Stage II

SAMPLE Question Paper

Fully Solved (Questions-Solutions)

MATHEMATICS

A Highly Simulated Practice Question Paper
for CBSE Class X Term II Examination (SA II)

Time : 3 hrs **Max. Marks : 90**

General Instructions

1. All questions are compulsory.
2. Draw neat labelled diagram whenever necessary to explain your answer.
3. Q. Nos. 1-8 are multiple choice questions, carrying 1 mark each.
4. Q. Nos. 9-14 are short answer type questions, carrying 2 marks each.
5. Q. Nos. 15-24 are short answer type questions, carrying 3 marks each.
6. Q. Nos. 25-34 are long answer type questions, carrying 4 marks each.

Section A

Que 1. If the roots of the quadratic equation $x^2+Kx+12=0$ are in the ratio $1:3$, then the values of K is

(a) ± 8 (b) ± 7
(c) ± 6 (d) ± 9

Que 2. If the surface areas of two spheres are in the ratio 4 : 9, then the ratio of their radii is

(a) 2 : 3 (b) 5 : 2
(c) 2 : 7 (d) 3 : 2

Que 3. If $3K+7, 15, 8K+12$ are three consecutive terms of an AP, then K is equal to

(a) 1 (b) 2
(c) -1 (d) -2

Que 4. A wire, in the shape of a square of side 88 cm, is bend so as to form a circular ring. The radius of the circle will be

(a) 54 cm (b) 50 cm
(c) 56 cm (d) 49 cm

Que 5. If the length of a tangent from a point A at a distance of 26 cm from the centre of the circle is 10 cm, then the radius of the circle is

(a) 22 cm (b) 24 cm
(c) 21 cm (d) 23 cm

Que 6. The distance between the points $(a\cos\theta, a\sin\theta)$ and $(-a\sin\theta, a\cos\theta)$ is

(a) $\sqrt{a}$ (b) a
(c) $2\sqrt{a}$ (d) $\sqrt{2}\,a$

Que 7. If the height and length of the shadow of a man are the same, then the angle of elevation of the Sun is

(a) 30° (b) 60°
(c) 45° (d) 15°

Que 8. A number x is selected from the numbers 1, 3, 4 and a number y is selected from 1, 2, 8. The probability that the product xy is less than 8, is

(a) $\frac{2}{9}$ (b) $\frac{3}{9}$ (c) $\frac{7}{9}$ (d) $\frac{5}{9}$

Section B

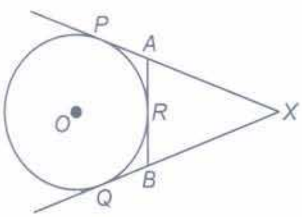
Que 9. One year ago, the father was 8 times as old as his son. Now, his age is the square of his son's age. Find their present ages.

Que 10. The internal and external diameters of a hollow hemispherical vessel are 24 cm and 25 cm respectively. If the cost of painting 1 cm^2 of the surface area is ₹ 0.05, find the total cost of painting the vessel all over. $\left[\text{take } \pi = \frac{22}{2}\right]$

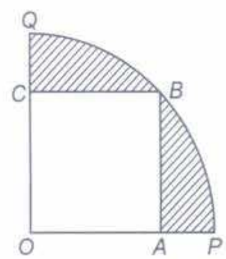
Que 11. Find the 10th term from end of the AP 4, 9, 14, …, 254.

Que 12. Find the volume and surface of a sphere whose radius is 7 cm.

Que 13. In the given figure, XP and XQ are tangents from X to the circle with centre O. R is a point on the circle. Prove that $XA + AR = XB + BR$.



Que 14. A square $OABC$ is inscribed in a quadrant $OPBQ$ of a circle as shown in figure. If $OA = 14$ cm. Find the area of the shaded region. $\left[\text{take } \pi = \frac{22}{7}\right]$



Section C

Que 15. A person on tour has ₹ 4200 for his expenses. If he extended his tour for 3 days, he has to cut down his daily expenses by ₹ 70. Find the original duration of the tour.

OR

The sum of the squares of two consecutive natural numbers is 421. Find the numbers.

Que 16. Neha has a cart whose wheels, are making 6 revolutions per second. If the diameter of the wheel is 77 cm, find the speed of the cart.

Que 17. Find the sum of n terms of the series $(a+b)^2 + (a^2+b^2) + (a-b)^2 + \ldots$

OR

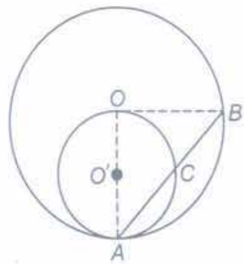
How many terms of the series $54 + 51 + 48 + 45 + \ldots$ must be taken to make 513? Explain the double answer.

Que 18. A median of a triangle divides it into two triangles of equal areas. Verify this result for ΔABC whose vertices are $A(4, -6)$, $B(3, -2)$ and $C(5, 2)$.

Que 19. A tree breaks due to strong wind and the broken part bends so that the top of the tree touches the ground making an angle of 30° with the ground. The distance between the foot of the tree and the point where the top touches the ground is 8 m. What is the height of the tree?

Que 20. The area of a triangle is 5. Two of its vertices are (2, 1) and (3, − 2). The third vertex is (x, y), where $y = x + 3$. Find the coordinates of third vertex.

Que 21. In figure, circles $C(O, r)$ and $C(O', r/2)$ touch internally at a point A and AB is a chord of the circle $C(O, r)$ intersecting $C(O', r/2)$ at C. Prove that $AC = CB$.



Que 22. An aeroplane, when 3000 m high, passes vertically above another aeroplane at an instant, when the angles of elevation of the two aeroplanes from the same point on the ground are 60° and 45° respectively. Find the vertical distance between the aeroplanes. [take $\sqrt{3} = 1.732$]

Que 23. Two dice are rolled once. Find the probability of getting such numbers on the two dices, whose product is 12.

OR

A bag contains tickets, numbered 11, 12, 13, ..., 30. A ticket is taken out from the bag at random. Find the probability that the number on the drawn ticket (i) is a multiple of 7 (ii) is greater than 15 and a multiple of 5.

Que 24. If the sum of 4th and 8th terms of an AP is 24 and the sum of 6th and 10th terms is 44. Find the first three terms of AP.

Section D

Que 25. Let the sum of ages of two friends Mohan and Sohan is 30 yr, where Mohan age is less than Sohan age. Three years ago, the product of their ages was 63 yr.

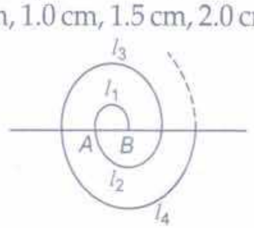
(a) Determine the present age of Mohan.
(b) Which mathematical concept is used to the solve the above problem?
(c) Is it possible that any one has age more than 30 yr?
(d) Discuss the benefits of true friendship?

Que 26. The radius of a solid iron sphere is 8 cm. Eight rings of iron plate of external radius $6\frac{2}{3}$ cm and thickness 3 cm are made by melting this sphere. Find the internal diameter of each ring.

Que 27. In an AP the sum of first n terms is $\frac{3n^2}{2} + \frac{5n}{2}$. Find its 25th term.

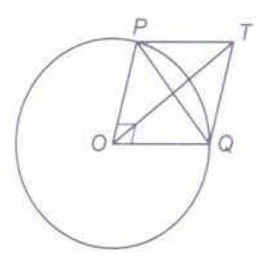
OR

A spiral is made up of successive semi-circles with centres alternately at A and B, starting with centre at A of radii 0.5 cm, 1.0 cm, 1.5 cm, 2.0 cm, ... as shown in figure. What is the total length of such a spiral made up of thirteen consecutive semi-circles? $\left[\text{take } \pi = \frac{22}{7}\right]$.



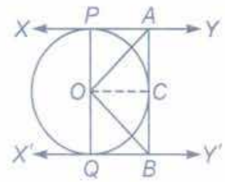
Que 28. The area of an equilateral triangle is 1732.05 cm^2. About each angular point as centre, a circle is described with radius equal to half the length of the side of the triangle. Find the area of the triangle not included in the circles. [take $\pi = 3.14$]

Que 29. In the figure, $PO \perp QO$. The tangent to the circle with centre O at P and Q intersect at a point T. Prove that PQ and OT are right bisectors of each other.



OR

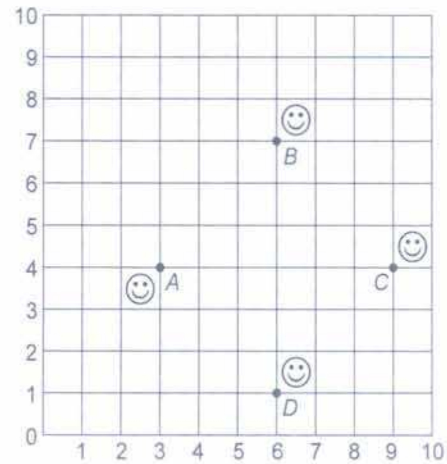
In figure, XY and $X'Y'$ are two parallel tangents to a circle with centre O and another tangent AB with point of contact C intersecting XY at A and $X'Y'$ at B. Prove that $\angle AOB = 90°$.



Que 30. *ABCD* is a flower bed. If $OA = 21$ m and $OC = 14$ m, find the area of the bed. $\left[\text{take } \pi = \frac{22}{7}\right]$

Que 31. Draw a right triangle in which the sides containing the right angle are 5 cm and 4 cm. Construct a similar triangle whose sides are $\frac{5}{3}$ times the sides of the above triangle.

Que 32. In a classroom, 4 friends are seated at the points *A*, *B*, *C* and *D* as shown in the figure. Priya and Jaanvi walk into the class and after observing for a few minutes Priya asks Jaanvi, "Don't you think *ABCD* is a square?" Jaanvi disagress. Using distance formula, find which of them is correct.



Que 33. A vertical tower subtends a right angle on the top of 10 m high flagstaff. If the distance between them is 20 m, then find the height of the tower.

Que 34. Cards with number 2 to 101 are placed in a box. A card is selected at random from the box. Find the probability that the card which is selected has a number which is a perfect square.

Solutions

1. (a) Let the roots of the equation $x^2 + Kx + 12 = 0$ be α and 3α, then

$$\alpha + 3\alpha = -K \Rightarrow 4\alpha = -K$$

$$\Rightarrow \quad \alpha = \frac{-K}{4} \quad \text{...(i)}$$

and $\quad \alpha \cdot (3\alpha) = 12 \Rightarrow \alpha^2 = 4 \quad$ ...(ii)

From Eqs. (i) and (ii), we get

$$\left(-\frac{K}{4}\right)^2 = 4 \Rightarrow K^2 = 4 \times 16 = 64 \Rightarrow K = \pm 8$$

2. (a) Given, ratio of surface areas $= 4 : 9$

$$\Rightarrow \quad \frac{S_1}{S_2} = \frac{4}{9} \Rightarrow \frac{4\pi r_1^2}{4\pi r_2^2} = \frac{4}{9}$$

$$\Rightarrow \quad \frac{r_1^2}{r_2^2} = \frac{4}{9} \Rightarrow \left(\frac{r_1}{r_2}\right)^2 = \left(\frac{2}{3}\right)^2$$

$$\Rightarrow \quad \frac{r_1}{r_2} = \frac{2}{3}$$

Thus, the ratio of their radii is 2 : 3.

3. (a) Given, $3K + 7, 15$ and $8K + 12$ are three consecutive terms of an AP, therefore,

$$15 - (3K + 7) = d \quad \text{and} \quad (8K + 12) - 15 = d$$

$$\Rightarrow \quad 8 - 3K = d \quad \text{and} \quad (8K - 3) = d$$

$$\therefore \quad 8 - 3K = 8K - 3$$

$$\Rightarrow \quad 11 = 11K$$

$$\Rightarrow \quad K = 1$$

4. (c) Given, side of square $= 88$ cm $\quad$ ...(i)

$\therefore$ Perimeter of a square $= (4 \times 88)$ cm

Let r be the radius of the circle, then

Circumference of the circle $= 2\pi r$ cm $\quad$ ...(ii)

From Eqs. (i) and (ii), we obtain

$$4 \times 88 = 2\pi r$$

$$\Rightarrow \quad 4 \times 88 = 2 \times \frac{22}{7} \times r$$

$$\Rightarrow \quad r = \frac{4 \times 88 \times 7}{44}$$

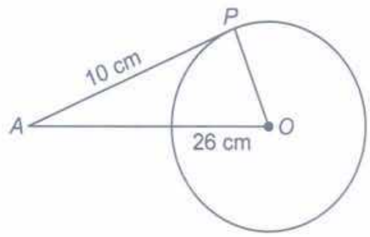
$$\Rightarrow \quad r = 4 \times 2 \times 7$$

$$\Rightarrow \quad r = 56 \text{ cm}$$

5. (b) Given, $OA = 26$ cm and $AP = 10$ cm

We know that, the tangent to a circle is perpendicular to the radius through the point of contact.

$\therefore \qquad \angle OPA = 90°$



In right angled ΔOPA, we have

$$OA^2 = OP^2 + AP^2$$

[by Pythagoras theorem]

$\Rightarrow \qquad (26)^2 = OP^2 + (10)^2$

$\Rightarrow \qquad OP^2 = 676 - 100 = 576 = (24)^2$

$\Rightarrow \qquad OP = 24$

Hence, the radius of the circle is 24 cm.

6. (d) Distance between the given points $(a \cos \theta, a \sin \theta)$ and $(-a \sin \theta, a \cos \theta)$

$= \sqrt{[a \cos \theta + a \sin \theta]^2 + [a \sin \theta - a \cos \theta]^2}$

$= \sqrt{a^2 [\cos^2 \theta + \sin^2 \theta + 2 \sin \theta \cos \theta] + a^2 [\sin^2 \theta + \cos^2 \theta - 2 \sin \cos \theta]}$

$= \sqrt{a^2 (1 + 2 \sin \theta \cos \theta) + a^2 (1 - 2 \sin \theta \cos \theta)}$

$= \sqrt{a^2 + a^2} = \sqrt{2a^2} = \sqrt{2} a$

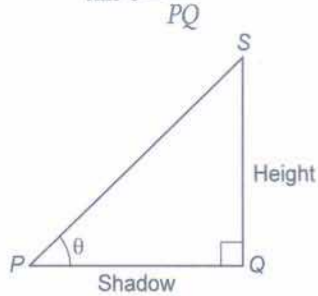
7. (c) Let SQ be the height of a man and his shadow is PQ.

Then, $\qquad PQ = SQ$

In right angled ΔPQS, $\quad \angle Q = 90°$

and angle of elevation, $\angle QPS = \theta$ [say]

$\therefore \qquad \tan \theta = \dfrac{QS}{PQ}$



$\Rightarrow \qquad \tan \theta = \dfrac{PQ}{PQ} \qquad [\because PQ = QS]$

$\Rightarrow \qquad \tan \theta = 1 \Rightarrow \theta = 45°$

Hence, angle of elevation of the Sun is 45°.

8. (d) Total number of products 'xy' are $3 \times 3 = 9$

$1 \times 1, 1 \times 2, 3 \times 1, 3 \times 2, 4 \times 1$, *i.e.*, 1, 2, 3, 6, 4 are less than 8.

So, the number of possible outcomes $= 5$

$\therefore \qquad$ Required probability $= \dfrac{5}{9}$

9. Let present age of the son $= x$ yr

$\therefore$ Present age of the father $= x^2$ yr

One year ago, age of the son $= (x - 1)$ yr

and age of the father $= (x^2 - 1)$ yr

According to question, $x^2 - 1 = 8(x - 1)$ **[1]**

$\Rightarrow \qquad x^2 - 1 - 8x + 8 = 0$

$\Rightarrow \qquad x^2 - 8x + 7 = 0$

$\Rightarrow \qquad x^2 - 7x - x + 7 = 0$

$\Rightarrow \qquad (x - 1)(x - 7) = 0$

$\Rightarrow$ Either $x - 1 = 0$ or $x - 7 = 0$

$\Rightarrow \qquad x = 1$ or $x = 7$

When $x = 1$, present age of son = present age of father Which is impossible.

$\therefore \qquad x = 7$

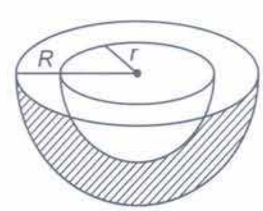
Hence, present age of son $= 7$ yr

and present age of father $= 49$ yr **[1]**

10. Internal radius of vessel, $r = \dfrac{24}{2} = 12$ cm

and external radius of vessel, $R = \dfrac{25}{2} = 12.5$ cm

Now, total surface area of the vessel = outer surface area + inner surface area + circular surface area of the edge



[1]

$= 2\pi R^2 + 2\pi r^2 + \pi(R^2 - r^2)$

$= 2\pi(r^2 + R^2) + \pi(R + r)(R - r)$

$= 2\pi(144 + 156.25) + \pi(12.5 + 12)(12.5 - 12)$

$= 2\pi(300.25) + \pi(24.5 \times 0.5) = \pi(600.50) + \pi(12.25)$

$= \pi(600.50 + 12.25) = \pi(612.75)$

$= \dfrac{22}{7} \times 612.75 = 1925.79 \text{ cm}^2$ **[1]**

11. Given, AP is 4, 9, 14, ..., 254.

Its common difference, $d = 9 - 4 = 5$

If we start from end and proceed towards begining, then also the numbers will be in AP but first term of this AP will be 254 (the last term) and common difference will be $-d$. **[1]**

Now, 10th term from end $= 254 + (10-1)(-d)$

$$= 254 - 9d$$
$$= 254 - 9 \times 5 = 209 \quad \textbf{[1]}$$

12. Given, radius of the sphere $= 7$ cm

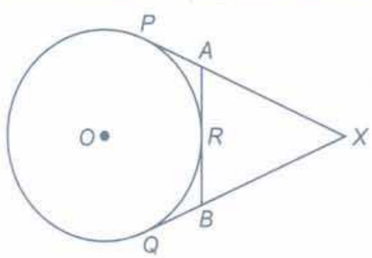
$\therefore$ Volume of the sphere $= \frac{4}{3}\pi r^3$

$$= \frac{4}{3} \times \frac{22}{7} \times (7)^3 \text{ cm}^3$$
$$= \frac{4}{3} \times \frac{22}{7} \times 7 \times 7 \times 7 \text{ cm}^3$$
$$= \frac{4312}{3} \text{ cm}^3 = 1437.3 \text{ cm}^3 \quad \textbf{[1]}$$

and surface area of the sphere $= 4\pi r^2$

$$= 4 \times \frac{22}{7} \times 7 \times 7 \text{ cm}^2 = 616 \text{ cm}^2 \quad \textbf{[1]}$$

13. *Given* XP and XQ are tangents from X to the circle having centre O and R is a point on the circle.



To prove $XA + AR = XB + BR$

Proof We know that the lengths of tangents from an external point to a circle are equal. **[1]**

$XP = XQ$ [tangent from X] ...(i)

$AP = AR$ [tangent from A] ...(ii)

and $BQ = BR$ [tangent from B] ...(iii)

Now, $XP = XQ$

$\Rightarrow XA + AP = XB + BQ$

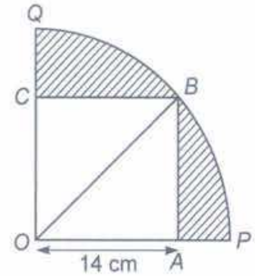
$\Rightarrow XA + AR = XB + BR$

[from Eqs. (ii) and (iii)]

14. Given, side of square, $OA = 14$ cm

Diagonal of square, $OB = \text{side}\sqrt{2} = 14\sqrt{2}$ cm $=$ radius

Area of the shaded region = Area of quadrant $OPBQ$ – Area of square $OABC$ **[1]**



$$= \frac{1}{4}\pi r^2 - (\text{side})^2 = \frac{1}{4} \times \frac{22}{7} \times (14\sqrt{2})^2 - (14)^2$$
$$= \frac{11}{14} \times 14 \times 14 \times 2 - 196 = 308 - 196 = 112 \text{ cm}^2 \quad \textbf{[1]}$$

15. Let the original duration of the tour $= x$ days

$\therefore$ The increased duration of the tour $= (x+3)$ days

According to the question,

$$\frac{4200}{x} - \frac{4200}{x+3} = 70 \quad \textbf{[1]}$$

$$\left[\because \text{Daily expenses} = \frac{\text{Total amount}}{\text{Number of days}}\right]$$

$$\Rightarrow \frac{4200(x+3) - 4200(x)}{x(x+3)} = 70$$
$$\Rightarrow \frac{4200(x+3-x)}{x(x+3)} = 70$$
$$\Rightarrow \frac{3}{x(x+3)} = \frac{70}{4200} = \frac{1}{60} \quad \textbf{[1/2]}$$
$$\Rightarrow x(x+3) = 180$$
$$\Rightarrow x^2 + 3x - 180 = 0$$
$$\Rightarrow x^2 + 15x - 12x - 180 = 0$$
$$\Rightarrow x(x+15) - 12(x+15) = 0$$
$$\Rightarrow (x-12)(x+15) = 0$$
$$\Rightarrow x - 12 = 0 \text{ or } x + 15 = 0$$
$$\Rightarrow x = 12 \text{ or } x = -15$$

But number of days cannot be negative $\therefore x = 12$

$\therefore$ Original duration of the tour $= 12$ days **[1½]**

OR

Let the two consecutive natural numbers be x and $x+1$.

Then, according to the question,

$$\therefore \quad x^2 + (x+1)^2 = 421$$
$$\Rightarrow x^2 + x^2 + 1 + 2x - 421 = 0 \quad \textbf{[1]}$$
$$\Rightarrow 2x^2 + 2x - 420 = 0$$
$$\Rightarrow x^2 + x - 210 = 0$$

$\Rightarrow \quad x^2 + 15x - 14x - 210 = 0$ **[1]**

$\Rightarrow \quad x(x+15) - 14(x+15) = 0$

$\Rightarrow \quad (x+15)(x-14) = 0$

$\Rightarrow \quad x = 14, \; x \neq -15$

[$\because$ x is natural number]

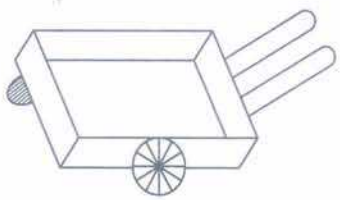
Hence, the required consecutive natural numbers are14 and 15. **[1]**

16. Given, diameter of the wheel $= 77$ cm

$\therefore$ Radius of the wheel $= \dfrac{77}{2}$ cm

Now, the distance travelled by the cart in one revolution $= 2\pi r$

$= 2 \times \dfrac{22}{7} \times \dfrac{77}{2}$ cm $= 242$ cm **[1]**



$\therefore$ The distance travelled by the cart in 6 revolutions

$= 6 \times 242$ cm $= 1452$ cm **[1]**

Thus, distance travelled in one second $= 1452$ cm

$\therefore$ The distance travelled in one hour $= 3600 \times 1452$ cm

[$\because$ 1 h $= 60 \times 60 = 3600$ s]

Speed (in km/hour) $= \dfrac{3600 \times 1452}{100 \times 1000}$ km/h $= 52.272$

Hence, required speed $= 52.272$ km/h **[1]**

17. Terms of given series are in AP whose first term $= (a+b)^2$ and common difference,

$d = a^2 + b^2 - (a+b)^2 = -2ab$ **[1]**

Now, sum of n terms of the given series,

$S_n = \dfrac{n}{2}[2(a+b)^2 + (n-1)(-2ab)]$

$= \dfrac{n}{2} \cdot 2[a^2 + b^2 + 2ab - (n-1)ab]$ **[1]**

$= n(a^2 + b^2) + nab(3-n)$

$= -abn^2 + n(a^2 + b^2 + 3ab)$ **[1]**

OR

Given series is an AP, where $a = 54$, $d = 51 - 54 = -3$

Let the sum of n terms of the given series be 513.

Then, $S_n = \dfrac{n}{2}[2a + (n-1)d]$

$\Rightarrow \quad 513 = \dfrac{n}{2}[2 \times 54 + (n-1)(-3)] = \dfrac{n}{2}(111 - 3n)$ **[1]**

$\Rightarrow \quad 1026 = 111n - 3n^2$

$\Rightarrow \quad 3n^2 - 111n + 1026 = 0$

$\Rightarrow \quad n^2 - 37n + 342 = 0$

$\Rightarrow \quad n^2 - 18n - 19n + 342 = 0$ **[1]**

$\Rightarrow \quad (n-18)(n-19) = 0 \Rightarrow n = 18, 19$

Now, 18th term $= 54 + (18-1)(-3) = 3$

and 19th term $= 54 + (19-1)(-3) = 0$

The sum of 18 terms and sum of 19 terms will be both 513, since 19th term is zero. So, n can take double values. **[1]**

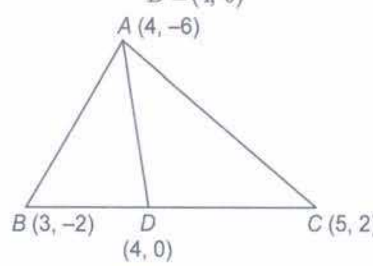
18. Given, ABC be a triangle such that

$A \equiv (4, -6)$, $B \equiv (3, -2)$ and $C \equiv (5, 2)$

Let AD be a median of ΔABC.

Then, D will be the mid-point of BC.

$\therefore \quad D \equiv (4, 0)$ **[1]**



Now, area of ΔABD

$= \dfrac{1}{2}|[4(-2-0) + 3(0-(-6)) + 4(-6+2)]|$

$= \dfrac{1}{2}|-8 + 18 - 16| = \dfrac{1}{2}|-6|$

$= \dfrac{1}{2} \times 6 = 3$ sq units

and area of ΔADC

$= \dfrac{1}{2}|[4(0-2) + 4(2+6) + 5(-6-0)]|$ **[1]**

$= \dfrac{1}{2}|(-8 + 32 - 30)| = \dfrac{1}{2}|-6|$

$= \dfrac{1}{2} \times 6 = 3$ sq units

Clearly, Area of ΔABD = Area of ΔADC **[1]**

19. Let AB be a tree. Let it be broken at point P in such a manner that the top A takes the position at point C on the ground. Let AP be x m, then

$PC = x$ m

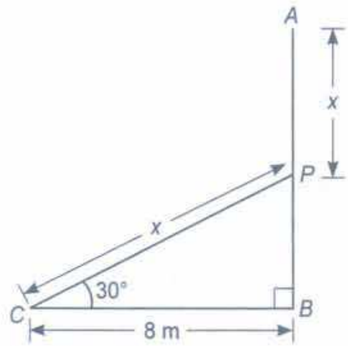
Then, according to the question,

$\angle PCB = 30°$ and $BC = 8$ m

In right angled ΔPBC,

$$\cos 30° = \frac{BC}{PC}$$

$$\Rightarrow \quad \frac{\sqrt{3}}{2} = \frac{8}{x}$$ **[1]**



$$\Rightarrow \quad \sqrt{3}\,x = 16$$

$$\Rightarrow \quad x = \frac{16}{\sqrt{3}} \text{ m} \quad \text{...(i)}$$

Again, $\tan 30° = \frac{PB}{BC}$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{PB}{8}$$

$$\Rightarrow \quad PB = \frac{8}{\sqrt{3}} \text{ m} \quad \text{...(ii)}$$ **[1]**

$\therefore$ Original height of the tree,

$$AB = AP + PB$$

$$= \frac{16}{\sqrt{3}} \text{ m} + \frac{8}{\sqrt{3}} \text{ m}$$

$$= \frac{24}{\sqrt{3}} \text{ m}$$

$$= \frac{24 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}} \text{ m} = 8\sqrt{3} \text{ m}$$ **[1]**

20. Let $A \equiv (x, y)$, $B \equiv (2, 1)$ and $C(3, -2)$.

Given, area of $\Delta ABC = 5$

$$\Rightarrow \left| \frac{1}{2}[x(1+2) + 2(-2-y) + 3(y-1)] \right| = 5$$ **[1]**

$$\Rightarrow \quad \frac{1}{2}|3x + y - 7| = 5$$

$$\Rightarrow \quad |3x + y - 7| = 10$$

$$\Rightarrow \quad 3x + y - 7 = \pm 10$$

$$\Rightarrow \quad 3x + y = 17 \quad \text{or} \quad 3x + y = -3$$

Case I When $3x + y = 17$ ...(i)

It is given that, $y = x + 3 \Rightarrow x - y = -3$ ...(ii) **[1]**

On solving Eqs. (i) and (ii), we get

$$x = \frac{7}{2} \text{ or } y = \frac{13}{2}$$

Case II When $3x + y = -3$ ...(iii)

On solving Eqs. (ii) and (iii), we get

$$x = -\frac{3}{2} \text{ or } y = \frac{3}{2}$$

Hence, coordinates of the third vertex are $\left[\frac{7}{2}, \frac{13}{2}\right]$ or $\left[-\frac{3}{2}, \frac{3}{2}\right]$. **[1]**

21. Here, $\angle OCA$ is the angle in a semi-circle.

$$\therefore \quad \angle OCA = 90°$$

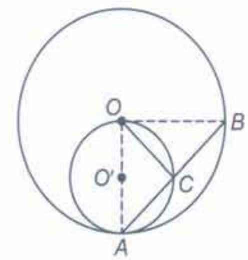
Since, AB is a straight line (chord)

$$\therefore \quad \angle OCB = 90° \quad [\because \angle OCA = 90°]$$ **[1]**

In right angled Δ's OCA and OCB, we have

$$\therefore \quad OA = OB \quad \text{[radii of circle]}$$

$$\angle OCA = \angle OCB = 90°$$



[1]

and $OC = OC$ [common]

So, by RHS, criterion of congruence, we get

$$\Delta OCA \cong \Delta OCB$$

$$\therefore \quad AC = CB$$ **[1]**

22. Let O be the point of observation. Let P and Q be the positions of the two aeroplanes.

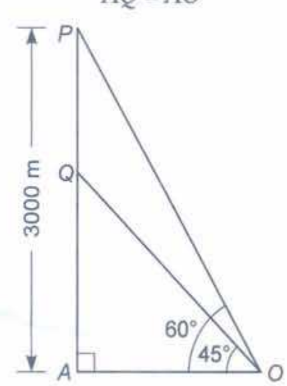
Given, $AP = 3000$ m, $\angle AOQ = 45°$

and $\angle AOP = 60°$

In right angled ΔQAO,

$$\tan 45° = \frac{AQ}{AO} = 1$$

$$\Rightarrow \quad AQ = AO \quad \text{...(i)}$$ **[1]**



[1]

Again, in right angled ΔPAO,

$$\tan 60^\circ = \frac{PA}{AO}$$

$$\Rightarrow \quad \sqrt{3} = \frac{3000}{AO} \Rightarrow AO = \frac{3000}{\sqrt{3}} \quad \text{...(ii)}$$

$\therefore$ From Eqs. (i) and (ii), we have

$$AQ = \frac{3000}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = 1000\sqrt{3}$$

$$= 1000 \times 1.732 = 1732 \text{ m}$$

Now, $PQ = AP - AQ = (3000 - 1732) \text{ m} = 1268 \text{ m}$

Hence, vertical distance between the aeroplanes is

$= 1268$ m **[1]**

23. When two dice are rolled once, then the possible outcomes of experiment are listed in the table

	1	2	3	4	5	6
1	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
2	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
3	(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
4	(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
5	(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
6	(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

So, the total number of outcomes $= 6 \times 6 = 36$ **[1½]**

Let A be the event of getting such numbers on the two dice, whose product is 12, then the outcomes favourable to A are

$$A = \{(2, 6), (6, 2), (3, 4), (4, 3)\}$$

$\therefore$ Favourable number of outcomes $= 4$

Hence, probability of getting such number on the two dice, whose product is 12.

$$P(A) = \frac{4}{36} = \frac{1}{9}$$ **[1½]**

OR

When a ticket is taken out from the tickets numbered 11, 12, 13, ..., 30

Total number of possible outcomes = 20

(i) Tickets with 'a multiple of 7' are 14, 21, 28.

$\therefore$ Number of tickets containing a number which is a multiple 7 $= 3$

$$\therefore \quad P(\text{a multiple of } 7) = \frac{3}{20}$$ **[1½]**

(ii) Number greater than 15 and multiples of 5 are 20, 25, 30.

$\therefore$ Number of tickets containing a number greater than 15 and a multiple of 5 $= 3$

$$\therefore \quad P(\text{a multiple of } 5 > 15) = \frac{3}{20}$$ **[1½]**

24. Let a be the first term and d be the common difference of given AP.

Given, $t_4 + t_8 = 24$

$\Rightarrow \quad (a + 3d) + (a + 7d) = 24$

$\Rightarrow \quad 2a + 10d = 24$ ...(i) **[1]**

Also, it is given that

$t_6 + t_{10} = 44$

$\Rightarrow \quad (a + 5d) + (a + 9d) = 44$

$\Rightarrow \quad 2a + 14d = 44$ ...(ii) **[1]**

On subtracting Eq. (i) from Eq. (ii), we get

$$4d = 20 \Rightarrow d = 5$$

From Eq. (i), $2a = 24 - 10d = 24 - 10 \times 5 = -26$

$\Rightarrow \quad a = -13$

Thus, $a = -13$ and $d = 5$

$\therefore$ First three terms of AP are $-13, -8, -3$

[add 5 to the preceding term] **[1]**

25. (a) Let the present age of Mohan be x yr. Then, the present age of Sohan be $(30 - x)$ yr.

[$\because$ the sum of their ages is 30 yr]

3 yr ago, Mohan's age was $(x - 3)$ yr

and Sohan's age was $(30 - x - 3)$ yr *i.e.*, $(27 - x)$ yr. **[1]**

According to the given condition,

$$(x - 3)(27 - x) = 63$$

$\Rightarrow \quad 27x - x^2 - 81 + 3x = 63$

$\Rightarrow \quad x^2 - 30x + 81 + 63 = 0$

$\Rightarrow \quad x^2 - 30x + 144 = 0$

$\Rightarrow \quad (x - 6)(x - 24) = 0$

$\Rightarrow \quad x = 6$ yr, 24 yr **[1]**

Hence, the present age of Mohan is 6 yr.

(b) Quadratic equation as a mathematical concept is used to solve the above problem.

(c) No, none of the persons have age more than 30 yr. **[1]**

(d) There are many benefits of true friendship

(i) You are not alone at any circumstances that comes across your life. A friend is there to help you out at the worst situations and share your happiness to spread it all around.

(ii) You can openly share your feelings with your friend. So that they may guide you of what is wrong or right without misguiding. **[1]**

26. Given, radius of sphere = 8 cm $\therefore$ volume of solid iron sphere $= \frac{4}{3}\pi \times 8^3 \text{ cm}^3 = \frac{2048}{3}\pi \text{ cm}^3$. External radius of each iron ring $= 6\frac{2}{3}$ cm $= \frac{20}{3}$ cm. Let the internal radius of each ring $= r$ cm. Since, each ring forms a hollow cylindrical shell of external and internal radii $\frac{20}{3}$ cm and r cm respectively and thickness 3 cm. **[1]**

$\therefore$ Volume of each ring $= \pi\left\{\left(\frac{20}{3}\right)^2 - r^2\right\} \times 3 \text{ cm}^3$ **[1]**

Then, volume of 8 such rings

$$= 8\pi\left\{\frac{400}{9} - r^2\right\} \times 3 \text{ cm}^3 = 24\pi\left[\frac{400}{9} - r^2\right] \text{cm}^3$$

Clearly, volume of 8 rings = volume of the sphere **[1]**

$$\Rightarrow \quad 24\pi\left[\frac{400}{9} - r^2\right] = \frac{2048}{3}\pi$$

$$\Rightarrow \quad \frac{400}{9} - r^2 = \frac{2048}{3}\pi \times \frac{1}{24\pi}$$

$$\Rightarrow \quad r^2 = \frac{400}{9} - \frac{256}{9} = \frac{144}{9} = 16$$

$$\Rightarrow \quad r = 4 \text{ cm}$$

Hence, internal diameter of each ring is $2 \times 4 = 8$ cm **[1]**

27. Let S_n denotes the sum of n terms of an AP whose nth term is a_n.

Given, $\quad S_n = \frac{3n^2}{2} + \frac{5n}{2}$

$\therefore \quad S_{n-1} = \frac{3}{2}(n-1)^2 + \frac{5}{2}(n-1)$

[replacing n by $(n-1)$] **[1]**

Now, $\quad S_n - S_{n-1} = \left\{\frac{3n^2}{2} + \frac{5n}{2}\right\} - \left\{\frac{3}{2}(n-1)^2 + \frac{5}{2}(n-1)\right\}$

$\Rightarrow \quad a_n = \frac{3}{2}\{n^2 - (n-1)^2\} + \frac{5}{2}\{n - (n-1)\}$ **[1]**

[$\because a_n = S_n - S_{n-1}$]

$\Rightarrow \quad a_n = \frac{3}{2}(2n-1) + \frac{5}{2}$

On putting $n = 25$, we get 25th term

$$a_{25} = \frac{3}{2}(2 \times 25 - 1) + \frac{5}{2} = \frac{3}{2} \times 49 + \frac{5}{2} = 76 \quad \textbf{[2]}$$

OR

Length of spiral made up of thirteen consecutive semi-circles.

$= (\pi \times 0.5 + \pi \times 1.0 + \pi \times 1.5 + \pi \times 2.0 + ... + \pi \times 6.5)$

$= \pi \times 0.5\,(1 + 2 + 3 + ... + 13)$ **[1]**

$= \pi \times 0.5 \times \frac{13}{2}[2 \times 1 + (13-1) \times 1]$

[which form an AP with first term, $a = 1$

common difference, $d = 2 - 1 = 1$

and number of terms, $n = 13$ **[1]**

Now, sum of n terms $= \frac{n}{2}[2a + (n-1)d]]$

$$= \frac{22}{7} \times \frac{5}{10} \times \frac{13}{2} \times 14$$

$$= 143 \text{ cm}$$

Hence, total length of spiral is 143 cm. **[2]**

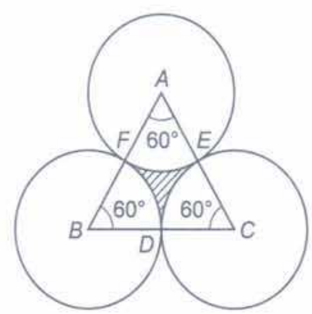
28. Let each side of the equilateral triangle be a cm and radius of each circle is $\frac{a}{2}$ cm.

Given, Area of triangle $= 1732.05 \text{ cm}^2$ **[1]**

$$\Rightarrow \quad \frac{\sqrt{3}}{4}a^2 = 1732.05$$

$$\Rightarrow \quad \frac{a^2}{4} = \frac{1732.05}{\sqrt{3}}$$

$$\Rightarrow \quad \left(\frac{a}{2}\right)^2 = \frac{1732.05}{\sqrt{3}} \quad ...(i)$$



[1]

Let A be the area of three sectors each of angle 60° in a circle with radius $\frac{a}{2}$ cm. Then,

$$A = 3\left\{\frac{60}{360} \times 3.14 \times \left(\frac{a}{2}\right)^2\right\} \text{cm}^2$$

$$\Rightarrow \quad A = \frac{1}{2} \times 3.14 \times \left(\frac{a}{2}\right)^2 = \frac{1}{2} \times 3.14 \times \frac{1732.05}{\sqrt{3}} \quad \textbf{[1]}$$

$$= 1570.04 \text{ cm}^2$$

Now, required area = area of ΔABC − area of three sector of angle 60° in a circle of radius $\frac{a}{2}$

$\Rightarrow$ Required area $= [1732.05 - 1570.04]\ \text{cm}^2$

$= 162.01\ \text{cm}^2$ **[1]**

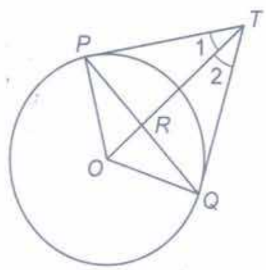
29. We know that, tangent to any circle is perpendicular to radius at point of contact.

$\therefore \quad \angle TPO = \angle TQO = 90°$

In right angled ΔTPO and ΔTQO, we have

$OP = OQ$ [radii of a circle]

$OT = OT$ [common side] **[1]**



$\therefore$ By RHS congruency, we have

$\Delta TPO \cong \Delta TQO$

$\angle 1 = \angle 2$ [CPCT] **[1]**

Again, in ΔPTR and ΔQTR

$\angle 1 = \angle 2$ [proved above]

$PT = QT$

[tangent from external point]

$TR = TR$ [common]

By SAS congruency, we have

$\Delta PTR \cong \Delta QTR$ **[1]**

$\angle PRT = \angle QRT$ [CPCT]

But $\angle PRT + \angle QRT = 180°$ [linear pair]

Thus, $\angle PRT = \angle QRT = 90°$

Also, $PR = QR$

$\therefore$ PQ and OT are right bisector of each other. **[1]**

OR

We know that, tangents drawn from an external point to a circle are equal.

$\therefore \quad AP = AC$

Thus, in ΔAPO and ΔACO **[1]**

$AP = AC$

$AO = AO$ [common]

$OP = OC$ [radius of circle]

$\therefore$ By SSC criterion of congruence, we have

$\Delta APO \cong \Delta ACO$

$\Rightarrow \quad \angle PAO = \angle OAC$

$\Rightarrow \quad \angle PAC = 2\angle CAO$ **[1]**

Similarly, we can prove that

$\angle CBO = \angle OBQ$

$\Rightarrow \quad \angle CBQ = 2\angle CBO$

Since, $xy \parallel x'y'$

$\therefore \quad \angle PAC + \angle QBC = 180°$

[sum of interior angles on the same side of transversal is 180°] **[1]**

$\therefore \quad 2\angle CAO + 2\angle CBO = 180°$

$\Rightarrow \quad \angle CAO + \angle CBO = 90°$ ...(i)

In ΔAOB, $\angle CAO + \angle CBO + \angle AOB = 180°$

$\angle CAO + \angle CBO = 180° - \angle AOB$ ...(ii)

Now, from Eqs. (i) and (ii), we get

$180° - \angle AOB = 90°$

$\Rightarrow \quad \angle AOB = 90°$ **[1]**

30. Given,

$OA = R = 21$ m and $OC = r = 14$ m

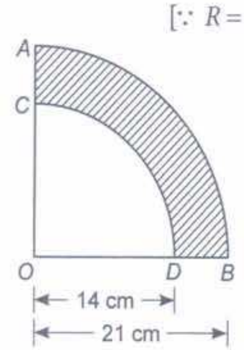
$\therefore$ Area of the flower bed = Area of quadrant of a circle of radius R − Area of a quadrant of a circle of radius r **[1]**

$= \frac{1}{4}\pi R^2 - \frac{1}{4}\pi r^2$

$= \frac{\pi}{4}(R^2 - r^2)$

$= \frac{1}{4} \times \frac{22}{7}(21^2 - 14^2)\ \text{cm}^2$ **[1]**

[$\because R = 21$ m and $r = 14$ m]



[1]

$= \left\{\frac{1}{4} \times \frac{22}{7} \times (21+14)(21-14)\right\}\ \text{m}^2$

$= \left\{\frac{1}{4} \times \frac{22}{7} \times 35 \times 7\right\}\ \text{m}^2 = 192.5\ \text{m}^2$ **[1]**

31. *Steps of construction*

Step I Draw a straight line $BC = 5$ cm

Step II From B draw a line $AB = 4$ cm making a right angle with BC.

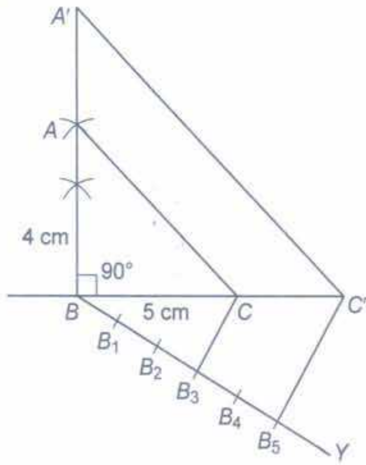
Step III Join AC, ΔABC is the given right angled triangle. **[1]**

Step IV From B draw any ray BY making an acute $\angle CBY$ downwards.

Step V On BY take five points B_1, B_2, B_3, B_4 and B_5 such that $BB_1 = B_1B_2 = B_2B_3 = B_3B_4 = B_4B_5$.

Step VI Join B_3C. **[1]**

Step VII From point B_5 draw $B_5C' \parallel B_3C$ producing BC to C'. **[1]**



Step VIII From point C' draw $C'A' \parallel CA$ producing BA to A'.

Thus, $\Delta A'BC'$ is the required triangle. **[1]**

32. From figure it is clear that

$A \equiv (3, 4)$, $B \equiv (6, 7)$, $C \equiv (9, 4)$ and $D \equiv (6, 1)$ **[1]**

Now, $AB = \sqrt{(3-6)^2 + (4-7)^2} = \sqrt{18} = 3\sqrt{2}$

$BC = \sqrt{(6-9)^2 + (7-4)^2} = \sqrt{18} = 3\sqrt{2}$

$CD = \sqrt{(9-6)^2 + (4-1)^2} = \sqrt{18} = 3\sqrt{2}$

and $DA = \sqrt{(6-3)^2 + (1-4)^2} = \sqrt{18} = 3\sqrt{2}$ **[1]**

Also, diagonals

$AC = \sqrt{(3-9)^2 + (4-4)^2} = 6$

and $BD = \sqrt{(6-6)^2 + (7-1)^2} = 6$

Since, $AB = BC = CD = DA$ and $AC = BD$

Therefore, $ABCD$ is a square.

Hence, Priya is correct. **[2]**

33. Let AB be the tower and CD be the flagstaff, we draw $PC \parallel BD$.

According to the question,

$\angle ACB = 90°$, $BD = PC = 20$ m

$PB = CD = 10$ m

In right angled ΔBDC,

$BC^2 = CD^2 + BD^2$

$= (10)^2 + (20)^2 = 100 + 400 = 500$ **[1]**

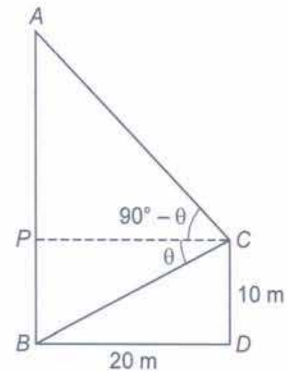
Again, in right angled ΔACB,

$AB^2 = AC^2 + BC^2$

$\Rightarrow \quad AB^2 = AP^2 + PC^2 + BC^2$

[$\because$ in right angled $\Delta APC = AP^2 + PC^2 = AC^2$]

$\Rightarrow \quad AB^2 = AP^2 + BD^2 + BC^2$ [$\because$ $PC = BD$] **[1]**



$\Rightarrow \quad AB^2 = (AB - BP)^2 + (20)^2 + 500$ **[1]**

$\Rightarrow \quad AB^2 = (AB - 10)^2 + 400 + 500$

$\Rightarrow \quad AB^2 = AB^2 - 20AB + 100 + 900$

$\Rightarrow \quad 20AB = 1000$

$\Rightarrow \quad AB = \frac{1000}{20} = 50$

Hence, height of the tower $= 50$ m **[1]**

34. Total number of cards in a box, with number 2 to 101 are 100 [*i.e.*, 101 − 1 = 100] **[1]**

$\therefore$ Total number of outcomes in which one card can be selected is 100.

Let A be the event that the number on the selected card is "a number which is a perfect square".

There are 9 perfect square numbers from 2 to 101 namely

$4 (= 2^2), 9 (= 3^2), 16 (= 4^2), 25 (= 5^2),$ **[1]**

$36 (= 6^2), 49 (= 7^2), 64 (= 8^2), 81 (= 9^2), 100 (= 10^2)$

$\therefore$ Number of outcomes favourable to event $A = 9$

Hence, the required probability, $P(A) = \frac{9}{100}$ **[2]**