CCE SAMPLE QUESTION PAPER

SECOND TERM (SA-II)

MATHEMATICS

(With Solutions)

CLASS X

Time Allowed : 3 Hours] *[Maximum Marks : 80*

General Instructions :

(i) *All questions are compulsory.*

(ii) *The question paper consists of 34 questions divided into four sections A, B, C and D. Section A comprises of 10 questions of 1 mark each. Section B comprises of 8 questions of 2 marks each. Section C comprises of 10 questions of 3 marks each and Section D comprises of 6 questions of 4 marks each.*

(iii) *Question numbers 1 to 10 in Section A are multiple choice questions where you are to select one correct option out of the given four.*

(iv) *There is no overall choice. However, internal choice has been provided in 1 question of two marks, 3 questions of three marks each and 2 questions of four marks each. You have to attempt only one of the alternatives in all such questions.*

(v) *Use of calculators is not permitted.*

Section 'A'

Question numbers 1 to 10 are of one mark each.

1. The values of k for which the quadratic equation $x^2 + k(4x + k - 1) + 2 = 0$ has equal roots are

(a) $-1, \frac{2}{3}$ (b) $-1, 2$

(c) $-1, \frac{3}{2}$ (d) $-1, -2$

Solution. Choice (a) is correct.

For equal roots : $D = b^2 - 4ac = 0$

$\Rightarrow \quad (4k)^2 - 4(1)(k^2 - k + 2) = 0 \qquad [x^2 + 4kx + k(k-1) + 2 = 0 \Rightarrow x^2 + 4kx + (k^2 - k + 2) = 0]$

$\Rightarrow \quad 16k^2 - 4k^2 + 4k - 8 = 0$

$\Rightarrow \quad 12k^2 + 4k - 8 = 0$

$\Rightarrow \quad 4(3k^2 + k - 2) = 0$

$\Rightarrow \quad 3k^2 + k - 2 = 0$

$\Rightarrow \quad 3k^2 + 3k - 2k - 2 = 0$

$\Rightarrow \quad 3k(k+1) - 2(k+1) = 0$

$\Rightarrow \quad (k+1)(3k-2) = 0$

$\Rightarrow \quad \mathbf{k = -1 \text{ or } k = \frac{2}{3}}$

2. If the sum and product of the roots of the quadratic equation $ax^2 - 5x + c = 0$ are both equal to 10, then the values of a and c are

(*a*) $\frac{1}{2}$ and -5 (*b*) $\frac{1}{2}$ and 5

(*c*) 5 and $\frac{3}{2}$ (*d*) $\frac{3}{2}$ and 5

Solution. Choice (*b*) is correct.

Since α and β are the roots of the equation $ax^2 - 5x + c = 0$, therefore

$$\alpha + \beta = -\left(\frac{-5}{a}\right) = \frac{5}{a}$$

and $$\alpha\beta = \frac{c}{a}$$

It is given that sum and product of the roots of the given equation are both equal to 10.

$$\therefore \quad \frac{5}{a} = 10 \quad \text{and} \quad \frac{c}{a} = 10$$

$$\Rightarrow \quad a = \frac{5}{10} \quad \text{and} \quad \frac{c}{a} = 10$$

$$\Rightarrow \quad a = \frac{1}{2} \quad \text{and} \quad c = 10a = 10 \times \frac{1}{2} = 5$$

$$\Rightarrow \quad a = \frac{1}{2} \quad \text{and} \quad c = 5$$

3. If the numbers a, b, c, d, e form an A.P., then the value of $a - 4b + 6c - 4d + e$ is

(*a*) 0 (*b*) 1

(*c*) 2 (*d*) -1

Solution. Choice (*a*) is correct.

Since a, b, c, d, e are in A.P., therefore

a = first term

b = second term = $a + d_1$, where d_1 is common difference

c = third term = $a + 2d_1$

d = fourth term = $a + 3d_1$

e = fifth term = $a + 4d_1$

Now, $a - 4b + 6c - 4d + e$

$$= a - 4(a + d_1) + 6(a + 2d_1) - 4(a + 3d_1) + (a + 4d_1)$$

$$= (a - 4a + 6a - 4a + a) + (-4d_1 + 12d_1 - 12d_1 + 4d_1)$$

$$= 0 + 0$$

$$= 0$$

4. The value of $a_{30} - a_{20}$ for the A.P. $-9, -14, -19, -24, \ldots\ldots$ is

(*a*) -30 (*b*) -40

(*c*) -50 (*d*) -60

Solution. Choice (*c*) is correct.

Given, A.P. is $-9, -14, -19, -24, \ldots\ldots$

$a_1 = -9$, $a_2 = -14$, d = common difference = $a_2 - a_1 = -14 - (-9) = -14 + 9 = -5$

Now, $a_{30} - a_{20} = [a_1 + (30 - 1)d] - [a_1 + (20 - 1)d]$

$\Rightarrow \quad a_{30} - a_{20} = (a_1 + 29d) - (a_1 + 19d)$

$\Rightarrow \quad a_{30} - a_{20} = 29d - 19d$

$\Rightarrow \quad a_{30} - a_{20} = 10d$

$\Rightarrow \quad a_{30} - a_{20} = 10 \times (-5)$

$\Rightarrow \quad a_{30} - a_{20} = \mathbf{-50}$

5. A card is drawn from a pack of cards numbered 1 to 52. The probability that the number on the card is a perfect square is

(*a*) $\frac{7}{52}$ (*b*) $\frac{5}{13}$

(*c*) $\frac{1}{13}$ (*d*) $\frac{3}{13}$

Solution. Choice (*a*) is correct.

Since there are 52 cards numbered 1 to 52 in a pack, therefore, the total number of possible outcomes = 52.

There are 7 perfect squares, *viz.*, $1^2, 2^2, 3^2, 4^2, 5^2, 6^2, 7^2$.

Let A be the event that "the number on the card is a perfect square", then the outcomes favourable to $A = 7$

So, $P(A) = \mathbf{\frac{7}{52}}$

6. If $P\left(\frac{a}{3}, 4\right)$ is the mid-point of the line segment joining the points $Q(-6, 5)$ and $R(-2, 3)$, then the value of a is

(*a*) -4 (*b*) -6

(*c*) -12 (*d*) 12

Solution. Choice (*c*) is correct.

Since P is the mid-point of the line segment joining points Q and R, then

$$\frac{-6-2}{2} = \frac{a}{3} \quad \text{and} \quad \frac{5+3}{2} = 4$$

$$\Rightarrow \quad -\frac{8}{2} = \frac{a}{3} \quad \text{and} \quad 4 = 4$$

$$\Rightarrow \quad \mathbf{a = -12}$$

7. The inner base radius of a cylinder and the base of a solid cone are equal. When this cone is fully immersed in the completely filled cylinder, $\frac{1}{4}$ of the volume of liquid flows out. The ratio of the height of cylinder and cone is

(*a*) 2 : 3 (*b*) 3 : 4

(*c*) 4 : 3 (*d*) 3 : 2

Solution. Choice (*c*) is correct.

Let r, h and V be the radius, height and volume of the cylinder and r_1, h_1 and V_1 be the radius, height and volume of the cone.

It is given that : $r = r_1$

$\therefore$ Volume of the cone $= \frac{1}{4}$ Volume of the cylinder

$\Rightarrow \quad \frac{1}{3}\pi r_1^2 h_1 = \frac{1}{4}(\pi r^2 h)$

$\Rightarrow \quad \frac{h}{h_1} = \frac{4}{3}\frac{\pi r_1^2}{\pi r^2}$

$\Rightarrow \quad \frac{h}{h_1} = \frac{4}{3}$ $\quad [\because r_1 = r]$

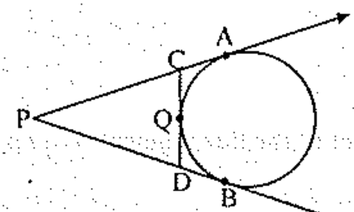
$\Rightarrow \quad h : h_1 = 4 : 3$

Thus, the ratio of the height of the cylinder and cone is **4 : 3.**

8. In the figure given below, *PA* and *PB* are tangents to the circle drawn from an external point *P*. *CD* is a third tangent touching circle at *Q*. If *PB* = 12 cm and *CQ* = 3 cm, then the length of *PC* is

(*a*) 6 cm (*b*) 8 cm

(*c*) 9 cm (*d*) 7 cm



Solution. Choice (*c*) is correct.

We have

$PB = PA = 12$ cm

and $CQ = CA = 3$ cm [$\because$ Tangents from external point to a circle are equal in length]

Now,

Length of $PC = PA - CA$

$= 12 - 3$

$=$ **9 cm.**

9. If the diameter of a semicircular protractor is 14 cm, then the perimeter of the protector is

(*a*) 36 cm (*b*) 14 cm

(*c*) 28 cm (*d*) 21 cm

Solution. Choice (*a*) is correct.

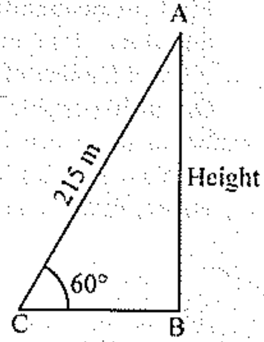
Diameter ($= 2r$) of a semicircular protractor $= 14$ cm.

$\Rightarrow \quad 2r = 14 \Rightarrow r = 7$ cm

$\therefore$ Perimeter of semicircular protractor = Perimeter of a semicircular portion of a protractor + Diameter of a protractor

$= \pi r + 2r = \left(\frac{22}{7} \times 7 + 2 \times 7\right)$ cm $=$ **36 cm.**

10. **A balloon is connected to a meteorological ground stand by a cable of length 215 m inclined at 60° to the horizontal. Assume that there is no slack in the cable. Then the height of the balloon from the ground is**



(*a*) $\frac{215\sqrt{3}}{2}$ m (*b*) $\frac{215}{\sqrt{3}}$ m

(*c*) $215\sqrt{3}$ m (*d*) $\frac{215}{2}$ m

Solution. Choice (*a*) is correct.

Length of cable $AC = 215$ m

Angle of elevation = 60°

In right angled triangle *ABC*, we have

$$\sin 60° = \frac{AB}{AC}$$

$$\Rightarrow \quad \frac{\sqrt{3}}{2} = \frac{AB}{215}$$

$$\Rightarrow \quad AB = \frac{215\sqrt{3}}{2} \text{ m}$$

$\Rightarrow$ Height of the balloon is $\mathbf{\frac{215\sqrt{3}}{2}}$ **m.**

Section 'B'

Question numbers 11 to 18 carry 2 marks each.

11. **The roots α and β of the quadratic equation $x^2 - 5x + 3(k-1) = 0$ are such that $\alpha - \beta = 11$. Find *k*.**

Solution. Since α and β are the roots of the given quadratic equation $x^2 - 5x + 3(k-1) = 0$, then

$$\alpha + \beta = -\left(\frac{-5}{1}\right) = 5 \quad \text{...(1)}$$

and $$\alpha\beta = \frac{3(k-1)}{1} = 3k - 3 \quad \text{...(2)}$$

But $\alpha - \beta = 11$ (given) ...(3)

Adding (1) and (3), we get

$$2\alpha = 16 \Rightarrow \alpha = 8 \quad \text{...(4)}$$

Putting $\alpha = 8$ in (1), we get

$$8 + \beta = 5 \Rightarrow \beta = 5 - 8 = -3 \quad \text{...(5)}$$

Substituting the values of α and β in (2), we get

$$(8)(-3) = 3k - 3$$

$$\Rightarrow \quad -24 = 3k - 3$$

$$\Rightarrow \quad 3k = -24 + 3$$

$$\Rightarrow \quad 3k = -21$$

$$\Rightarrow \quad \mathbf{k = -7}$$

12. There are 30 cards, of same size, in a bag on which numbers 1 to 30 are written. One card is taken out of the bag at random.

Find the probability that the number on the selected card is not divisible by 3.

Solution. Total number of cards from numbers 1 to 30 are 30.

∴ Total number of outcomes in which one card is taken out at random are 30.

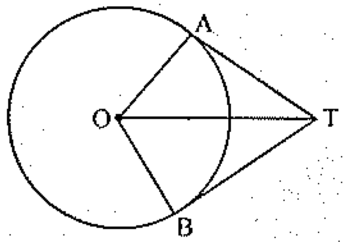
Out of 30 possible outcomes, 10 outcomes (3, 6, 9, 12, 15, 18, 21, 24, 27, 30) are favourable to the number on the selected card is divisible by 3.

∴ 20 (= 30 – 10) outcomes are favourable to the number on the selected card is not divisible by 3.

Hence, P(that a number on the selected card is not divisible by 3)

$$= \frac{20}{30} = \frac{2}{3}$$

13. In figure, if $\angle ATO = 40°$, find $\angle AOB$.



Solution. In ΔTAO and ΔTBO, we have

$TA = TB$ [Tangents from an external point T are equal]

$OA = OB$ [Radii of a circle]

$OT = OT$ [Common]

∴ $\Delta TAO \cong \Delta TBO$ [By SSS Theorem of congruence]

∴ $\angle ATO = \angle BTO$ [CPCT]

But $\angle ATO = 40°$

∴ $\angle ATB = \angle ATO + \angle BTO = 2\angle ATO = 2 \times 40° = 80°$

Since the angle between two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line-segments joining the points of contact at the centre, *i.e.*,

$\angle AOB + \angle ATB = 180°$

$\Rightarrow \quad \angle AOB = 180° - \angle ATB$

$\Rightarrow \quad \angle AOB = 180° - 80°$

$\Rightarrow \quad \angle AOB = \mathbf{100°}$.

14. For what value of p, are the points (2, 1), (p, –1) and (–1, 3) collinear ?

Solution. Since the given points are collinear, therefore, the area of the triangle formed by them must be zero, *i.e.*,

$$\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] = 0, \text{ where, } x_1 = 2, y_1 = 1, x_2 = p, y_2 = -1, x_3 = -1, y_3 = 3$$

$$\Rightarrow \frac{1}{2}[2(-1 - 3) + p(3 - 1) + (-1)(1 + 1)] = 0$$

$$\Rightarrow \quad \frac{1}{2}[-8 + 2p - 2] = 0$$

$\Rightarrow \quad \frac{1}{2}[-10+2p]=0$

$\Rightarrow \quad -5+p=0$

$\Rightarrow \quad p=5.$

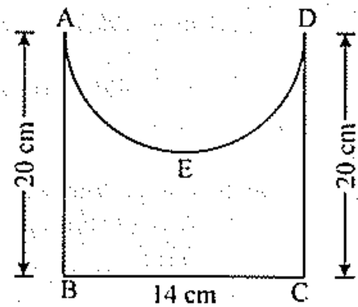
Verification :

$$\text{Area of } \Delta = \frac{1}{2}[2(-1-3)+5(3-1)+(-1)(1+1)]$$

$$= \frac{1}{2}[-8+10-2] = 0.$$

15. Find the perimeter of figure, where $\widehat{AED}$ is a semi-circle and $ABCD$ is a rectangle.

Solution. Perimeter of figure



$= AB + BC + CD + \text{length of } \widehat{AED}$

$= 20 + 14 + 20 + \frac{1}{2}(\text{Circumference of a circle})$

$= 54 + \frac{1}{2}(2\pi r)$

$= 54 + \frac{1}{2}\pi(2r)$

$= 54 + \frac{1}{2}\pi(14)$ $\quad [\because 2r = BC = 14 \text{ cm}]$

$= 54 + 7\pi$

$= 54 + 7 \times \frac{22}{7}$

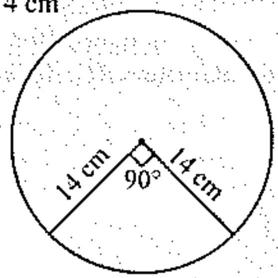
$= 54 + 22$

$= \mathbf{76}$ **cm.**

Or

A chord of a circle of radius 14 cm subtends a right angle at the centre. What is the area of the minor sector ?

Solution. Area of the minor sector of an angle 90° in a circle of radius 14 cm



$= \frac{\theta}{360°} \times \pi r^2$

$= \frac{90°}{360°} \times \frac{22}{7} \times (14)^2 \text{ cm}^2$

$= \frac{1}{4} \times 22 \times 14 \times 2 \text{ cm}^2$

$= \mathbf{154 \text{ cm}^2}$

16. A 4 cm cube is cut into 1 cm cubes. Calculate the total surface area of all the small cubes.

Solution. Each edge of a small cube = 1 cm

Surface area of each small cube = $6 \times (1)^2 = 6 \text{ cm}^2$

Volume of a bigger cube = $(4)^3 = 64 \text{ cm}^3$

Volume of a small cube = $(1)^3 = 1\ cm^3$.

$\therefore$ Number of small cubes = $\dfrac{\text{Volume of bigger cube}}{\text{Volume of small cube}}$

$$= \frac{64}{1} = 64$$

$\therefore$ Total surface area of 64 small cubes = $64 \times 6 =$ **384 cm²**

17. *ABCD* is a field in the shape of a trapezium. *AB* ‖ *DC* and $\angle ABC = 90°$, $\angle DAB = 60°$. Four sectors are formed with centres *A*, *B*, *C* and *D* (see figure). The radius of each sector is 17.5 m. Find the total area of the four sectors.

Solution. Since *AB* ‖ *CD* and $\angle ABC = 90°$, therefore $\angle BCD = 90°$.

Also $\angle DAB = 60°$ [given]

$\therefore$ $\angle ABC + \angle BCD + \angle DAB + \angle CDA = 360°$

[Sum of interior ∠s of a trapezium (or a quad) is 360°]

$\Rightarrow$ $90° + 90° + 60° + \angle CDA = 360°$

$\Rightarrow$ $240° + \angle CDA = 360°$

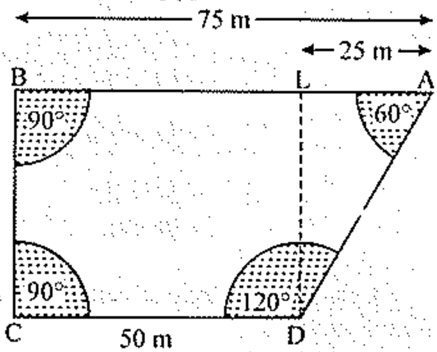
$\Rightarrow$ $\angle CDA = 360° - 240° = 120°$

Total area of the four sectors

= Area of sector at *A* + Area of sector at *B* + Area of sector at *C* + Area of sector at *D*

$$= \frac{60°}{360°} \times \pi \times (17.5)^2 + \frac{90°}{360°} \times \pi \times (17.5)^2 + \frac{90°}{360°} \times \pi \times (17.5)^2 + \frac{120°}{360°} \times \pi \times (17.5)^2$$

$$= \pi \times (17.5)^2 \left[\frac{60°}{360°} + \frac{90°}{360°} + \frac{90°}{360°} + \frac{120°}{360°}\right]$$



$$= \pi \times (17.5)^2 \left[\frac{360°}{360°}\right]$$

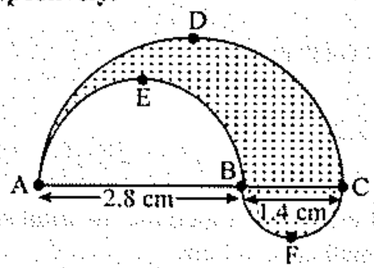
$$= \pi \times (17.5)^2 \times 1$$

$$= \frac{22}{7} \times \frac{35}{2} \times \frac{35}{2}$$

$$= \frac{1925}{2} m^2$$

= **962.5 m².**

18. In figure, find the perimeter of shaded region where *ADC*, *AEB* and *BFC* are semi-circles on diameters *AC*, *AB* and *BC* respectively.



Solution. In figure, diameters of semi-circles *AEB*, *BFC* and *ADC* are $AB = 2.8$ cm, $BC = 1.4$ cm and $AC = AB + BC = (2.8 + 1.4)$ cm $= 4.2$ cm respectively.

Perimeter of the shaded region

= Perimeter of the semi-circle on AC as diameter
+ Perimeter of the semi-circle on AB as diameter
+ Perimeter of the semi-circle on BC as diameter

$$= \pi\left(\frac{\text{Diameter } AC}{2}\right) + \pi\left(\frac{\text{Diameter } AB}{2}\right) + \pi\left(\frac{\text{Diameter } BC}{2}\right)$$

$$\left[\because \text{Perimeter of a semi-circle} = \pi r = \pi\left(\frac{2r}{2}\right) = \pi\left(\frac{\text{Diameter}}{2}\right)\right]$$

$$= \frac{\pi}{2}[\text{Diameter } AC + \text{Diameter } AB + \text{Diameter } BC]$$

$$= \frac{\pi}{2}[4.2 + 2.8 + 1.4] \text{ cm}$$

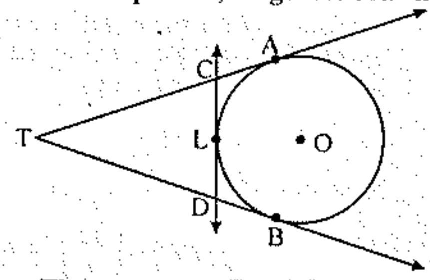
$$= \frac{\pi}{2}[8.4] \text{ cm}$$

$$= 4.2\,\pi \text{ cm} = 4.2 \times \frac{22}{7} \text{ cm} = \mathbf{13.2 \text{ cm}}.$$

Section 'C'

Question numbers 19 to 28 carry 3 marks each.

19. TA and TB are tangents from T to the circle with centre O. At a point L, tangent is drawn cutting TA at C and TB at D. Prove that $CD = CA + DB$.



Solution. We know that the lengths of tangents drawn from an external point to a circle are equal.

$TA = TB$ ...(1) [Tangents from T]

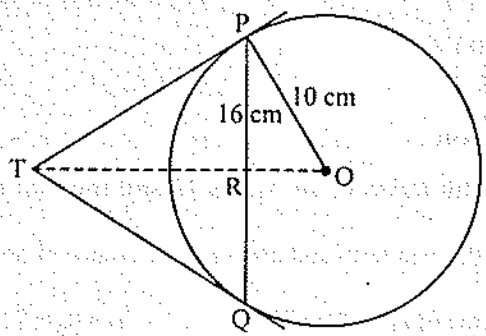
$CA = CL$ ...(2) [Tangents from C]

$BD = DL$ ...(3) [Tangents from D]

Now, $CD = CL + DL$

$\Rightarrow$ $CD = CA + DB$ [using (2) and (3)]

20. PQ is a chord of length 16 cm of a circle of radius 10 cm. The tangents at P and Q intersect at a point T (see figure). Find the length of TP.



Solution. Since OT is perpendicular bisector of PQ, therefore

$$PR = RQ \quad \text{...(1)}$$

But $PQ = 16$ cm (given)

$\Rightarrow \quad PR + RQ = 16$

$\Rightarrow \quad PR + PR = 16$ [using (1)]

$\Rightarrow \quad PR = 8$

$\Rightarrow \quad RQ = PR = 8$ cm ...(2) [using (1)]

In right triangle *ORP*, we have

$$OP^2 = OR^2 + PR^2$$

$\Rightarrow \quad OR^2 = OP^2 - PR^2$

$\Rightarrow \quad OR^2 = 100 - 64 = 36$

$\Rightarrow \quad OR = 6$ cm ...(3)

Since *TP* is a tangent to circle with centre *O* and *OP* is its radius, therefore,

$$OP \perp TP$$

[$\because$ The tangent at any point of a circle is perpendicular to the radius through the point of contact]

$\therefore \quad \angle OPT = 90°$

In right triangle *OPT*, we have

$$OT^2 = PT^2 + OP^2$$

$\Rightarrow \quad (TR + OR)^2 = PT^2 + 100$

$\Rightarrow \quad (TR + 6)^2 = PT^2 + 100$ ...(4)

In right triangle *PRT*, we have

$$PT^2 = TR^2 + PR^2$$

$\Rightarrow \quad PT^2 = TR^2 + 64$ ...(5) [using (2)]

From (4) and (5), we have

$$(TR + 6)^2 = (TR^2 + 64) + 100$$

$\Rightarrow TR^2 + 36 + 12TR = TR^2 + 164$

$\Rightarrow \quad 12TR = 128$

$\Rightarrow \quad TR = \frac{32}{3}$ ...(6)

Now, from (5) and (6), we get

$$PT^2 = \left(\frac{32}{3}\right)^2 + 64 = \frac{1024}{9} + 64$$

$$\Rightarrow \quad PT^2 = \frac{1024 + 576}{9} = \frac{1600}{9}$$

$$\Rightarrow \quad TP^2 = \left(\frac{40}{3}\right)^2$$

$$\Rightarrow \quad \mathbf{TP = \frac{40}{3}\ cm}$$

21. The sum of 5th and 9th terms of an A.P. is 72 and the sum of 7th and 12th terms is 97. Find the A.P.

Solution. Let *a* and *d* be the first term and common difference of an A.P. $a, a + d, a + 2d, \ldots\ldots$

It is given that :

The sum of 5th and 9th terms of an A.P. is 72, *i.e.*, $t_5 + t_9 = 72$ ...(1)

and the sum of 7th and 12th terms of an A.P. is 97, *i.e.*, $t_7 + t_{12} = 97$ ...(2)

Re-writing (1) and (2) as

$$(a + 4d) + (a + 8d) = 72$$

$$\Rightarrow \quad 2a + 12d = 72 \qquad \ldots(3)$$

and $(a + 6d) + (a + 11d) = 97$

$$\Rightarrow \quad 2a + 17d = 97 \qquad \ldots(4)$$

Subtracting (3) from (4), we get

$$(2a + 17d) - (2a + 12d) = 97 - 72$$

$$\Rightarrow \quad 5d = 25$$

$$\Rightarrow \quad d = 5$$

Substituting $d = 5$ in (3), we get

$$2a + 12(5) = 72$$

$$\Rightarrow \quad 2a = 72 - 60$$

$$\Rightarrow \quad 2a = 12$$

$$\Rightarrow \quad a = 6$$

Thus, the A.P. is **6, 11, 16, 21, ...**

22. If (– 5) is a root of the quadratic equation $2x^2 + px - 15 = 0$ and the quadratic equation $p(x^2 + x) + k = 0$ has equal roots, then find the values of p and k.

Solution. Since – 5 is a root of the quadratic equation

$$2x^2 + px - 15 = 0, \text{ therefore}$$

$$2(-5)^2 + p(-5) - 15 = 0$$

$$\Rightarrow \quad 50 - 5p - 15 = 0$$

$$\Rightarrow \quad 35 - 5p = 0$$

$$\Rightarrow \quad 5p = 35$$

$$\Rightarrow \quad p = 7$$

Substituting $p = 7$ in given equation : $p(x^2 + x) + k = 0$, we get

$$7(x^2 + x) + k = 0$$

$$\Rightarrow \quad 7x^2 + 7x + k = 0$$

Here, $a = 7$, $b = 7$ and $c = k$

This equation will have equal roots if

Discriminant $= b^2 - 4ac = 0$

$$\Rightarrow \quad (7)^2 - 4(7)(k) = 0$$

$$\Rightarrow \quad 49 - 28k = 0$$

$$\Rightarrow \quad k = \frac{49}{28} = \frac{7}{4}$$

Hence, the values of p and k are **7** and $\mathbf{\frac{7}{4}}$ respectively.

Or

Solve the following quadratic equation for x :

$$\mathbf{x^2 - 2(a + 2)x + (a + 1)(a + 3) = 0.}$$

Solution. The given quadratic equation is

$$x^2 - 2(a + 2)x + (a + 1)(a + 3) = 0$$

$$\Rightarrow \quad x^2 - (2a + 4)x + (a + 1)(a + 3) = 0$$

$$\Rightarrow \quad x^2 - [(a + 1) + (a + 3)]x + (a + 1)(a + 3) = 0$$

$\Rightarrow \quad x^2 - (a+1)x - (a+3)x + (a+1)(a+3) = 0$

$\Rightarrow \quad [x^2 - (a+1)x] + [-(a+3)x + (a+1)(a+3)] = 0$

$\Rightarrow \quad x[x-(a+1)] - (a+3)[x-(a+1)] = 0$

$\Rightarrow \quad [x-(a+1)][x-(a+3)] = 0$

$\Rightarrow$ Either $x - (a+1) = 0$ or $x - (a+3) = 0$

$\Rightarrow$ Either $x = (a+1)$ or $x = (a+3)$

Hence, $x = a + 1$ or $x = a + 3$.

23. Draw a pair of tangents to a circle of radius 5 cm which are inclined to each other at an angle of 60°.

Solution. Steps of Construction :

1. Take a point O on the plane of the paper and draw a circle of radius $OA = 5$ cm.
2. Extend OA to B such that $OA = AB = 5$ cm.
3. With A as centre draw a circle of radius $OA = AB = 5$ cm. Suppose it intersect the circle drawn in step 1 at the points P and Q.
4. Join BP and BQ.

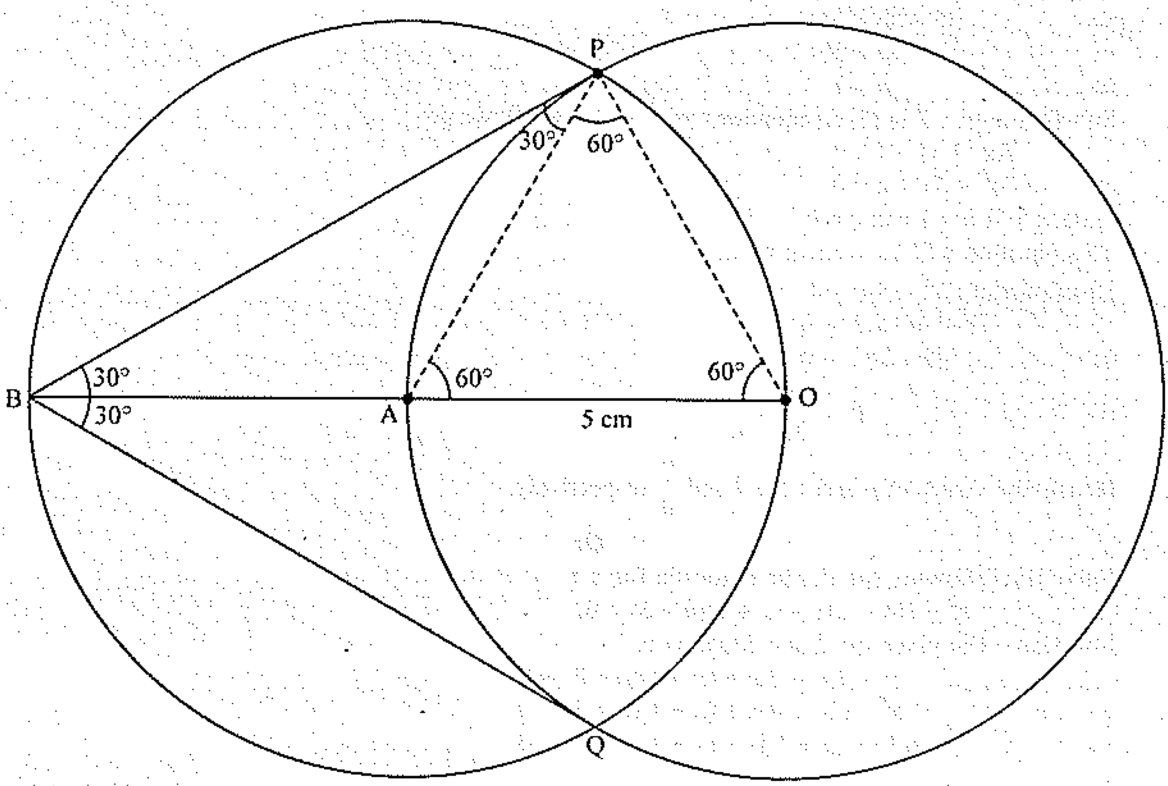
Then BP and BQ are the required tangents which are inclined to each other at an angle of 60° (see figure).

For justification of the construction :

In ΔOAP, we have

$OA = OP = 5$ cm (= Radius)

Also, $AP = 5$ cm (= Radius of circle with centre A).



ΔOAP is equilateral

$\Rightarrow \quad \angle PAO = 60°$

$\Rightarrow \quad \angle BAP = 120°$

In ΔBAP, we have

$$AB = AP \text{ and } \angle BAP = 120°$$

$\therefore \quad \angle ABP = \angle APB = 30°$

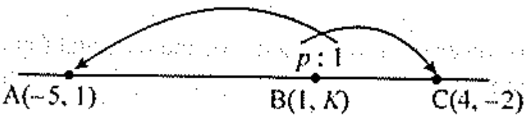
Similarly we can prove that

$$\angle ABQ = \angle AQB = 30°$$

$\Rightarrow \quad \angle PBQ = 60°.$

24. Find the value of K for which the points $A(-5, 1)$, $B(1, K)$ and $C(4, -2)$ are collinear. Also find the ratio in which B divides AC.

Solution. Let the point $B(1, K)$ divides AC in the ratio $p : 1$, where the points A and C are $A(-5, 1)$ and $C(4, -2)$.



By using the section formula, we have

$$1 = \frac{4p + (-5)}{p+1} \quad \text{and} \quad K = \frac{-2p+1}{p+1}$$

$$\Rightarrow \quad p + 1 = 4p - 5 \quad \text{and} \quad K = \frac{-2p+1}{p+1}$$

$$\Rightarrow \quad 4p - p = 1 + 5 \quad \text{and} \quad K = \frac{-2p+1}{p+1}$$

$$\Rightarrow \quad 3p = 6 \quad \text{and} \quad K = \frac{-2p+1}{p+1}$$

$$\Rightarrow \quad p = 2 \quad \text{and} \quad K = \frac{-2 \times 2 + 1}{2+1}$$

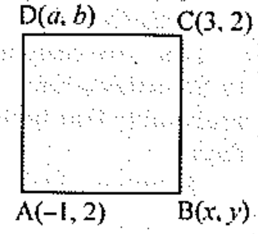
$$\Rightarrow \quad p = 2 \quad \text{and} \quad K = \frac{-3}{3} = -1$$

Hence, the required ratio is $p : 1$, *i.e.*, **2 : 1** and the value of K is **– 1**.

Or

The two opposite vertices of a square are (– 1, 2) and (3, 2). Find the coordinates of other two vertices.

Solution. Let $ABCD$ be a square and let $A(-1, 2)$ and $C(3, 2)$ be the opposite vertices of a square. Let $B(x, y)$ be the unknown vertex. Then



$$AB = BC \quad \text{[All sides of a square are equal]}$$

$\Rightarrow \quad AB^2 = BC^2$

$\Rightarrow \quad (x+1)^2 + (y-2)^2 = (x-3)^2 + (y-2)^2$

$\Rightarrow \quad (x^2 + 2x + 1) = x^2 - 6x + 9$ [Cancelling $(y-2)^2$ from both sides]

$\Rightarrow \quad 2x + 6x = 9 - 1$

$\Rightarrow \quad 8x = 8$

$\Rightarrow \quad x = 1$

Also, in right triangle ABC, we have

$$AC^2 = AB^2 + BC^2$$

$\Rightarrow (-1-3)^2 + (2-2)^2 = (x+1)^2 + (y-2)^2 + (x-3)^2 + (y-2)^2$

$\Rightarrow \quad 16 + 0 = (1+1)^2 + (y^2 - 4y + 4) + (1-3)^2 + (y^2 - 4y + 4) \quad [\because x = 1]$

$\Rightarrow 2y^2 - 8y + 8 + 4 + 4 = 16$

$\Rightarrow \quad 2y^2 - 8y = 0$

$\Rightarrow \quad 2y(y-4) = 0$

$\Rightarrow \quad$ Either $y = 0$ or $y = 4$.

Thus the vertices are **(1, 0)** and **(1, 4).**

25. The angles of elevation of the top of a tower from two points at a distance of 4 m and 9 m from the base of the tower and in the same straight line with it complementary. Prove that the height of the tower is 6 m.

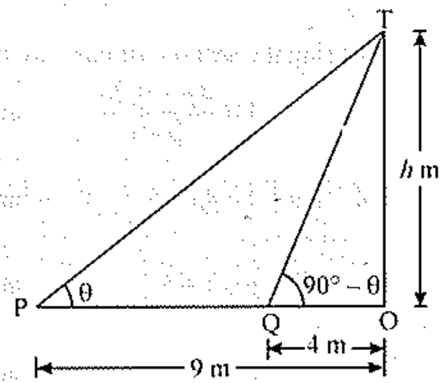
Solution. Let OT be the tower such that O and T be the base and top of the tower respectively. Let P and Q be the points at distances of 4 m and 9 m, respectively from the base O of the tower OT.

Then $OP = 9$ m, $OQ = 4$ m

Let $\angle TPO = \theta$, then $\angle TQO = 90° - \theta$

Let the height of the tower be h m *i.e.*, $OT = h$ m

In right ΔOPT, we have



$$\tan\theta = \frac{OT}{OP}$$

$\Rightarrow \quad \tan\theta = \dfrac{h}{9}$

$\Rightarrow \quad h = 9\tan\theta \quad ...(1)$

Again in right ΔOQT, we have

$$\tan(90° - \theta) = \frac{OT}{OQ}$$

$\Rightarrow \quad \cot\theta = \dfrac{h}{4}$

$\Rightarrow \quad h = 4\cot\theta \quad ...(2)$

Multiplying (1) and (2), we get

$$h^2 = (9\tan\theta) \times (4\cot\theta)$$

$\Rightarrow \quad h^2 = 36\tan\theta\cot\theta$

$\Rightarrow \quad h^2 = 36$

$\Rightarrow \quad h = 6$

Thus, the height of the tower is **6 m.**

26. Two customers Shyam and Ekta are visiting a particular shop in the same week (Tuesday to Saturday). Each is equally likely to visit the shop on any day as another day. What is the probability that both will visit the shop on (*i*) the same day ? (*ii*) consecutive days ? (*iii*) different days ?

Solution. Each customer can visit a shop in the same week namely Tuesday to Saturday *i.e.*, Tuesday, Wednesday, Thursday, Friday and Saturday.

Two customers Shyam and Ekta can visit a shop in $5 \times 5 = 25$ ways

$\therefore$ Total number of outcomes = 25

Shop	*Tuesday* (Tu)	*Wednesday* (W)	*Thursday* (Th)	*Friday* (F)	*Saturday* (S)
Tuesday (Tu)	(Tu, Tu)	(Tu, W)	(Tu, Th)	(Tu, F)	(Tu, S)
Wednesday (W)	(W, Tu)	(W, W)	(W, Th)	(W, F)	(W, S)
Thursday (Th)	(Th, Tu)	(Th, W)	(Th, Th)	(Th, F)	(Th, S)
Friday (F)	(F, Tu)	(F, W)	(F, Th)	(F, F)	(F, S)
Saturday (S)	(S, Tu)	(S, W)	(S, Th)	(S, F)	(S, S)

(*i*) Let A denote the event that two customers will visit the shop on the same day, then

$$A = \{(Tu, Tu), (W, W), (Th, Th), (F, F), (S, S)\}$$

$\therefore$ Favourable number of outcomes of two customers visit the shop on the same day = 5

So, required probability $= P(A) = \frac{5}{25} = \mathbf{\frac{1}{5}}$

(*ii*) Let B denote the event that two customers will visit the shop on consecutive days, then

$$B = \{(Tu, W), (W, Th), (Th, F), (F, S)\}$$

$\therefore$ Favourable number of outcomes of two customers visit the shop on consecutive days = 4

So, required probability $= P(B) = \mathbf{\frac{4}{25}}$

(*iii*) Let $\overline{A}$ denote the event that two customers visit the shop on different days, then

$$P(\overline{A}) = 1 - P(A) = 1 - \frac{1}{5} = \mathbf{\frac{4}{5}}.$$

Or

A lot consists of 144 ball pens of which 20 are defective and the others are good. Nuri will buy a pen if it is good, but will not buy if it is defective. The shopkeeper draws one pen at random and gives it to her. What is the probability that

(*i*) she will buy it ? (*ii*) she will not buy it ?

Solution. Total number of ball pens in a lot = 144 = Total outcomes.

Number of defective ball pens in a lot = 20

$\therefore$ Number of good ball pens = 144 − 20 = 124

(*i*) Let G the event "the ball pen is good", then the number of outcomes favourable to $G = 124$.

$\therefore$ Probability that she will buy the ball pen is $P(G)$, *i.e.*,

$$P(G) = \frac{124}{144} = \mathbf{\frac{31}{36}}$$

(*ii*) Let D be the event "the ball pen is defective", then the number of outcomes favourable to $D = 20$.

$\therefore$ Probability that she will not buy the ball pen is $P(D)$, *i.e.*,

$$P(D) = \frac{20}{144} = \frac{\mathbf{5}}{\mathbf{36}}.$$

27. How many silver coins, 1.75 cm in diameter and of thickness 2 mm, must be melted to form a cuboid of dimensions 5.5 cm × 10 cm × 3.5 cm ?

Solution. Diameter of a silver coin = 1.75 cm

$$\text{Radius of a silver coin } (r) = \frac{1.75}{2} \text{ cm}$$

$$= \frac{175}{200} = \frac{7}{8} \text{ cm}$$

$$\text{Thickness of a coin} = \text{Height } (h) = 2 \text{ mm}$$

$$= \frac{2}{10} = \frac{1}{5} \text{ cm}$$

$\therefore$ Volume of a silver coin of radius $\frac{7}{8}$ cm and thickness $\frac{1}{5}$ cm

$$= \text{Volume of a cylinder of radius } \frac{7}{8} \text{ cm and height } \frac{1}{5} \text{ cm}$$

$$= \pi r^2 h$$

$$= \frac{22}{7} \times \frac{7}{8} \times \frac{7}{8} \times \frac{1}{5} \text{ cm}^3$$

$$= \frac{22 \times 7}{8 \times 8 \times 5} \text{ cm}^3 \qquad \ldots(1)$$

Volume of a cuboid of dimensions 5.5 cm × 10 cm × 3.5 cm

$$= \frac{55}{10} \times 10 \times \frac{7}{2} \text{ cm}^3$$

$$= \frac{55 \times 7}{2} \text{ cm}^3 \qquad \ldots(2)$$

$\therefore$ Required number of silver coins

$$= \frac{\text{Volume of a cuboid}}{\text{Volume of a silver coin}}$$

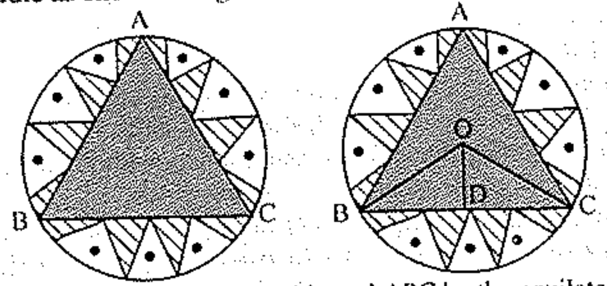
$$= \frac{\frac{55 \times 7}{2} \text{ cm}^3}{\frac{22 \times 7}{8 \times 8 \times 5} \text{ cm}^3} \qquad \text{[using (1) and (2)]}$$

$$= \frac{55 \times 7 \times 8 \times 8 \times 5}{2 \times 22 \times 7}$$

$$= 5 \times 2 \times 8 \times 5$$

$$= \mathbf{400}$$

28. In a circular table cover of radius 32 cm, a design is formed leaving an equilateral triangle *ABC* in the middle as shown in figure. Find the area of the design (shaded region).



Solution. Let O be the centre of a circular table and ABC be the equilateral triangle.

From O, draw $OD \perp BC$.

In ΔOBD, we have

$$\cos 60^\circ = \frac{OD}{OB} \Rightarrow \frac{1}{2} = \frac{OD}{32} \qquad [\because OB = \text{radius} = 32 \text{ cm}]$$

$$\Rightarrow \quad OD = 16 \text{ cm}$$

$$\sin 60^\circ = \frac{BD}{OB} \Rightarrow \frac{\sqrt{3}}{2} = \frac{BD}{32} \Rightarrow BD = 16\sqrt{3} \text{ cm}$$

$$\therefore \quad BC = 2\,BD = 2(16\sqrt{3}) = 32\sqrt{3} \text{ cm}$$

Area of the design (shaded region)

$$= \text{Area of the circle of radius 32 cm} - \text{Area of equilateral } \Delta ABC$$

$$= \left[\pi \times (32)^2 - \frac{\sqrt{3}}{4} \times \left(32\sqrt{3}\right)^2\right] \text{cm}^2 \qquad [\because BC = 32\sqrt{3} \text{ cm}]$$

$$= \left[\frac{22}{7} \times 32 \times 32 - \frac{\sqrt{3}}{4} \times 32 \times 32 \times 3\right] \text{cm}^2$$

$$= 32 \times 32\left[\frac{22}{7} - \frac{3\sqrt{3}}{4}\right] \text{cm}^2$$

$$= 1024\left[\frac{22}{7} - \frac{3\sqrt{3}}{4}\right] \text{cm}^2$$

$$= \left(\frac{22528}{7} - 768\sqrt{3}\right) \text{cm}^2.$$

Section 'D'

Question numbers 29 to 34 carry 4 marks each.

29. Derive the formula for the sum of first *n* terms of an A.P. whose first term is '*a*' and the common difference is '*d*'.

Solution. Let 'a' be the first term and 'd' the common difference of an A.P.

$a, a + d, a + 2d, \ldots\ldots$ ...(1)

The nth term of this A.P. is

$a_n = a + (n - 1)d$

Let us denote the sum upto n terms of the given A.P. (1) by S_n, then

$S_n = a + (a + d) + \ldots\ldots + [a + (n - 2)d] + [a + (n - 1)d]$ ...(2)

Also, writing the terms, starting from the last and finishing with the first, we have

$S_n = [a + (n - 1)d] + [a + (n - 2)d] + \ldots\ldots + (a + d) + a$ ...(3)

Adding the corresponding terms of equation (2) and equation (3), we have

$$2S_n = [(a) + \{a + (n-1)d\}] + [(a + d) + \{a + (n-2)d\}] + \ldots\ldots + [\{a + (n-2)d\} + (a + d)] + [\{a + (n-1)d\} + (a)]$$

$$= \underbrace{[2a + (n-1)d + [2a + (n-1)d] + \ldots\ldots + [2a + (n-1)d] + [2a + (n-1)d]}_{\text{Number of terms } = n}$$

$\Rightarrow\ 2S_n = n \times [2a + (n - 1)d]$

$\Rightarrow\ S_n = \frac{n}{2} \times [2a + (n - 1)d]$ ...(A)

From (A), we have

$$S_n = \frac{n}{2} \times [2a + (n-1)d]$$

$$= \frac{n}{2} \times [a + \{a + (n-1)d\}]$$

$= \frac{n}{2} \times [a + l]$, where l = last term of the A.P. $= a + (n - 1)d$

$= \frac{n}{2} \times [a_1 + a_n]$ ...(B). [Where a_1 = first term, a_n = last term]

$= \frac{n}{2} \times$ (1st term + last term)

30. From the top of a building 100 m high, the angles of depression of the top and bottom of a tower are observed to be 45° and 60° respectively. Find the height of the tower. Also find the distance between the foot of the building and the bottom of the tower.

Solution. Let AD (= 100 m) be the building and CE be the tower. Let $BC = DE = y$ m, be the distance between the foot of the building and the bottom of the tower and $CE = BD = x$ m be the height of the tower.

Then, $AB = AD - BD$

$\Rightarrow\ AB = (100 - x)$ m

be the difference of height between the building and the tower.

In right triangle ABC, we have

$$\tan 45° = \frac{AB}{BC}$$

$$\Rightarrow\ 1 = \frac{100 - x}{y}$$

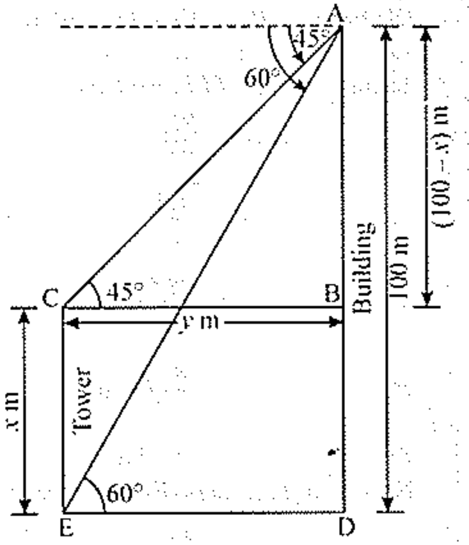
$\Rightarrow \quad y = (100 - x)$ m ...(1)

In right triangle ADE, we have

$$\tan 60^\circ = \frac{AD}{DE}$$

$$\Rightarrow \quad \sqrt{3} = \frac{100}{y}$$

$$\Rightarrow \quad y = \frac{100}{\sqrt{3}} \text{ m} \quad ...(2)$$



From (1) and (2), we get

$$100 - x = \frac{100}{\sqrt{3}}$$

$$\Rightarrow \quad 100\sqrt{3} - \sqrt{3}x = 100$$

$$\Rightarrow \quad \sqrt{3}x = 100\sqrt{3} - 100$$

$$\Rightarrow \quad \sqrt{3}x = 100(\sqrt{3} - 1)$$

$$\Rightarrow \quad x = \frac{100(\sqrt{3} - 1)}{\sqrt{3}}$$

$$\Rightarrow \quad x = \frac{100}{3}(\sqrt{3} - 1)\sqrt{3} \text{ m}$$

$$\Rightarrow \quad x = \frac{100}{3}(3 - \sqrt{3}) \text{ m}$$

Thus, the distance between the foot of the building and the bottom of the tower is $\frac{100}{\sqrt{3}} = \frac{100\sqrt{3}}{3}$ m $=$ **57.73 m** and the height of the tower is $\frac{100}{3}(3 - \sqrt{3})$ m $=$ **42.27 m.**

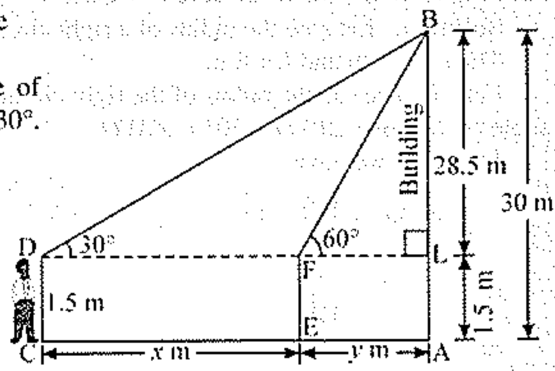
Or

A 1.5 m tall boy is standing at some distance from a 30 m tall building. The angle of elevation from his eyes to the top of the building increases from 30° to 60° as he walks towards the building. Find the distance he walked towards the building.

Solution. Let $AB = 30$ m be the height of the building. Let $CD = 1.5$ m be a tall boy.

Let D be the point of observation of the angle of elevation of the top of the building such that $\angle BDL = 30^\circ$.

Let $EF = 1.5$ m be the position of a tall boy after walking a distance of $CE = x$ m towards the building along the same horizontal line, such that F be the point of observation of the angle of elevation of the top of the building such that $\angle BFL = 60^\circ$.



We have $BL = AB - AL = 30 \text{ m} - 1.5 \text{ m} = 28.5 \text{ m}$ $[\because AL = 1.5 \text{ m} = EF = CD]$

Let $EA = FL = y$ m

In right triangle BFL, we have

$$\tan 60° = \frac{BL}{FL}$$

$$\Rightarrow \quad \sqrt{3} = \frac{28.5}{y} \quad [\because BL = 28.5 \text{ m}]$$

$$\Rightarrow \quad y = \frac{28.5}{\sqrt{3}} \text{ m}$$

$$\Rightarrow \quad y = \frac{28.5\sqrt{3}}{3} \text{ m}$$

$$\Rightarrow \quad y = 9.5\sqrt{3} \text{ m} \quad \ldots(1)$$

Now in right triangle BDL, we have

$$\tan 30° = \frac{BL}{DL}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{28.5}{DF + FL} \quad [\because BL = 28.5 \text{ m}]$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{28.5}{CE + EA}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{28.5}{x + y}$$

$$\Rightarrow \quad x + y = 28.5\sqrt{3}$$

$$\Rightarrow \quad x = 28.5\sqrt{3} - 9.5\sqrt{3} \quad [\text{using } (1)]$$

$$\Rightarrow \quad x = (28.5 - 9.5)\sqrt{3} \text{ m}$$

$$\Rightarrow \quad \mathbf{x = 19\sqrt{3} \text{ m.}}$$

31. The interior of a building is in the form of a right circular cylinder of radius 7 m and height 6 m surmounted by a right circular cone of vertical angle 60°. Find the cost of painting the building from inside at the rate of ₹ 30/m².

Solution. Let r be the radius of a right circular cylinder and h be its height.

Then, $r = 7$ m and $h = 6$ m.

Let r (= 7 m) be the radius of the right circular cone and l_1 be its slant height and the semi-vertical angle of the cone $\angle BVO = 30° = \angle AVO$.

In ΔVOA, we have

$$\sin 30° = \frac{OA}{VA}$$

$$\Rightarrow \quad \frac{1}{2} = \frac{7}{VA}$$

$$\Rightarrow \quad VA = 14 \text{ m} = l$$

Internal curved surface area of the building

$= \text{Curved surface area of the cylinder} + \text{Curved surface area of the cone}$

$= 2\pi rh + \pi rl$

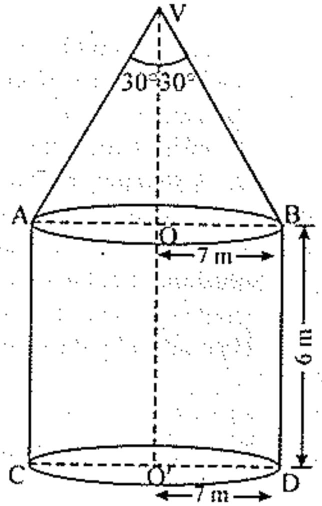
$= \pi(2rh + rl)$

$= \frac{22}{7}(2 \times 7 \times 6 + 7 \times 14) \text{ m}^2$

$= \frac{22}{7} \times 7(12 + 14) \text{ m}^2$

$= 22 \times 26 \text{ m}^2$

$= 572 \text{ m}^2$



Now, cost of painting the building from inside = ₹ 30/m^2

So, total cost of painting the building from inside of 572 m^2

= ₹ (572 × 30)

= **₹ 17160.**

Or

From a solid cylinder whose height is 8 cm and radius 6 cm, a conical cavity of height 8 cm and of base radius 6 cm, is hollowed out. Find the volume of the remaining solid correct to two places of decimals. Also find the total surface area of the remaining solid. [Take $\pi = 3.1416$]

Solution. We have

Volume of the remaining solid

$= \text{Volume of the cylinder} - \text{Volume of the cone}$

$= \pi r^2 h - \frac{1}{3}\pi r^2 h$

$= \pi r^2 h\left(1 - \frac{1}{3}\right)$

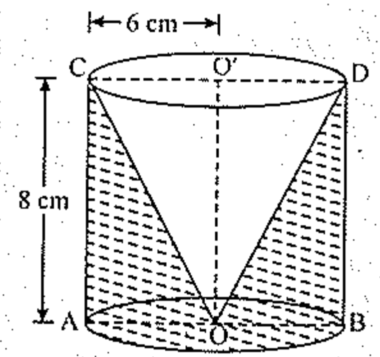
$= 3.1416 \times 6 \times 6 \times 8 \left(1 - \frac{1}{3}\right) \text{ cm}^3$

$= 3.1416 \times 6 \times 6 \times 8 \times \frac{2}{3} \text{ cm}^3$

$= 3.1416 \times 192 \text{ cm}^3$

$= 603.1872 \text{ cm}^3$

$= \mathbf{603.19 \text{ cm}^3}$ (upto two places of decimals)



Slant height of the cone $OC = \sqrt{(OO')^2 + (O'C)^2}$

$= \sqrt{(8)^2 + (6)^2} \text{ cm}$

$= \sqrt{64 + 36} \text{ cm}$

$= 10 \text{ cm}$

Total surface area of the remaining solid

= Curved surface area of the cylinder + Area of the base of the cylinder + Curved surface area of cone

$= 2\pi rh + \pi r^2 + \pi rl$

$= \pi r [2h + r + l]$
$= 3.1416 \times 6 [2 \times 8 + 6 + 10]$ cm^2
$= 3.1416 \times 6 \times 32$ cm^2
$= 603.1872$ cm^2
$=$ **603.19 cm^2** (upto two places of decimals).

32. A journey of 192 km from Mumbai to Pune takes 2 hours less by a superfast train than that by an ordinary passenger train. If the average speed of the slower train is 16 km/h less than that of the faster train, determine their average speeds.

Solution. Let the average speed of the slower train (*i.e.*, ordinary passenger train) be x km/h, then, the average speed of the faster train (*i.e.*, superfast train) is $(x + 16)$ km/h

Time taken by the ordinary passenger train for a journey of 192 km

$$= \frac{192}{x} \text{ hours}$$

Time taken by the faster train for a journey of 192 km

$$= \frac{192}{(x+16)} \text{ hours}$$

It is given that a journey of 192 km from Mumbai to Pune takes 2 hours less by a superfast train than by an ordinary passenger train.

$$\therefore \quad \frac{192}{x} - \frac{192}{(x+16)} = 2$$

$$\Rightarrow \quad 192 \times \left[\frac{1}{x} - \frac{1}{x+16}\right] = 2$$

$$\Rightarrow \quad 192 \times \left[\frac{(x+16) - x}{x(x+16)}\right] = 2$$

$$\Rightarrow \quad 192 \times 16 = 2x(x + 16)$$

$$\Rightarrow \quad 192 \times 8 = x^2 + 16x$$

$$\Rightarrow \quad x^2 + 16x - 1536 = 0$$

$$\Rightarrow \quad x^2 + 48x - 32x - 1536 = 0$$

$$\Rightarrow \quad x(x + 48) - 32(x + 48) = 0$$

$$\Rightarrow \quad (x + 48)(x - 32) = 0$$

$\Rightarrow$ Either $x + 48 = 0$ or $x - 32 = 0$

$\Rightarrow$ Either $x = -48$ or $x = 32$

$\Rightarrow \quad x = 32$ [$\because$ x, being the speed, cannot be negative]

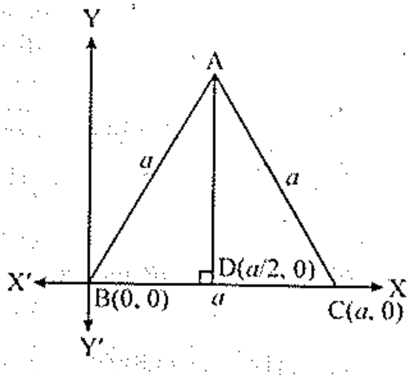
Hence, the average speed of ordinary passenger train is **32 km/h** and the average speed of faster train is **48 km/h**.

33. If a is the length of one of the sides of an equilateral triangle ABC, base BC lies on x-axis and vertex B, is at origin. Find the coordinates of the vertices of the triangle ABC.

Solution. Since the base BC of an equilateral ΔABC lies on x-axis and vertex B, is at origin. Therefore, the coordinates of B and C are $(0, 0)$ and $(a, 0)$. Let AD be the perpendicular from A on BC, then D is the mid-point of side of BC. Therefore, coordinates of D are

$$\left(\frac{0+a}{2}, \frac{0+0}{2}\right) = \left(\frac{a}{2}, 0\right)$$

In right triangle ADB, we have



$$AB^2 = AD^2 + BD^2$$

$$\Rightarrow \quad a^2 = AD^2 + \left(\frac{a}{2} - 0\right)^2 + (0-0)^2$$

$$\Rightarrow \quad AD^2 = a^2 - \frac{a^2}{4}$$

$$\Rightarrow \quad AD^2 = \frac{3a^2}{4} = \left(\frac{\sqrt{3}a}{2}\right)^2$$

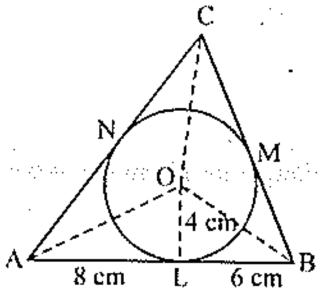
$$\Rightarrow \quad AD = \frac{\sqrt{3}a}{2}$$

Thus, the coordinates of point A are $\left(\frac{a}{2}, \frac{\sqrt{3}a}{2}\right)$.

Hence, the vertices of the triangle ABC are $A\left(\frac{a}{2}, \frac{\sqrt{3}a}{2}\right)$, $B(0, 0)$, $C(a, 0)$.

34. The radius of the incircle of a triangle is 4 cm and the segments into which one side is divided by the point of contact are 6 cm and 8 cm. Determine the other two sides of the triangle.

Solution. Let ABC be a triangle. A circle with centre O and radius 4 cm is inscribed in the triangle ABC.



The perpendicular OL from the centre O to the side AB of ΔABC divides the side AB by the point of contact L into two segments $AL = 8$ cm and $BL = 6$ cm.

In figure, AL and AN are the two tangents from an external point A to the circle with centre O.

$\therefore \quad AN = AL = 8$ cm [$\because AL = 8$ cm]

[$\because$ The lengths of tangents drawn from an external point to a circle are equal]

Again, BL and BM are the two tangents from an external point B to the circle with centre O.

$\therefore \quad BM = BL = 6$ cm [$\because BM = 6$ cm]

[Same reasoning as above]

Further, CM and CN are the two tangents from an external point C to the circle with centre O.

$\therefore \quad CM = CN = x$ cm, say.

Now, $\quad AB = AL + BL = 8 + 6 = 14$ cm

$\quad BC = BM + CM = 6 + x = (6 + x)$ cm

and $\quad CA = CN + AN = 8 + x = (8 + x)$ cm

$$2s = \text{Perimeter of a triangle } ABC$$
$$= AB + BC + CA = 14 + (6 + x) + (8 + x) = (28 + 2x) \text{ cm} \quad \ldots(1)$$
$$\Rightarrow \quad s = (14 + x) \text{ cm}$$
$$\text{Area of } \Delta ABC = \sqrt{s(s - AB)(s - BC)(s - CA)}$$
$$= \sqrt{(14 + x)(14 + x - 14)(14 + x - 6 - x)(14 + x - 8 - x)}$$
$$= \sqrt{(14 + x)(x)(8)(6)} \quad \ldots(2)$$

Now, join OA, OB and OC.

$\therefore$ Area of ΔABC = Area of ΔOAB + Area of ΔOBC + Area of ΔOCA.

$$\Rightarrow \sqrt{(14 + x)(x)(8)(6)} = \frac{1}{2} \times AB \times r + \frac{1}{2} \times BC \times r + \frac{1}{2} \times CA \times r \quad \text{[using (2)]}$$
$$= \frac{1}{2} r[AB + BC + CA]$$
$$= \frac{1}{2} \times 4 \times (28 + 2x) \quad \text{[using (1) and } r = 4 \text{ cm]}$$
$$\Rightarrow \sqrt{(14 + x) \times x \times 48} = 2 \times (28 + 2x)$$
$$\Rightarrow \sqrt{(14 + x) \times x \times 48} = 2 \times 2 \times (14 + x)$$

Squaring both sides, we get

$$(14 + x) \times x \times 48 = 16 \times (14 + x)^2$$
$$\Rightarrow \quad x \times 3 = 14 + x$$
$$\Rightarrow \quad 3x - x = 14$$
$$\Rightarrow \quad 2x = 14$$
$$\Rightarrow \quad x = 7 \text{ cm}$$

Hence, the other two sides of a triangle are **13 cm** ($BC = 6 + 7 = 13$) and **15 cm** ($CA = 8 + 7 = 15$) respectively.