio 4 : 9, then the ratio of their radii is

(*a*) 2 : 3 (*b*) 3 : 2

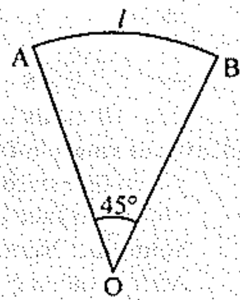
(*c*) 1 : 3 (*d*) 3 : 1

Solution. Choice (*a*) is correct.

$$\frac{S_1}{S_2}=\frac{4}{9} \Rightarrow \frac{4\pi r_1^2}{4\pi r_2^2}=\frac{4}{9} \Rightarrow \frac{4}{9}=\frac{r_1^2}{r_2^2} \Rightarrow \left(\frac{2}{3}\right)^2=\left(\frac{r_1}{r_2}\right)^2 \Rightarrow \frac{r_1}{r_2}=\frac{2}{3}$$

Thus, the ratio of their radii is **2 : 3.**

9. If the following figure is a sector of a circle of radius 14 cm, then the perimeter of this sector is



(*a*) 35 cm (*b*) 36 cm

(*c*) 39 cm (*d*) 38 cm

Solution. Choice (*c*) is correct.

The arc length $l = AB$ of a sector of an angle 45° in a circle of radius 14 cm is given by

$$l=\frac{\theta}{360^\circ}\times 2\pi r$$

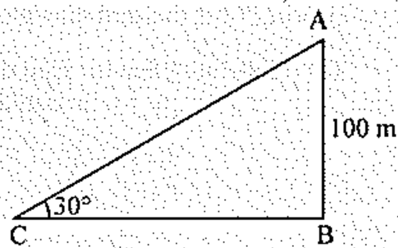
$$\Rightarrow \quad l=\left(\frac{45^\circ}{360^\circ}\times 2\times\frac{22}{7}\times 14\right)\text{cm}$$

$$\Rightarrow \quad l=\left(\frac{1}{8}\times 2\times 22\times 2\right)\text{cm}$$

$$\Rightarrow \quad l = 11 \text{ cm}$$

$\therefore$ Perimeter of sector $OAB = OA + OB + \text{arc } AB = 14 + 14 + 11 =$ **39 cm.**

10. The height of a tower is 100 m when the angle of elevation of the sun is 30°, then the shadow of the tower is



(*a*) $100\sqrt{3}$ m (*b*) 100 m

(*c*) $100(\sqrt{3}-1)$ m (*d*) $\frac{100}{\sqrt{3}}$ m

Solution. Choice (*a*) is correct.

Let AB be the tower and BC be its shadow.

In ΔABC, we have

$$\tan 30^\circ=\frac{AB}{BC}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}}=\frac{100}{BC}$$

$$\Rightarrow \quad BC = \mathbf{100\sqrt{3}\ m}$$

Section 'B'

Question numbers 11 to 18 carry 2 marks each.

11. Find the values of x for which the distance between the points $P(2, -3)$ and $Q(x, 5)$ is 10 units.

Solution. Here $P(2, -3)$ and $Q(x, 5)$ be the given points. Then

$$PQ = 10 \text{ units (given)}$$

$$\Rightarrow \quad \sqrt{(x-2)^2+(5+3)^2}=10$$

$$\Rightarrow \quad (x-2)^2 + 64 = (10)^2$$
$$\Rightarrow \quad x^2 + 4 - 4x = 100 - 64$$
$$\Rightarrow \quad x^2 - 4x + 4 = 36$$
$$\Rightarrow \quad x^2 - 4x - 32 = 0$$
$$\Rightarrow \quad x^2 - 8x + 4x - 32 = 0$$
$$\Rightarrow \quad x(x-8) + 4(x-8) = 0$$
$$\Rightarrow \quad (x-8)(x+4) = 0$$
$$\Rightarrow \quad x - 8 = 0 \text{ or } x + 4 = 0$$
$$\Rightarrow \quad \mathbf{x = 8} \text{ or } \mathbf{x = -4}$$

12. All cards of ace, jack and queen are removed from a deck of playing cards. One card is drawn at random from the remaining cards, find the probability that the card drawn is

(*a*) a face card

(*b*) not a face card.

Solution. Number of all ace, jack and queen cards = 4 + 4 + 4 = 12 cards.

After removing all ace, jack and queen cards from a deck of 52 playing cards, there are 40 (= 52 – 12) cards left in the deck.

Out of these 40 cards, one card is drawn in 40 ways.

∴ Total number of possible outcomes = 40.

(*a*) There are 4 face cards, *i.e.*, 4 kings in the deck containing 40 cards. Out of these 4 kings one king can be drawn in 4 ways.

∴ Favourable number of outcomes = 4

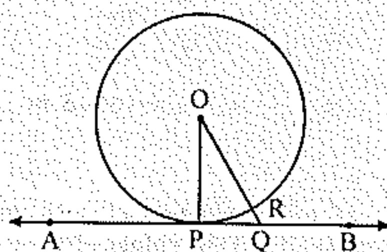
Hence, $P(\text{that the card drawn is a face card}) = \frac{4}{40} = \mathbf{\frac{1}{10}}$.

(*b*) Probability that the card drawn is not a face card

$$= 1 - \text{Probability that the card drawn is a face card}$$
$$= 1 - \frac{1}{10}$$
$$= \mathbf{\frac{9}{10}}.$$

13. Prove that a line drawn through the end point of a radius and perpendicular to it is a tangent to the circle.

Solution. Given : A radius *OP* of a circle with centre *O* and a line *APB*, perpendicular to *OP*.



To prove : *AB* is a tangent to the circle at the point *P*.

Proof : Take a point *Q*, different from *P*, on the line *AB*.

Now, $OP \perp AB$.

⇒ Among all the line segments joining *O* to a point on *AB*, *OP* is the shortest.

$$\Rightarrow \quad OP < OQ$$
$$\Rightarrow \quad OQ > OP$$

⇒ *Q* lies outside the circle.

Thus, every point on *AB*, other than *P*, lies outside the circle. This shows that *AB* meets the circle only at the point *P*.

Hence, *AB* is a **tangent** to the circle at *P*.

14. Solve for x : $9x^2 - 6ax + (a^2 - b^2) = 0$

Solution. We have

$$9x^2 - 6ax + (a^2 - b^2) = 0 \qquad ...(1)$$

Here, constant term $= a^2 - b^2 = (a - b)(a + b)$

and, coefficient of middle term $= -6a$

$$= -[3(a + b) + 3(a - b)]$$

$\therefore$ Rewriting (1) as

$$9x^2 - [3(a + b) + 3(a - b)]x + (a - b)(a + b) = 0$$

$\Rightarrow$ $[9x^2 - 3(a + b)x] - [3(a - b)x - (a - b)(a + b)] = 0$

$\Rightarrow$ $3x[3x - (a + b)] - (a - b)[3x - (a + b)] = 0$

$\Rightarrow$ $[3x - (a + b)][3x - (a - b)] = 0$

$\Rightarrow$ Either $3x - (a + b) = 0$ or $3x - (a - b) = 0$

$\Rightarrow$ Either $x = \dfrac{a + b}{3}$ or $x = \dfrac{a - b}{3}$

Or

The sum of two numbers is 15. If the sum of their reciprocals is $\frac{3}{10}$, find the numbers.

Solution. Let the required numbers be x and $15 - x$, then

$$\frac{1}{x} + \frac{1}{15 - x} = \frac{3}{10}$$

$\Rightarrow$ $\dfrac{15 - x + x}{x(15 - x)} = \dfrac{3}{10}$

$\Rightarrow$ $\dfrac{15}{15x - x^2} = \dfrac{3}{10}$

$\Rightarrow$ $45x - 3x^2 = 150$

$\Rightarrow$ $3x^2 - 45x + 150 = 0$

$\Rightarrow$ $x^2 - 15x + 50 = 0$

$\Rightarrow$ $x^2 - 10x - 5x + 50 = 0$

$\Rightarrow$ $(x^2 - 10x) - (5x - 50) = 0$

$\Rightarrow$ $x(x - 10) - 5(x - 10) = 0$

$\Rightarrow$ $(x - 10)(x - 5) = 0$

$\Rightarrow$ Either $x - 10 = 0$ or $x - 5 = 0$

$\Rightarrow$ Either $x = 10$ or $x = 5$

When $x = 10$, then $15 - x = 15 - 10 = 5$

When $x = 5$, then $15 - x = 15 - 5 = 10$

Hence, the two numbers are **10** and **5** or **5** and **10.**

15. The perimeter of a sector of a circle of radius 5.2 cm is 16.4 cm. Find the area of the sector.

Solution. Let OAB be the given sector, then

Perimeter of sector $OAB = 16.4$ cm (given)

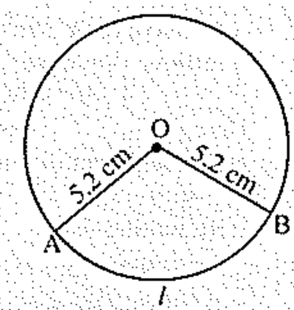
$\Rightarrow$ $OA + OB + \text{arc } AB = 16.4$ cm

$\Rightarrow \quad 5.2 + 5.2 + \text{arc } AB = 16.4 \text{ cm}$

$\Rightarrow \quad 10.4 + l = 16.4 \text{ cm}$

$\Rightarrow \quad l = (16.4 - 10.4) \text{ cm}$

$\Rightarrow \quad l = 6 \text{ cm}$

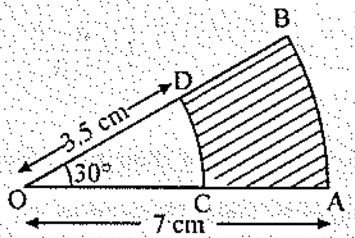


$\therefore$ Area of a sector $OAB = \frac{1}{2} lr$

$= \left[\frac{1}{2} \times (6)(5\cdot2)\right] \text{ cm}^2$

$= (3 \times 5.2) \text{ cm}^2 = \mathbf{15.6 \text{ cm}^2}$

16. In the figure, sectors of two concentric circles of radii 7 cm and 3.5 cm are given. Find the area of the shaded region.



Solution. We have

Shaded area = Area of sector OAB – Area of sector OCD

$\Rightarrow$ Shaded area $= \left(\frac{30°}{360°} \times \frac{22}{7} \times 7 \times 7 - \frac{30°}{360°} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}\right) \text{ cm}^2$

$\Rightarrow$ Shaded area $= \frac{30°}{360°} \times \frac{22}{7} \times 7 \times 7\left(1 - \frac{1}{2} \times \frac{1}{2}\right) \text{ cm}^2$

$= \frac{1}{12} \times 22 \times 7\left(1 - \frac{1}{4}\right) \text{ cm}^2$

$= \frac{11 \times 7}{6} \times \frac{3}{4} \text{ cm}^2$

$= \frac{77}{8} \text{ cm}^2$

$= \mathbf{9.625 \text{ cm}^2}$

17. Three cubes whose edges measures 3 cm, 4 cm and 5 cm respectively to form a single cube. Find its edge. Also, find the surface area of the new cube.

Solution. Let x cm be the edge of the new cube, then

Volume of the new cube = Sum of the volumes of three cubes

x^3 = Volume of a cube of edge 3 cm + Volume of a cube of edge 4 cm + Volume of a cube of edge 5 cm

$\Rightarrow \quad x^3 = 3^3 + 4^3 + 5^3$

$\Rightarrow \quad x^3 = 27 + 64 + 125$

$\Rightarrow \quad x^3 = 216$

$\Rightarrow \quad x^3 = (6)^3$

$\Rightarrow \quad x = 6 \text{ cm}$

∴ Edge of a new cube is **6 cm.**

Surface area of the new cube $= 6x^2$

$$= 6 \times (6)^2 \text{ cm}^2$$

$$= \mathbf{216 \text{ cm}^2}$$

18. A right circular cone is of height 8.4 cm and radius of its base is 2.1 cm. It is melted and recast into a sphere. Find the radius of the sphere.

Solution. We have

Radius of the base of the cone $= r = 2.1$ cm

Height of the cone $= h = 8.4$ cm

∴ Volume of the cone $= \frac{1}{3}\pi r^2 h$

$$= \left[\frac{1}{3}\pi \times (2.1)^2 \times 8.4\right] \text{cm}^3$$

Let R be the radius of the sphere obtained by recasting the melted cone.

∴ Volume of a sphere $= \frac{4}{3}\pi R^3$

Since the volume of the material in the form of cone and sphere remains the same, therefore

Volume of a sphere = Volume of the cone

$$\Rightarrow \quad \frac{4}{3}\pi R^3 = \frac{1}{3} \times \pi \times (2.1)^2 \times (8.4)$$

$$\Rightarrow \quad 4R^3 = (2.1)^2 \times 8.4$$

$$\Rightarrow \quad R^3 = (2.1)^2 \times 2.1$$

$$\Rightarrow \quad R^3 = (2.1)^3$$

$$\Rightarrow \quad R = 2.1 \text{ cm}$$

Hence, the radius of the sphere is **2.1 cm.**

Section 'C'

Question numbers 19 to 28 carry 3 marks each.

19. If the 10th term of an A.P. is 47 and its first term is 2, find the sum of its 15 terms.

Solution. Let $a = 2$ be the first term and d be the common difference of an A.P.

$2, 2 + d, 2 + 2d, \ldots\ldots\ldots$

It is given that :

The 10th term of an A.P. is 47

∴ $t_{10} = 2 + (10 - 1)d$ $\quad [\because t_n = a + (n-1)d]$

$\Rightarrow$ $47 = 2 + 9d$

$\Rightarrow$ $9d = 45$

$\Rightarrow$ $d = 5$

Now, $S_{15} = \frac{15}{2}[2a + (15 - 1)d]$ $\quad \left[\because S_n = \frac{n}{2}[2a + (n-1)d]\right]$

$$= \frac{15}{2}[2 \times 2 + 14 \times 5]$$

$$= \frac{15}{2}[4 + 70]$$

$$= \frac{15}{2} \times 74$$
$$= 15 \times 37 = 555$$

Hence, the sum of 15 terms of an A.P. is **555.**

Or

The sum of first six terms of an arithmetic progression is 42. The ratio of its 10th term to its 30th term is 1 : 3. Calculate the first and the thirteenth term of the A.P.

Solution. Let a and d be the first term and common difference of an A.P. $a, a+d, a+2d, \ldots$.

(*i*) It is given that the sum of first six terms of an A.P. is 42.

$\therefore$ Sum of first six terms of an A.P. $(S_6) = \frac{6}{2}[2a + (6-1)d] = 42$ (given)

$$\Rightarrow \quad 3(2a + 5d) = 42$$
$$\Rightarrow \quad 2a + 5d = 14 \quad \ldots(1)$$

(*ii*) It is also given that the ratio of 10th term of an A.P. is to 30th term of an A.P. is 1 : 3.

$$\therefore \quad \frac{a_{10}}{a_{30}} = \frac{1}{3}$$
$$\Rightarrow \quad \frac{a+(10-1)d}{a+(30-1)d} = \frac{1}{3}$$
$$\Rightarrow \quad \frac{a+9d}{a+29d} = \frac{1}{3}$$
$$\Rightarrow \quad 3a + 27d = a + 29d$$
$$\Rightarrow \quad 2a = 2d$$
$$\Rightarrow \quad a = d \quad \ldots(2)$$

From (1) and (2), we have

$$2a + 5a = 14 \Rightarrow 7a = 14 \Rightarrow a = 2 \quad \ldots(3)$$

From (2) and (3), we get $d = 2$

Now, the thirteenth term of an A.P.

$$= a + (13-1)d$$
$$= 2 + 12(2)$$
$$= 2 + 24$$
$$= 26$$

Hence, the first term and the thirteenth term of the A.P. are **2** and **26**, respectively.

20. Solve for x :

$$2\left(\frac{2x-1}{x+3}\right) - 3\left(\frac{x+3}{2x-1}\right) = 5; \text{ given that } x \neq -3, x \neq \frac{1}{2}$$

Solution. The given equation is

$$2\left(\frac{2x-1}{x+3}\right) - 3\left(\frac{x+3}{2x-1}\right) = 5 \quad \ldots(1)$$

Putting $\frac{2x-1}{x+3} = y$, then (1) becomes

$$2y - \frac{3}{y} = 5$$

$$\Rightarrow \quad 2y^2 - 3 = 5y$$

$$\Rightarrow \quad 2y^2 - 5y - 3 = 0$$

$$\Rightarrow \quad 2y^2 - 6y + y - 3 = 0$$

$$\Rightarrow \quad 2y(y - 3) + (y - 3) = 0$$

$$\Rightarrow \quad (y - 3)(2y + 1) = 0$$

$\Rightarrow$ Either $\quad y - 3 = 0 \quad$ or $\quad 2y + 1 = 0$

$\Rightarrow$ Either $\quad y = 3 \quad$ or $\quad y = -\frac{1}{2}$

When $y = 3$, then

$$\frac{2x - 1}{x + 3} = 3$$

$$\Rightarrow \quad 2x - 1 = 3x + 9$$

$$\Rightarrow \quad 3x - 2x = -1 - 9$$

$$\Rightarrow \quad x = -10$$

When $y = -\frac{1}{2}$, then

$$\frac{2x - 1}{x + 3} = -\frac{1}{2}$$

$$\Rightarrow \quad 4x - 2 = -x - 3$$

$$\Rightarrow \quad 4x + x = -3 + 2$$

$$\Rightarrow \quad 5x = -1$$

$$\Rightarrow \quad x = -\frac{1}{5}$$

Hence, $x = -10$ and $x = -\frac{1}{5}$ are the required solutions.

21. A bag contains 35 balls out of which x are blue.

(*i*) If one ball is drawn at random, what is the probability that it will be a blue ball ?

(*ii*) If 7 more blue balls are put in the bag, the probability of drawing a blue ball will be double than that in (*i*).

Find x.

Solution. Random drawing of balls ensures equally likely.

Total number of outcomes (*i.e.*, balls) = 35

Out of 35 outcomes (*i.e.*, balls), favourable outcomes (*i.e.*, blue balls) = x

(*i*) Since the bag contains x blue balls, therefore

$$P(\text{drawing a blue ball}) = \frac{x}{35}$$

(*ii*) When 7 more blue balls are put in the bag, then
the total number of outcomes (*i.e.*, balls) = 35 + 7 = 42
Out of 42 outcomes (*i.e.*, balls), favourable are (*i.e.*, blue balls) = $x + 7$

$$P(\text{drawing a blue ball}) = \frac{x + 7}{42}$$

It is given that the probability of drawing a blue ball in case (*ii*) is double than that in case (*i*).

$$\therefore \quad \frac{x + 7}{42} = 2 \times \frac{x}{35}$$

$\Rightarrow \quad \frac{x+7}{x} = \frac{2 \times 42}{35}$

$\Rightarrow \quad \frac{x+7}{x} = \frac{12}{5}$

$\Rightarrow \quad 5x + 35 = 12x$

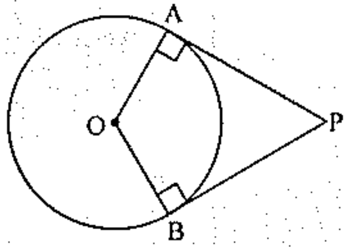
$\Rightarrow \quad 12x - 5x = 35$

$\Rightarrow \quad 7x = 35$

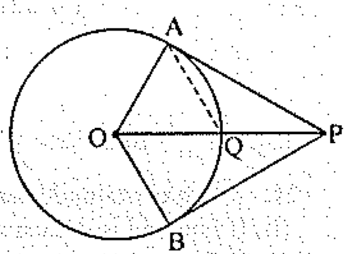
$\Rightarrow \quad x = 5$

Hence, the value of x is **5.**

22. In figure, OP is equal to diameter of the circle. Prove that ABP is a equilateral triangle.



Solution. Join OP. Suppose OP meets the circle at Q. Join AQ.



We have

OP = diameter [given]

$\Rightarrow \quad OQ + PQ$ = diameter

$\Rightarrow$ Radius + PQ = diameter [$\because OQ$ = radius]

$\Rightarrow \quad PQ$ = diameter – radius

$\Rightarrow \quad PQ$ = radius

Thus, $OQ = PQ$ = radius

Thus, OP is the hypotenuse of right triangle OAP and Q is the mid-point of OP.

$\therefore \quad OA = AQ = OQ$

[$\because$ Mid-point of hypotenuse of a right triangle is equidistant from the vertices]

$\Rightarrow \Delta OAQ$ is equilateral

$\Rightarrow \quad \angle AOQ = 60°$

So, $\quad \angle APO = 30°$

$\therefore \quad \angle APB = 2\angle APO = 2 \times 30° = 60°$

Also, $\quad PA = PB$

$\Rightarrow \quad \angle PBA = \angle PAB.$

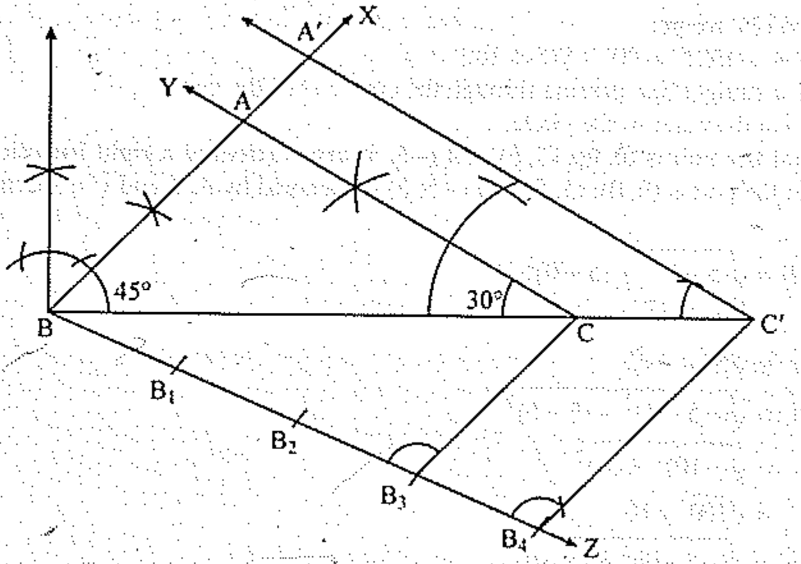
But, $\angle APB = 60°$. Therefore, $\angle PAB = \angle PBA = 60°$

Hence, ΔABP is an **equilateral triangle.**

23. Draw a triangle ABC with side BC = 7 cm, $\angle B = 45°$, $\angle A = 105°$. Then construct a triangle whose sides are $\frac{4}{3}$ times the corresponding sides of ΔABC.

Solution. Steps of Construction :

1. Draw a line segment BC = 7 cm.
2. At B construct $\angle CBX = 45°$.
3. At C construct $\angle BCY = 180° - (45° + 105°) = 30°$.

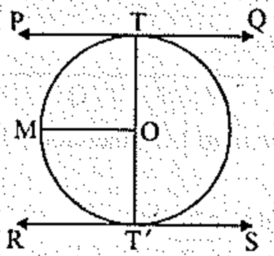


4. Suppose BX and CY intersect at A. Then, ABC is the given triangle.
5. Construct an acute angle CBZ at B on opposite sides of vertex A of ΔABC.
6. Locate 4 points (the greater of 4 and 3 in $\frac{4}{3}$) B_1, B_2, B_3, B_4 on BZ such that $BB_1 = B_1B_2 = B_2B_3 = B_3B_4$.
7. Join B_3 (the 3rd point) to C and draw a line through B_4 parallel to B_3C intersecting the extended line segment BC at C'.
8. Draw a line through C' parallel to CA intersecting the extended line segment BA at A'.

Triangle $A'BC'$, so obtained is the required triangle.

24. Prove that the perpendicular at the point of contact to the tangent to a circle passes through the centre.

Solution. Let PTQ and $RT'S$ be two parallel tangents to a circle with centre O at points T and T' respectively. Join OT and OT'. Draw $OM \parallel PTQ$.



Now, $PTQ \parallel OM$ and a transversal TO intersects them

$\therefore$ $\angle PTO + \angle MOT = 180°$ [$\because$ Sum of the angles on the same side of a transversal is 180°]

$\Rightarrow$ $90° + \angle MOT = 180°$ [$\because$ $\angle PTO$ = angle between a tangent and radius = 90°]

$\Rightarrow$ $\angle MOT = 180° - 90° = 90°$ ...(1)

Similarly, $\angle MOT' = 90°$ ...(2)

Adding (1) and (2), we get

$$\angle MOT + \angle MOT' = 90^\circ + 90^\circ = 180^\circ$$

Thus, TOT' is a straight line passing through the centre O of the circle.

Hence, TOT' is a diameter of the circle.

25. Prove that the points (0, 0); (5, 5) and (–5, 5) are vertices of a right isosceles triangle.

Solution. Let the points (0, 0); (5, 5) and (– 5, 5) be denoted by A, B and C respectively.

Then,

$$AB = \sqrt{(5-0)^2 + (5-0)^2} = \sqrt{25+25} = \sqrt{50} = 5\sqrt{2} \quad \text{...(1)}$$

$$BC = \sqrt{(-5-5)^2 + (5-5)^2} = \sqrt{(-10)^2 + (0)^2} = \sqrt{100} = 10 \quad \text{...(2)}$$

and

$$AC = \sqrt{(-5-0)^2 + (5-0)^2} = \sqrt{25+25} = \sqrt{50} = 5\sqrt{2} \quad \text{...(3)}$$

From (1), (2) and (3), we get

$AB = 5\sqrt{2} = AC \Rightarrow$ The triangle is an isosceles triangle

and $BC^2 = 100 = 50 + 50$

$\Rightarrow$ $BC^2 = AB^2 + AC^2$

$\Rightarrow$ The triangle ABC is right triangle.

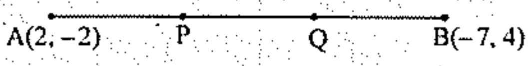
Hence, the given points (0, 0); (5, 5) and (– 5, 5) are the vertices of a **right isosceles triangle.**

Or

Find the coordinates of the points of trisection (*i.e.*, points dividing in three equal parts) of the line segment joining the points $A(2, -2)$ and $B(-7, 4)$.

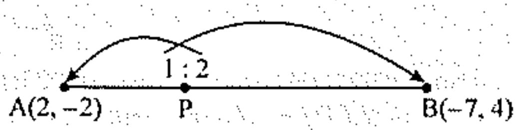
Solution. Let P and Q be the points of trisection of the line AB joining the points $A(2, -2)$ and $B(-7, 4)$, *i.e.*,

$$AP = PQ = QB \text{ (see figure)}$$



Since, P divides AB internally in the ratio 1 : 2, therefore, the coordinates of P, by using the section formula, are

$$P\left\{\frac{1(-7)+2(2)}{1+2}, \frac{1(4)+2(-2)}{1+2}\right\}$$

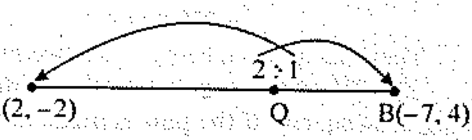


i.e., $$P\left(\frac{-7+4}{3}, \frac{4-4}{3}\right)$$

i.e., $\mathbf{P(-1, 0)}$

Also, since Q divides AB internally in the ratio 2 : 1, therefore, the coordinates of Q, by using section formula, are

$$\therefore \quad Q\left(\frac{2(-7)+1(2)}{2+1}, \frac{2(4)+1(-2)}{2+1}\right)$$



i.e., $$Q\left(\frac{-14+2}{3}, \frac{8-2}{3}\right)$$

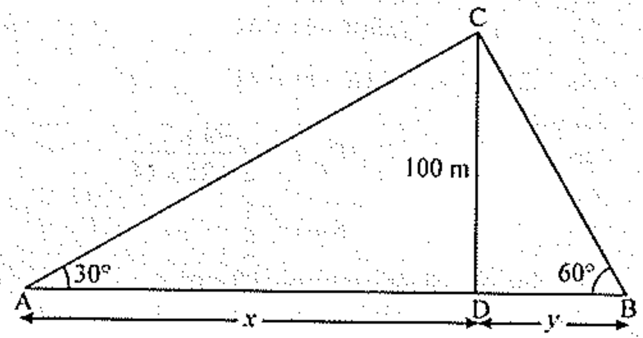
i.e. $$Q(-4, 2)$$

26. Two men on either side of a cliff 100 m high observe the angles of elevation of the top of the cliff to be 30° and 60° respectively. Find the distance between the two men. [Use $\sqrt{3} = 1.732$]

Solution. Let $CD = 100$ m be the height of the cliff. Let A and B be the points of observations such that the angle of elevation at A is 30° and the angle of elevation at B is 60°.

$\therefore \quad \angle CAD = 30°$ and $\angle CBD = 60°$.

Let $AD = x$ m and $DB = y$ m.



In right triangle ADC, we have

$$\tan 30° = \frac{CD}{AD}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{100}{x}$$

$$\Rightarrow \quad x = 100\sqrt{3} \text{ m} \quad \ldots(1)$$

In right triangle BDC, we have

$$\tan 60° = \frac{CD}{DB}$$

$$\Rightarrow \quad \sqrt{3} = \frac{100}{y}$$

$$\Rightarrow \quad y = \frac{100}{\sqrt{3}} \text{ m} \quad \ldots(2)$$

The distance between the two men is AB, *i.e.*,

$$AB = AD + DB$$

$$= x + y$$

$$= \left(100\sqrt{3} + \frac{100}{\sqrt{3}}\right) \text{ m} \quad \text{[using (1) and (2)]}$$

$$= \left(\frac{300+100}{\sqrt{3}}\right) \text{ m}$$

$$= \frac{400}{\sqrt{3}} \text{ m}$$

$$= \frac{400\sqrt{3}}{3} \text{ m}$$

$$= \frac{400 \times (1.732)}{3} \text{ m}$$

$= (400 \times 0.577)$ m
$= 4 \times 57.7$ m
$= 230.8$ m

Thus, the distance between the two men is **230.8 m.**

27. The difference between outside and inside surface of a cylindrical metallic pipe 14 cm long is 44 sq. cm. If the pipe is made of 99 cubic cm of metal, find the outer and inner radii of the pipe.

Solution. Let the outer radius of the cylindrical metallic pipe be R cm and the inner radius of the cylindrical pipe be r cm.

Length of a cylindrical metallic pipe $(h) = 14$ cm

$\therefore$ Difference between the outside and inner surface of a cylindrical metallic pipe $= 44 \text{ cm}^2$ [given]

$\Rightarrow \quad 2\pi Rh - 2\pi rh = 44$

$\Rightarrow \quad 2\pi h(R - r) = 44$

$\Rightarrow \quad 2 \times \frac{22}{7} \times 14 \times (R - r) = 44$

$\Rightarrow \quad R - r = \frac{44 \times 7}{2 \times 22 \times 14}$

$\Rightarrow \quad R - r = \frac{1}{2}$ ...(1)

Volume of cylindrical pipe $= 99 \text{ cm}^3$ [given]

$\Rightarrow \quad \pi R^2 h - \pi r^2 h = 99$

$\Rightarrow \quad \pi h(R^2 - r^2) = 99$

$\Rightarrow \quad \frac{22}{7} \times 14 \times (R^2 - r^2) = 99$

$\Rightarrow \quad 44(R - r)(R + r) = 99$

$\Rightarrow \quad 44 \times \frac{1}{2}(R + r) = 99$ [using (1)]

$\Rightarrow \quad 22 \times (R + r) = 99$

$\Rightarrow \quad R + r = \frac{99}{22}$

$\Rightarrow \quad R + r = \frac{9}{2}$ ...(2)

Adding and subtracting (2) and (1), we get

$2R = \frac{9}{2} + \frac{1}{2}$ and $2r = \frac{9}{2} - \frac{1}{2}$

$\Rightarrow \quad 2R = \frac{10}{2} = 5$ and $2r = \frac{8}{2} = 4$

$\Rightarrow \quad R = 5 \div 2 = 2.5$ and $r = 4 \div 2 = 2$

Hence, the outer radius of the pipe = **2.5 cm** and inner radius of the pipe = **2 cm.**

28. A hemispherical bowl of internal diameter 30 cm is containing some liquid. This liquid is to be filled into cylindrical shaped bottles each of diameter 5 cm and height 6 cm. Find the number of bottles necessary to empty the bowl.

Solution. Diameter of hemispherical bowl = 30 cm
Radius of hemispherical bowl (R) = 30 ÷ 2 = 15 cm
Diameter of a cylindrical shaped bottle = 5 cm
Radius of a cylindrical shaped bottle (r) = 5 ÷ 2 = 2.5 cm
Height of the cylindrical shaped bottle (h) = 6 cm
Volume of the liquid in a cylindrical shaped bottle = $\pi r^2 h$

$$= [\pi \times (2.5)^2 \times 6] \text{ cm}^2$$

$$= \left[\pi \times \frac{25}{4} \times 6\right] \text{cm}^2 \qquad \left[\because (2.5)^2 = \left(\frac{5}{2}\right)^2 = \frac{25}{4}\right]$$

$$= \frac{75\pi}{2} \text{ cm}^2 \qquad \ldots(1)$$

Volume of liquid in a hemispherical bowl

$$= \frac{2}{3}\pi R^3$$

$$= \left[\frac{2\pi}{3} \times (15)^3\right] \text{ cm}^3$$

$$= [2\pi \times 5 \times 15 \times 15] \text{ cm}^3$$

$$= 2250\pi \text{ cm}^3 \qquad \ldots(2)$$

$$\therefore \text{ Required number of bottles} = \frac{\text{Volume of liquid in hemispherical bowl}}{\text{Volume of liquid in cylindrical bottle}}$$

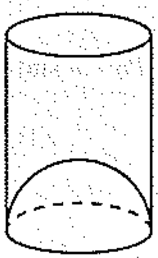
$$= \frac{2250\pi}{75\pi/2}$$

$$= \frac{2250 \times 2}{75}$$

$$= \mathbf{60.}$$

Or

A juice seller serves his customers using a glass as shown in figure. The inner diameter of the cylindrical glass is 5 cm, but the bottom of the glass has a hemispherical portion raised which reduces the capacity of the glass. If the height of the glass is 10 cm, find the apparent capacity of the glass and its actual capacity. [Use π = 3.14]



Solution. It is given that :
The inner diameter of the cylindrical glass = 5 cm
So, the inner radius of the cylindrical glass (r) = 2.5 cm

Height of the glass (h) = 10 cm

Thus, the apparent capacity of the glass $= \pi r^2 h$

$$= 3.14\,(2.5)^2(10)\text{ cm}^3 \qquad [\because \pi = 3.14 \text{ (given)}]$$

$$= 3.14 \times 62.5 \text{ cm}^3 = \mathbf{196.25\ cm^3} \qquad \ldots(1)$$

But the actual capacity of the glass is less by the volume of the hemispherical at the base of the glass, *i.e.*,

Actual capacity of the glass = Apparent capacity of a glass – Volume of the hemispherical portion.

$$= (196.25 - \frac{2}{3}\pi r^3)\text{cm}^3 \qquad [\text{using } (1)]$$

$$= 196.25 \text{ cm}^3 - \frac{2}{3} \times 3.14 \times (2.5)^3 \text{ cm}^3$$

$$= \left(196.25 - \frac{6.28}{3} \times 15.625\right)\text{cm}^3$$

$$= \left(196.25 - \frac{98.125}{3}\right)$$

$$= (196.25 - 32.71)\text{ cm}^3$$

$$= \mathbf{163.54\ cm^3}.$$

Section 'D'

Question numbers 29 to 34 carry 4 marks each.

29. The angle of elevation of a jet fighter from a point A on the ground is 60°. After a flight of 10 seconds, the angle of elevation changes to 30°. If the jet is flying at a speed of 648 km/hour, find the constant height at which the jet is flying. [Use $\sqrt{3} = 1.732$]

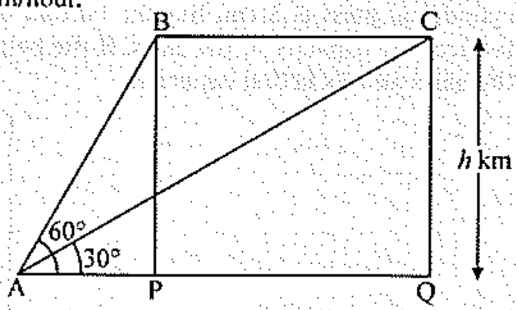
Solution. Let B and C be the two positions of a jet fighter as observed from a point A on the ground. Let APQ be the horizontal line through A.

It is given that the angles of elevation of a jet fighter in two positions B and C as observed from a point A are 60° and 30°, respectively.

$\therefore \quad \angle BAP = 60°$ and $\angle CAQ = 30°$

Let the constant height of a jet fighter be h km, *i.e.*, $BP = CQ = h$ km.

It is also given that a jet is flying at a speed of 648 km/hour.



In right-angled ΔAPB, we have

$$\tan 60° = \frac{BP}{AP}$$

$$\Rightarrow \quad \sqrt{3} = \frac{h}{AP} \qquad [\because QC = PB = h \text{ km}]$$

$$\Rightarrow \quad AP = \frac{h}{\sqrt{3}} \qquad \ldots(1)$$

In right-angled ΔAQC, we have

$$\tan 30° = \frac{CQ}{AQ}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{AQ} \qquad [\because CQ = h \text{ km}]$$

$\Rightarrow \quad AQ = \sqrt{3}\,h$ ...(2)

Now, $\quad BC = PQ = AQ - AP$

$\Rightarrow \quad BC = \left(\sqrt{3}h - \frac{h}{\sqrt{3}}\right)$ km

$\Rightarrow \quad BC = \left(\frac{3h - h}{\sqrt{3}}\right)$ km

$\Rightarrow \quad BC = \frac{2h}{\sqrt{3}}$ km ...(3)

As the jet fighter travels BC km in 10 seconds.

$\therefore$ Time taken in hours, the jet fighter travels BC km $= \left(\frac{10}{60 \times 60}\right)$ hour.

Now, Distance = Speed × Time taken

$\Rightarrow \quad BC = \left(648 \times \frac{10}{60 \times 60}\right)$ km

$\Rightarrow \quad \frac{2h}{\sqrt{3}} = \frac{108}{60}$ km

$\Rightarrow \quad h = \sqrt{3} \times \frac{9}{10}$ km

$\Rightarrow \quad h = 0.9 \times (1.732)$ km

$\Rightarrow \quad h = 1.5588$ km

Thus, the constant height at which the jet fighter is flying = **1.5588 km or 1558.8 m.**

30. A two digit number is such that the product of its digits is 15. If 18 is added to the number, the digits interchange their places. Find the number.

Solution. Let the unit place digit be x and ten's place digit be y, then according to the given condition

$$yx = 15$$

$$\Rightarrow \quad y = \frac{15}{x} \quad \text{...(1)}$$

Original number $= 10y + x$

$= 10\left(\frac{15}{x}\right) + x$ [using (1)]

$= \left(\frac{150}{x}\right) + x$

When digits interchange their places, then

New number $= 10x + y$

$= 10x + \frac{15}{x}$ [using (1)]

According to the given condition, we have

Original number + 18 = New number

$$\Rightarrow \quad \frac{150}{x} + x + 18 = 10x + \frac{15}{x}$$

$$\Rightarrow \quad (10x - x) + \left(\frac{15}{x} - \frac{150}{x}\right) - 18 = 0$$

$$\Rightarrow \quad 9x - \frac{135}{x} - 18 = 0$$

$$\Rightarrow \quad 9x^2 - 135 - 18x = 0$$

$$\Rightarrow \quad 9(x^2 - 2x - 15) = 0$$

$$\Rightarrow \quad x^2 - 2x - 15 = 0$$

$$\Rightarrow \quad x^2 - 5x + 3x - 15 = 0$$

$$\Rightarrow \quad x(x - 5) + 3(x - 5) = 0$$

$$\Rightarrow \quad (x - 5)(x + 3) = 0$$

$\Rightarrow$ Either $x - 5 = 0$ or $x + 3 = 0$

$\Rightarrow$ Either $x = 5$ or $x = -3$

$\Rightarrow$ $x = 5$ [$\because$ Digits can't be negative]

Thus, unit place digit = 5 and ten's place digit = $\frac{15}{x} = \frac{15}{5} = 3$

Hence, the required number is **35**.

Or

Two water taps together can fill the tank in $9\frac{3}{8}$ hours. The tap of larger diameter takes 10 hours less than the smaller one to fill the tank separately. Find the time in which each tap can separately fill the tank.

Solution. Let the time taken by the tap of smaller diameter = x hours

$\therefore$ Time taken by the tap of larger diameter = $(x - 10)$ hours

$\therefore$ Work done by the tap of smaller diameter in one hour = $\frac{1}{x}$

and that the work done by tap of larger diameter in hour = $\frac{1}{(x-10)}$

Thus, the work done by the two taps together in 1 hour

$$= \frac{1}{x} + \frac{1}{x-10}$$

$$= \frac{(x-10) + x}{x(x-10)}$$

$$= \frac{2x - 10}{x(x-10)}$$

The two taps together can fill the tank in $\frac{x(x-10)}{2x-10}$ hours.

According to the given information,

$$\frac{x(x-10)}{2x-10} = 9\frac{3}{8} \text{ hours (given)}$$

$$\Rightarrow \quad \frac{x^2 - 10x}{2x - 10} = \frac{75}{8}$$

$$\Rightarrow \quad 8x^2 - 80x = 150x - 750$$

$$\Rightarrow \quad 8x^2 - 230x + 750 = 0$$

$$\Rightarrow \quad 4x^2 - 115x + 375 = 0 \qquad \text{[Dividing the whole equation by 2]}$$

$$\Rightarrow \quad 4x^2 - 100x - 15x + 375 = 0$$

$$\Rightarrow \quad 4x(x - 25) - 15(x - 25) = 0$$

$$\Rightarrow \quad (x - 25)(4x - 15) = 0$$

$$\Rightarrow \quad \text{Either } x - 25 = 0 \quad \text{or} \quad 4x - 15 = 0$$

$$\Rightarrow \quad \text{Either } x = 25 \quad \text{or} \quad x = \frac{15}{4}$$

When $x = 25$, then the tap of smaller diameter can fill the tank in **25 hours** and the tap of larger diameter can fill the tank in $25 - 10 =$ **15 hours.**

When $x = \frac{15}{4}$, then $x - 10 = \frac{15}{4} - 10 = \frac{-25}{4}$ which is rejected as time to fill the tank cannot be negative.

31. The coordinates of the centroid of a triangle are (1, 3) and two of its vertices are (– 7, 6) and (8, 5). Find the third vertex. Also, find the coordinates of the centroid of the triangle when the third vertex is (2, 4).

Solution. Let the two vertices of the triangle be $A(-7, 6)$ and $B(8, 5)$ and let the third vertex of the triangle ABP be $P(x, y)$, the centroid of the triangle is given as $G(1, 3)$.

$$\therefore \quad \frac{x_1 + x_2 + x_3}{3} = 1 \quad \text{and} \quad \frac{y_1 + y_2 + y_3}{3} = 3$$

$$\Rightarrow \quad \frac{(-7) + 8 + x}{3} = 1 \quad \text{and} \quad \frac{6 + 5 + y}{3} = 3$$

$$\Rightarrow \quad 1 + x = 3 \quad \text{and} \quad 11 + y = 9$$

$$\Rightarrow \quad x = 3 - 1 \quad \text{and} \quad y = 9 - 11$$

$$\Rightarrow \quad x = 2 \quad \text{and} \quad y = -2$$

Thus, the coordinates of the third vertex are **(2, –2).**

Let the centroid of the triangle ABC be $G_1(x, y)$.

Also, let the three vertices of the triangle ABC are $A(-7, 6)$, $B(8, 5)$ and $C(2, 4)$, respectively.

$$\text{Then} \quad x = \frac{(-7) + 8 + 2}{3} \quad \text{and} \quad y = \frac{6 + 5 + 4}{3}$$

$$\Rightarrow \quad x = \frac{3}{3} \quad \text{and} \quad y = \frac{15}{3}$$

$$\Rightarrow \quad x = 1 \quad \text{and} \quad y = 5$$

Thus, the coordinates of the centroid are **(1, 5).**

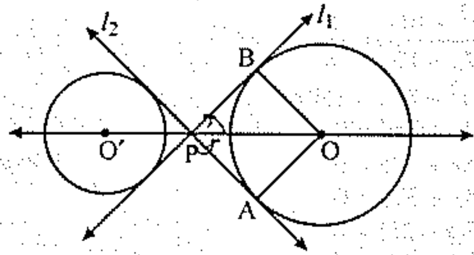
32. Find the locus of centres of circles which touch two intersecting lines.

Solution. Let l_1 and l_2 be two intersecting lines which intersect at point P.

Let O be the centre of the circle which touches both l_1 and l_2.

In triangles OAP and OBP, we have

$$OA = OB \qquad \text{[Each equal to radius]}$$



$$PA = PB \quad \text{[Tangents drawn from an external point to a circle are equal]}$$

and $\quad OP = OP \quad$ [Common]

So, by SSS-congruence criterion, we have

$\Delta OAP \cong \Delta OBP$

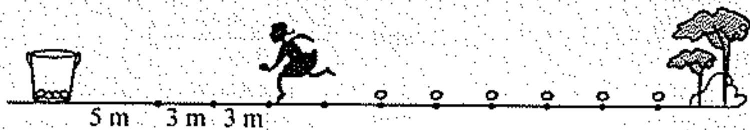
$\Rightarrow \quad \angle APO = \angle BPO$

$\Rightarrow \quad OP$ is the bisector of $\angle APB$

$\Rightarrow \quad O$ lies on the bisector of the angle between l_1 and l_2.

Hence, the required locus is the line bisecting the angle between the given lines.

33. In a potato race, a bucket is placed at the starting point, which is 5 m from the first potato, and the other potatoes are placed 3 m apart in a straight line. There are ten potatoes in the line (see figure). A competitor starts from the bucket, picks up the nearest potato, runs back with it, drops it in the bucket, runs back to pick up the next potato, runs to the bucket to drop it in the bucket, and she continues in the same way until all the potatoes are in the bucket. What is the total distance the competitor has to run ?



Solution. We have,

d_1 : Distance run by the competitor to pick up first potato $= 2 \times 5$ m

d_2 : Distance run by the competitor to pick up second potato $= 2(5 + 3)$ m

d_3 : Distance run by the competitor to pick up third potato $= 2(5 + 2 \times 3)$ m

d_4 : Distance run by the competitor to pick up fourth potato $= 2(5 + 3 \times 3)$ m

$\vdots$

d_{10} : Distance run by the competitor to pick up tenth potato $= 2(5 + 9 \times 3)$ m

$\therefore$ Total distance run by the competitor to pick up 10 potatoes

$= d_1 + d_2 + d_3 + d_4 \ldots + d_{10}$

$= 2 \times 5 + 2(5 + 3) + 2(5 + 2 \times 3) + 2(5 + 3 \times 3) + \ldots + 2(5 + 9 \times 3)$

$= 2[5 + \{5 + 3\} + \{5 + (2 \times 3)\} + \{5 + (3 \times 3\} + \ldots + \{5 + 9 \times 3\}]$

$= 2[\underbrace{(5 + 5 + \ldots + 5}_{10 \text{ times}} + \{3 + (2 \times 3) + (3 \times 3) + \ldots + (9 \times 3)\}]$

$= 2[5 \times 10 + 3\{1 + 2 + 3 + \ldots + 9\}]$

$= 2\left[50 + 3 \times \frac{9}{2}(1 + 9)\right] \quad \left[\text{using} : S_n = \frac{n}{2}(a + l)\right]$

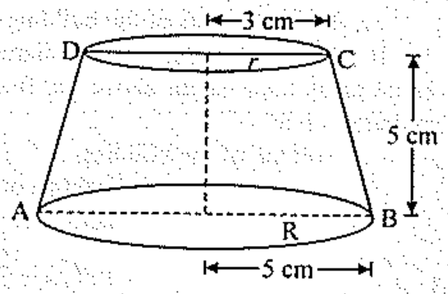
$= 2\left[50 + 3 \times \frac{9}{2} \times 10\right]$
$= 2[50 + 135]$
$= 2[185]$
$=$ **370 metres.**

34. The diameters of the bottom of a frustum of right circular cone is 10 cm, and that of the top is 6 cm and height is 5 cm. Find out the area of the total surface and volume of the frustum.

$\left[\textbf{Take } \pi = \frac{22}{7}\right]$

Solution. Here : $r = \frac{6}{2} = 3$ cm and $R = \frac{10}{2} = 5$ cm be the radii of the top and bottom of a frustum of a right circular cone. Also, $h = 5$ cm (given) be the height of a frustum of a cone.

$\therefore$ Slant height (l) of the frustum of a cone



$= \sqrt{h^2 + (R-r)^2}$
$= \sqrt{(5)^2 + (5-3)^2}$
$= \sqrt{25+4}$ cm
$= \sqrt{29}$ cm
$= 5.38$ cm

Total surface area of a frustum of a cone

= Curved surface area of a frustum + Area of top circular section + Area of bottom circular section

$= \pi(R + r)l + \pi r^2 + \pi R^2$
$= \pi[(5 + 3)5.38 + (3)^2 + (5)^2]$ cm^2
$= \frac{22}{7}[43.04 + 9 + 25]$ cm^2
$= \frac{22}{7}[77.04]$ cm^2
$= (22 \times 11.01)$ cm^2 (approx.)
$=$ **242.22 cm^2**

Volume of the frustum of a cone

$= \frac{\pi h}{3}[R^2 + r^2 + Rr]$
$= \frac{22}{7} \times \frac{5}{3}[(5)^2 + (3)^2 + 5 \times 3]$ cm^3
$= \frac{110}{7 \times 3}[25 + 9 + 15]$ cm^3
$= \frac{110}{7 \times 3}[49]$ cm^3

$$= \frac{110 \times 7}{3} \text{ cm}^3$$

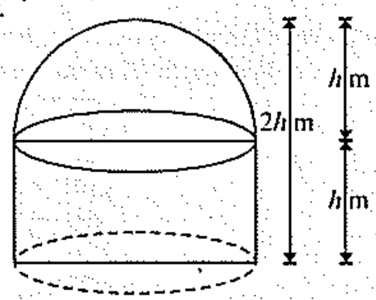
$$= \frac{770}{3} \text{ cm}^3$$

$$= \mathbf{256.67 \text{ cm}^3}$$

Or

A building is in the form of a cylinder surmounted by a hemispherical vaulted dome and contains $41\frac{19}{21}$ m^3 of air. If the internal diameter of the building is equal to its total height above the floor, find the height of the building. $\left[\text{Take } \pi = \frac{22}{7}\right]$

Solution. Let $2h$ m be the height of the building above the floor.



$\therefore$ Internal diameter of the building = $2h$ m

$\Rightarrow$ Internal radius of the building = h m

It is given that the internal diameter of the building is equal to its total height above the floor. Then, height of the cylinder = h m.

$\therefore$ Volume of the building

= Volume of cylinder + Volume of the hemispherical vaulted dome

$$= \pi(h)^2 h + \frac{2}{3}\pi(h)^3$$

[$\because$ Radius of the cylinder = h m and radius of the hemispherical vaulted dome = h m]

$$= \pi h^3 + \frac{2}{3}\pi h^3$$

$$= \frac{5}{3}\pi h^3$$

But the building contains $41\frac{19}{21}$ m^3 = $\frac{880}{21}$ m^3 of air

$$\therefore \quad \frac{5}{3}\pi h^3 = \frac{880}{21}$$

$$\Rightarrow \quad \frac{5}{3} \times \frac{22}{7} \times h^3 = \frac{880}{21}$$

$$\Rightarrow \quad \frac{110}{21} h^3 = \frac{880}{21}$$

$$\Rightarrow \quad h^3 = \frac{880}{21} \times \frac{21}{110}$$

$$\Rightarrow \quad h^3 = 8 = (2)^3$$

$$\Rightarrow \quad h = 2 \text{ m}$$

$$\Rightarrow \quad 2h = 4 \text{ m}$$

Hence the height of the building is **4 m.**