Stage II

Sample QUESTION PAPER

Fully Solved **(Questions-Solutions)**

MATHEMATICS

A Highly Simulated Practice Question Papers for **CBSE Class X**

Term I *Examination* (SA I)

Time : 3 hrs **Max. Marks :** 90

General Instructions

1. All questions are compulsory.
2. Draw neat labelled diagram whenever necessary to explain your answer.
3. Question Numbers 1- 8 are multiple choice questions, carrying 1 mark each.
4. Question Numbers 9-14 are short answer type questions, carrying 2 marks each.
5. Question Numbers 15-24 are short answer type questions, carrying 3 marks each.
6. Question Numbers 25-34 are long answer type questions, carrying 4 marks each.

Section A

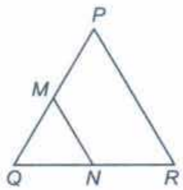
Que 1. n^2-1 is divisible by 8, if n is

(a) an integer
(b) a natural number
(c) an odd integer
(d) an even integer

Que 2. Graph of $p(x)=7x+1$ is

(a) a parabola
(b) intersects x-axis in two points
(c) a straight line intersecting x-axis in exactly one point
(d) None of the above

Que 3. In the given figure, $MN \parallel PR$, then



(a) $\frac{PR}{MN}=\frac{PQ}{QR}$ (b) $QN = NR$

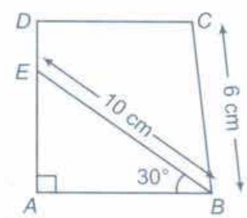
(c) $\frac{PR}{QR}=\frac{QM}{MP}$ (d) $\frac{PM}{MQ}=\frac{RN}{NQ}$

Que 4. If $\sin A = \frac{9}{15}$ and $\cos A = \frac{12}{15}$, then $\tan A$ is equal to

(a) $\frac{12}{9}$ (b) $\frac{9}{12}$

(c) $\frac{12}{9}$ (d) $\frac{15}{9}$

Que 5. In the given figure, length of AE is equal to



(a) 4 cm (b) 5 cm
(c) 6 cm (d) 8 cm

Que 6. Median of the observation

x_i	5	6	7	8	9	10
f_i	4	5	7	9	7	6

(a) 9 (b) 10 (c) 7 (d) 8

Que 7. ΔABC is a right angled at B. AC is known as

(a) side adjacent to $\angle A$ (b) side adjacent to $\angle B$
(c) hypotenuse (d) side opposite to $\angle C$

Que 8. If the angle remains the same, then the value of the trigonometric ratios of the angle

(a) vary with the length of the sides of the triangle

(b) do not vary with the length of the sides of the triangle

(c) vary with the change in length of hypotenuse only

(d) do not vary with the change in length of hypotenuse only

Section B

Que 9. $\frac{163}{150}$ will have a terminating decimal expansion. State true or false. Justify your answer.

Que 10. Prove that $\frac{\cos\theta}{1-\sin\theta} = \sec\theta + \tan\theta$.

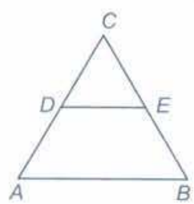
OR

Without using trigonometric tables, find the value of

$$\frac{\sin 50^\circ}{\cos 40^\circ} + \frac{\text{cosec}\, 40^\circ}{\sec 50^\circ} - 4\cos 50^\circ \,\text{cosec}\, 40^\circ.$$

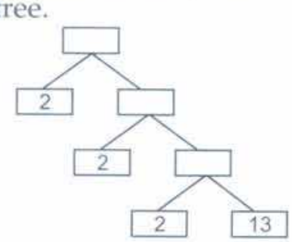
Que 11. A cricketer has a mean score of 58 runs in nine innings. Find out how many runs are to be scored in the tenth inning to raise the mean score to 61.

Que 12. In the given figure, $\angle A = \angle B$ and $AD = BE$, show that $DE \parallel AB$.



Que 13. Evaluate $\frac{2\tan 30^\circ}{1+\tan^2 30^\circ}$.

Que 14. Find the missing numbers in prime factors tree.



Section C

Que 15. Using Euclid's division algorithm, find HCF of 216 and 1176.

Que 16. Draw the graph of the equations $x-y+1=0$ and $3x+2y-12=0$. Determine the coordinates of the vertices of the triangle formed by these lines and the x-axis and shade the triangular region.

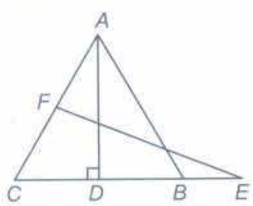
Que 17. Find the value of

$$\frac{\cos^2 20° + \cos^2 70°}{\sin^2 31° + \sin^2 59°} + \sin^2 64° + \cos 64° \cdot \sin 26°.$$

Que 18. Two isosceles triangles have equal angles and their areas are in the ratio 81 : 25. Find the ratio of their corresponding heights.

Que 19. In the adjoining figure, E is a point on side BC produced of an isosceles ΔABC with $AB = AC$.

If $AD \perp BC$ and $EF \perp AC$. Prove that $\Delta ABD \sim \Delta ECF$.



OR

In an isosceles ΔABC with $AB = AC$, BD is perpendicular from B to the side AC. Prove that $BD^2 - CD^2 = 2\, CD \cdot AD$.

Que 20. Show that $\dfrac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta} = \tan\theta$.

Que 21. Find the zeroes of the polynomial $y^2 + \frac{3}{2}\sqrt{5}y - 5$ by factorisation method and verify the relations between the zeroes and the coefficients of the polynomial.

OR

Prove that $\dfrac{1+\cos A}{\sin A} + \dfrac{\sin A}{1+\cos A} = 2 \operatorname{cosec} A$.

Que 22. If the mean of the following data is 18.75. Find the value of p.

x_i	10	15	p	25	30
f_i	5	10	7	8	2

Que 23. Find the sum of the deviations of the variate values 3, 4, 6, 7, 8 and 14 from their mean.

Que 24. Show that square of an odd positive integer can be of the form $6q+1$ or $6q+3$ for some integer q.

OR

Prove that, if x and y are both odd positive integers, then $x^2 + y^2$ is even but not divisible by 4.

Section D

Que 25. If $\tan\theta + \sec\theta = l$, then prove that $\sec\theta = \dfrac{l^2+1}{2l}$.

Que 26. CD and GH are respectively the bisectors of $\angle ACB$ and $\angle EGF$ such that D and H lie on sides AB and FE of ΔABC and ΔEFG, respectively. If $\Delta ABC \sim \Delta FEG$. Show that

(i) $\dfrac{CD}{GH} = \dfrac{AC}{FG}$ (ii) $\Delta DCB \sim \Delta HGE$

Que 27. It can take 12 h to fill a swimming pool using two pipes. If the pipe of larger diameter is used for 4 h and the pipe of smaller diameter is used for 9 h, only half of the pool can be filled. How Long would it take for each pipe to fill the pool separately?

OR

A man start his job with a certain monthly salary and earns a fixed increment every year. If his salary was ₹ 1500 after 4 yr of service and ₹ 1800 after 10 yr of service, what was his starting salary and what is the annual increment?

Que 28. The following table gives the weekly consumption of electricity of 56 families

Weekly consumption	0-10	10-20	20-30	30-40	40-50
Number of families	16	12	18	6	4

Find the mean.

Que 29. If 2 and 3 are zeroes of polynomial $3x^2 - 2kx + 2m$, find the values of k and m.

OR

α and β are zeroes of the polynomial $x^2 - 6x + a$, find the value of a, if $3\alpha + 2\beta = 20$.

Que 30. Prove that

$$\frac{\cos A}{1 - \sin A} + \frac{\sin A}{1 - \cos A} + 1 = \frac{\sin A \cos A}{(1 - \sin A)(1 - \cos A)}.$$

Que 31. Solve for x and y

$$6x + 3y = 8x + 9y - 5 = 10x + 12y - 8.$$

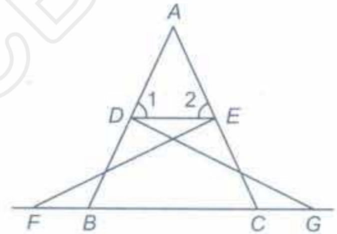
Que 32. Ram went to the market to purchase some T-shirt and shoes. When her father asked his how many of each he had bought, he answered, "The number of shoes is 4 less than 4 times the number of T-shirts purchased. Also, the number of shoes is 8 less than 8 times the number of T-shirts purchased." Help his father to find

(i) How many T-shirts and shoes Ram bought?

(ii) Which mathematical concept is used to solve the question?

(iii) Which value (s) are hidden behind conductivity in the question?

Que 33. In figure, $\Delta FEC \cong \Delta GBD$ and $\angle 1 = \angle 2$. Prove that $\Delta ADE \sim \Delta ABC$.



Que 34. If the mean of the following frequency distribution is 65.6. Find the missing frequencies f_1 and f_2.

Class interval	Frequency
10-30	5
30-50	8
50-70	f_1
70-90	20
90-110	f_2
110-130	2
Total	50

Solutions

1. (*c*) Let $a=n^2-1$, where $n=1, 3, 5, \ldots$

At $n=1$, $a=1-1=0$

At $n=3$, $a=9-1=8$

At $n=5$, $a=25-1=24$ ... etc,

which is divisible by 8.

2. (*c*) Graph of $p(x)=7x+1$ is a straight line intersecting x-axis in exactly one point. Because it is a linear equation in one variable.

3. (*d*) By basic proportionality theorem $\frac{PM}{MQ}=\frac{RN}{NQ}$.

4. (*b*) We know that,

$$\tan A = \frac{\sin A}{\cos A} = \frac{\frac{9}{15}}{\frac{12}{15}}$$

$$\left[\because \sin A = \frac{9}{15}, \cos A = \frac{12}{15}, \text{given}\right]$$

$$= \frac{9}{15} \times \frac{15}{12} = \frac{9}{12}$$

5. (*b*) From figure, $\sin 30° = \frac{P}{H} = \frac{AE}{EB}$

$$\Rightarrow \sin 30° = \frac{AE}{10}$$

$$\Rightarrow \frac{1}{2} = \frac{AE}{10} \quad \left[\because \sin 30° = \frac{1}{2}\right]$$

$$\Rightarrow AE = \frac{10}{2}$$

$$\Rightarrow AE = 5 \text{ cm}$$

6. (*d*)

x	*f*	*cf*
5	4	4
6	5	9
7	7	16
8	9	25
9	7	32
10	6	38
Total	38	

Here, $N=38$

$$\Rightarrow \frac{N}{2} = \frac{38}{2} = 19$$

$\therefore$ Median $= 8$

7. (*c*) *AC* is known as hypotenuse.

8. (*b*) If the angle remains the same, the value of the trigonometric ratios of the angle do not vary with the length of the sides of the triangle.

9. $\frac{163}{150} = \frac{163}{2^1 \times 3^1 \times 5^2}$ **[1]**

It has non-terminating decimal expansion, because denominator is of the form $2^1 \times 3^1 \times 5^2$. So, given statement is false. **[1]**

10. Taking LHS $= \frac{\cos\theta}{1-\sin\theta}$

On multiplying by $(1+\sin\theta)$ in numerator and denominator, we get

$$= \frac{\cos\theta}{(1-\sin\theta)} \times \frac{(1+\sin\theta)}{(1+\sin\theta)} \quad \textbf{[1]}$$

$$= \frac{\cos\theta(1+\sin\theta)}{(1-\sin^2\theta)}$$

$$= \frac{\cos\theta(1+\sin\theta)}{\cos^2\theta} \quad \left[\begin{array}{l}\because \sin^2\theta + \cos^2\theta = 1 \\ \Rightarrow \cos^2\theta = 1-\sin^2\theta\end{array}\right]$$

$$= \frac{1+\sin\theta}{\cos\theta} = \sec\theta + \tan\theta$$

$=$ RHS **[1]**

OR

Sol. Given,

$$\frac{\sin 50°}{\cos 40°} + \frac{\operatorname{cosec} 40°}{\sec 50°} - 4\cos 50° \operatorname{cosec} 40°$$

$$= \frac{\sin 50°}{\cos(90°-50°)} + \frac{\operatorname{cosec} 40°}{\sec(90°-40°)} - 4\cos(90°-40°)\operatorname{cosec} 40° \quad \textbf{[1]}$$

$$= \frac{\sin 50°}{\sin 50°} + \frac{\operatorname{cosec} 40°}{\operatorname{cosec} 40°} - 4\sin 40° \cdot \operatorname{cosec} 40°$$

$$= 1+1-4\sin 40° \cdot \frac{1}{\sin 40°} \quad \left[\because \operatorname{cosec}\theta = \frac{1}{\sin\theta}\right]$$

$$= 2-4 = -2 \quad \textbf{[1]}$$

11. Mean score of nine innings $= 58$

$\therefore$ Total score of nine innings $= 9 \times 58 = 522$ **[1]**

Mean score of ten innings $= 61$

$\therefore$ Total score of ten innings $= 61 \times 10 = 610$

Runs to be scored of tenth innings $= 610 - 522$
$= 88$ runs **[1]**

12. Given, $\angle A = \angle B$ [ ]

$\therefore$ $AC = BC$ ...(i)

Also, $AD = BE$ ...(ii) **[1]**

On dividing Eq. (i) from Eq. (ii), we get

$$\frac{AC}{AD} = \frac{BC}{BE}$$

$\therefore$ $DE \parallel AB$ **[1]**

[using converse of basic proportionality theorem]

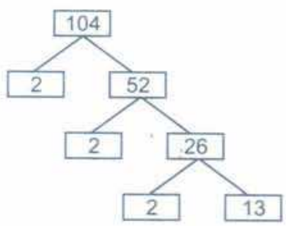
13. Given,

$$\frac{2\tan 30^\circ}{1+\tan^2 30^\circ}=\frac{2\times\frac{1}{\sqrt{3}}}{1+\left(\frac{1}{\sqrt{3}}\right)^2} \qquad \left[\because \tan 30^\circ=\frac{1}{\sqrt{3}}\right] \text{ [1]}$$

$$=\frac{\frac{2}{\sqrt{3}}}{1+\frac{1}{3}}=\frac{\frac{2}{\sqrt{3}}}{\frac{3+1}{3}}$$

$$=\frac{2}{\sqrt{3}}\times\frac{3}{4}=\frac{\sqrt{3}}{2}$$ **[1]**

14.



$\left[1\frac{1}{2}\right]$

i.e., 104, 52 and 26 are answers. **[1/2]**

15. Since, 1176 > 216, we apply the Euclid's division algorithm,

$$1176 = 216\times 5+96$$ **[1]**

$$216 = 96\times 2+24$$ **[1]**

$$96 = 24\times 4+0$$ **[1]**

Hence, the HCF (1176, 216) = 24

16. Given,

$$x-y+1=0 \quad \text{...(i)}$$

and $3x+2y-12=0$ ...(ii)

From Eq. (i),

$$-y=-(x+1)$$

$\Rightarrow$ $y=x+1$

If $x=0$, then $y=0+1=1$

If $x=1$, then $y=1+1=2$

If $x=-1$, then $y=-1+1=0$

If $x=2$, then $y=2+1=3$

x	0	1	−1	2
y	1	2	0	3

[1/2]

From Eq. (ii),

$$2y=12-3x$$

$\Rightarrow$ $y=\frac{12-3x}{2}$

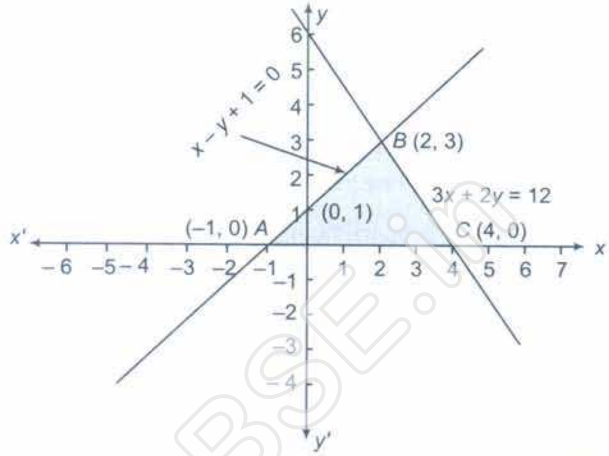
If $x=0$, then $y=\frac{12-3\times 0}{2}=6$

If $x=2$, then $y=\frac{12-3\times 2}{2}=3$

If $x=4$, then $y=\frac{12-3\times 4}{2}=0$

x	0	4	2
y	6	0	3

[1/2]



$\left[1\frac{1}{2}\right]$

$\therefore$ Required vertices are $A(-1, 0)$, $B(2, 3)$ and $C(4, 0)$. **[1/2]**

17. Given,

$$\frac{\cos^2 20^\circ+\cos^2 70^\circ}{\sin^2 31^\circ+\sin^2 59^\circ}+\sin^2 64^\circ+\cos 64^\circ\cdot\sin 26^\circ$$

$$=\frac{\cos^2(90^\circ-70^\circ)+\cos^2 70^\circ}{\sin^2(90^\circ-59^\circ)+\sin^2 59^\circ}$$ **[1]**

$$+\sin^2 64^\circ+\cos 64^\circ\cdot\sin(90^\circ-64^\circ)$$

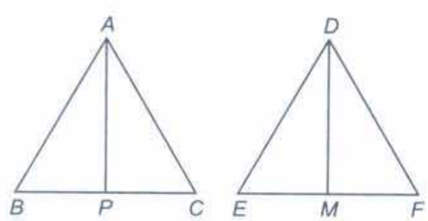
$$=\frac{\sin^2 70^\circ+\cos^2 70^\circ}{\cos^2 59^\circ+\sin^2 59^\circ}+\sin^2 64^\circ+\cos 64^\circ\cdot\cos 64^\circ$$ **[1]**

$$\left[\begin{array}{l}\because \cos\theta=\sin(90^\circ-\theta)\\ \sin\theta=\cos(90^\circ-\theta)\end{array}\right]$$

$$=\frac{1}{1}+\sin^2 64^\circ+\cos^2 64^\circ$$

$$=1+1=2$$ **[1]**

18. In ΔABC and ΔDEF,



$$\frac{AB}{AC}=\frac{DE}{DF}=1$$

[triangles are isosceles and have equal angles]

As $\angle A=\angle D$

$\therefore$ $\Delta ABC \sim \Delta DEF$

$$\Rightarrow \quad \frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta DEF)} = \frac{BC^2}{EF^2} \quad \textbf{[1]}$$

$$\Rightarrow \quad \frac{81}{25} = \frac{BC^2}{EF^2}$$

$$\therefore \quad \frac{BC}{EF} = \frac{9}{5}$$

Also, $$\frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta DEF)} = \frac{\frac{1}{2} \times BC \times AP}{\frac{1}{2} \times EF \times DM} \quad \textbf{[1]}$$

$$\Rightarrow \quad \frac{81}{25} = \frac{9}{5} \times \frac{AP}{DM}$$

$$\Rightarrow \quad \frac{AP}{DM} = \frac{81}{25} \times \frac{5}{9}$$

$$\Rightarrow \quad \frac{AP}{DM} = \frac{9}{5}$$

$$\therefore \quad AP : DM = 9 : 5 \quad \textbf{[1]}$$

19. ΔABC is an isosceles triangle.

$AB = AC$ [given]

$\therefore \angle B = \angle C$ [angles opposite to equal sides] **[1]**

Now, in ΔABD and ΔECF, we have

$\angle ABD = \angle ECF$ [$\because \angle B = \angle C$] **[1]**

$\angle ADB = \angle EFC = 90°$ [$\because$ $AD \perp BC$ and $EF \perp AC$]

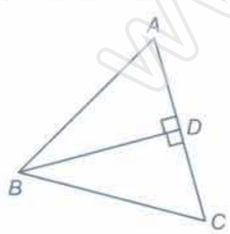
$\therefore \Delta ABD \sim \Delta ECF$ [by AAA similarity criterion] **[1]**

Hence proved.

OR

Sol. Given ΔABC is an isosceles triangle with $AB = AC$ and $BD \perp AC$.

To prove $BD^2 - CD^2 = 2\,CD \cdot AD$ **[1]**



Proof In right angled ΔADB,

Using Pythagoras theorem,

$$AB^2 = BD^2 + AD^2$$

$$\Rightarrow \quad AC^2 = BD^2 + AD^2 \quad [\because AB = AC]$$

$$\Rightarrow \quad (AD + CD)^2 = BD^2 + AD^2 \quad \textbf{[1]}$$

$[\because AC = AD + CD]$

$$\Rightarrow \quad AD^2 + CD^2 + 2AD \cdot CD = BD^2 + AD^2$$

$[\because (a+b)^2 = a^2 + b^2 + 2ab]$

$$\Rightarrow \quad 2AD \cdot CD = BD^2 + AD^2 - AD^2 - CD^2$$

$$\Rightarrow \quad 2AD \cdot CD = BD^2 - CD^2$$

$$\Rightarrow \quad BD^2 - CD^2 = 2AD \cdot CD \quad \textbf{[1]}$$

Hence proved.

20. We have,

$$\frac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta} = \tan\theta$$

$$\text{LHS} = \frac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta}$$

$$= \frac{\sin\theta(1 - 2\sin^2\theta)}{\cos\theta(2\cos^2\theta - 1)} \quad \textbf{[1]}$$

$$= \frac{\sin\theta(\sin^2\theta + \cos^2\theta - 2\sin^2\theta)}{\cos\theta[2\cos^2\theta - (\sin^2\theta + \cos^2\theta)]}$$

$[\because \sin^2\theta + \cos^2\theta = 1]$

$$= \frac{\sin\theta(\cos^2\theta - \sin^2\theta)}{\cos\theta(2\cos^2\theta - \sin^2\theta - \cos^2\theta)} \quad \textbf{[1]}$$

$$= \frac{\sin\theta(\cos^2\theta - \sin^2\theta)}{\cos\theta(\cos^2\theta - \sin^2\theta)}$$

$$= \frac{\sin\theta}{\cos\theta}$$

$$= \tan\theta = \text{RHS} \quad \textbf{[1]}$$

21. Let $f(y) = y^2 + \frac{3}{2}\sqrt{5}y - 5$

$$= 2y^2 + 3\sqrt{5}y - 10$$
$$= 2y^2 + 4\sqrt{5}y - \sqrt{5}y - 10$$
$$= 2y(y + 2\sqrt{5}) - \sqrt{5}(y + 2\sqrt{5})$$
$$= (y + 2\sqrt{5})(2y - \sqrt{5}) \quad \textbf{[1]}$$

So, the value of $y^2 + \frac{3}{2}\sqrt{5}y - 5$ is zero when $(y + 2\sqrt{5})$ or $(2y - \sqrt{5})$ *i.e.*, when $y = -2\sqrt{5}$ or $y = \frac{\sqrt{5}}{2}$.

So, the zeroes of $2y^2 + 3\sqrt{5}y - 10$ are $-2\sqrt{5}$ and $\frac{\sqrt{5}}{2}$. **[1]**

$$\therefore \text{ Sum of zeroes} = -2\sqrt{5} + \frac{\sqrt{5}}{2} = \frac{-3\sqrt{5}}{2}$$

$$= -\frac{(\text{Coefficient of } x)}{(\text{Coefficient of } x^2)} = -\frac{(3\sqrt{5})}{2} \quad \textbf{[1/2]}$$

and product of zeroes $= -2\sqrt{5} \times \frac{\sqrt{5}}{2} = -5$

$$= \frac{\text{Constant term}}{\text{Coefficient of } x^2} = -\frac{10}{2} = -5 \quad \textbf{[1/2]}$$

OR

Sol. $$\text{LHS} = \frac{1 + \cos A}{\sin A} + \frac{\sin A}{1 + \cos A}$$

$$= \frac{(1 + \cos A)^2 + \sin^2 A}{\sin A(1 + \cos A)}$$

$$= \frac{1 + 2\cos A + \cos^2 A + \sin^2 A}{\sin A(1 + \cos A)}$$ **[1]**

$$[\because (a+b)^2 = a^2 + b^2 + 2ab]$$

$$= \frac{1 + 2\cos A + 1}{\sin A(1 + \cos A)} \quad [\because \sin^2 A + \cos^2 A = 1]$$

$$= \frac{2 + 2\cos A}{\sin A(1 + \cos A)}$$ **[1]**

$$= \frac{2(1 + \cos A)}{\sin A(1 + \cos A)}$$

$$= \frac{2}{\sin A} = 2\operatorname{cosec} A \left[\because \operatorname{cosec} A = \frac{1}{\sin A}\right]$$ **[1]**

= RHS

22. Given, mean = 18.75

x_i	f_i	$f_i \times x_i$
10	5	50
15	10	150
p	7	$7p$
25	8	200
30	2	60
Total	$\sum f_i = 32$	$\sum f_i x_i = 460 + 7p$

[1]

$\sum f_i = 32$ and $\sum f_i x_i = 460 + 7p$

$\therefore \quad \text{Mean} = \frac{\sum f_i x_i}{\sum f_i}$ **[1]**

$\Rightarrow \quad 18.75 = \frac{460 + 7p}{32}$

$\Rightarrow \quad 18.75 \times 32 = 460 + 7p$

$\Rightarrow \quad 600 = 460 + 7p$

$\Rightarrow \quad 600 - 460 = 7p$

$\Rightarrow \quad 140 = 7p$

$\therefore \quad p = \frac{140}{7} = 20$ **[1]**

23. Variate are 3, 4, 6, 7, 8 and 14.

$\therefore \quad \text{Mean}\ (\bar{x}) = \frac{3 + 4 + 6 + 7 + 8 + 14}{6}$ **[1]**

$= \frac{42}{6} = 7$

Sum of deviations from $\bar{x} = 7$ is

$(3-7) + (4-7) + (6-7) + (7-7) + (8-7) + (14-7)$

$= -4 + (-3) + (-1) + 0 + 1 + 7$ **[1]**

$= -4 - 3 - 1 + 8$

$= -8 + 8 = 0$ **[1]**

24. Let $a = 6q + 1$ be any odd positive integer.

On squaring both sides, we get

$a^2 = (6q + 1)^2$

$\Rightarrow \quad a^2 = 36q^2 + 1 + 12q$ **[1]**

$\Rightarrow \quad a^2 = 36q^2 + 12q + 1$

$\Rightarrow \quad a^2 = 6(6q^2 + 2q) + 1$

$\Rightarrow \quad a^2 = 6m + 1$, where $m = 6q^2 + 2q$

$\therefore$ Square of any odd positive integer is of the form $6m + 1$.

Similarly, for $6q + 3$. **[2]**

OR

Sol. Let $x = 2m+1$ and $y = 2m+3$ are odd positive integers, for every positive integer m. **[1]**

Then, $x^2 + y^2 = (2m+1)^2 + (2m+3)^2$

$= 4m^2 + 1 + 4m + 4m^2 + 9 + 12m$

$= 8m^2 + 16m + 10 =$ even **[1]**

$= 2(4m^2 + 8m + 5)$

or $4(2m^2 + 4m + 2) + 1$

$\therefore\ x^2 + y^2$ is even for every positive integer m.

But not divisible by 4. **[1]**

25. Given, $\tan\theta + \sec\theta = l$

On squaring both sides, we get

$(\tan\theta + \sec\theta)^2 = l^2$ **[1]**

$\Rightarrow \quad \tan^2\theta + \sec^2\theta + 2\tan\theta \cdot \sec\theta = l^2$

$[\because (a+b)^2 = a^2 + b^2 + 2ab]$

$\Rightarrow \quad \sec^2\theta - 1 + \sec^2\theta + 2\tan\theta \cdot \sec\theta = l^2$

$[\because \tan^2\theta = \sec^2\theta - 1]$

$\Rightarrow \quad 2\sec^2\theta + 2\tan\theta \cdot \sec\theta = l^2 + 1$ **[2]**

$\Rightarrow \quad 2\sec\theta(\sec\theta + \tan\theta) = l^2 + 1$

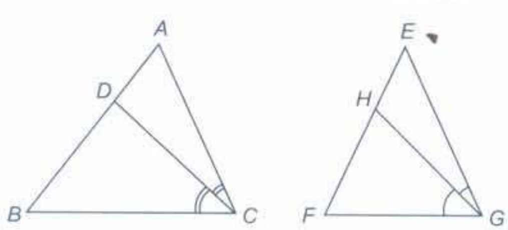
$\Rightarrow \quad 2\sec\theta \cdot l = l^2 + 1$

$\Rightarrow \quad 2\sec\theta = \frac{l^2 + 1}{l}$

$\therefore \quad \sec\theta = \frac{l^2 + 1}{2l}$ **[1]**

Hence proved.

26. Draw two ΔABC and ΔEFG along that draw two bisectors CD and GH of $\angle ACB$ and $\angle EGF$.



Since, $\Delta ABC \sim \Delta EFG$

(i) In ΔACD and ΔFGH

$\angle CAD = \angle GFH$ ...(i)

$\begin{cases} \because \Delta ABC \sim \Delta FEG \\ \therefore \angle CAB = \angle GFE \\ \Rightarrow \angle CAD = \angle GFH \end{cases}$

$\angle ACD = \angle FGH$ ...(ii) **[1]**

$\begin{cases} \because \Delta ABC \sim \Delta FEG \\ \therefore \angle ACB = \angle FGE \\ \Rightarrow \frac{1}{2}\angle ACB = \frac{1}{2}\angle FGE \end{cases}$

$\begin{cases} \text{Halves of equals are equal} \\ \Rightarrow \angle ACD = \angle FGH \end{cases}$

From Eqs. (i) and (ii), we get

$\Delta ACD \sim \Delta FGH$ [$\because$ AA similarity criterion]

$\therefore \quad \frac{CD}{GH} = \frac{AC}{FG}$

[$\because$ corresponding sides of two similar triangles are proportional] **[1]**

(ii) In ΔDCB and ΔHGE,

$\angle DBC = \angle HEG$ ...(iii)

$\begin{cases} \because \Delta ABC \sim \Delta FEG \\ \therefore \angle ABC = \angle FEG \\ \Rightarrow \angle DBC = \angle HEG \end{cases}$

$\angle DCB = \angle HGE$ ...(iv) **[1]**

$\begin{cases} \because \Delta ABC \sim \Delta FEG \\ \therefore \angle ACB = \angle FGE \\ \Rightarrow \frac{1}{2}\angle ACB = \frac{1}{2}\angle FGE \end{cases}$

[Halves of equals are equal]

$\Rightarrow \quad \angle DCB = \angle HGE$

From Eqs. (iii) and (iv), we get

$\Delta DCB \sim \Delta HGE$ **[1]**

[$\because$ AA similarity criterion]

27. Let time taken by larger pipe = y

and time taken by smaller pipe = x

Ist condition, $\frac{12}{x} + \frac{12}{y} = 1$ ...(i) **[1/2]**

2nd condition, $\frac{4}{y} + \frac{9}{x} = \frac{1}{2}$ ...(ii) **[1/2]**

On multiplying Eq. (ii) by 3 and then subtracting from Eq. (i), we get

$$\frac{12}{x} + \frac{12}{y} = 1$$

$$\frac{12}{y} + \frac{27}{x} = \frac{3}{2}$$

$$\frac{12}{x} - \frac{27}{x} = 1 - \frac{3}{2}$$

$\Rightarrow \quad -\frac{15}{x} = -\frac{1}{2}$

$\Rightarrow \quad \therefore \quad x = 30$

On putting the value of x in Eq. (i), we get

$$\frac{12}{30} + \frac{12}{y} = 1$$ **[2]**

$\Rightarrow \quad \frac{12}{y} = 1 - \frac{12}{30}$

$\Rightarrow \quad \frac{12}{y} = \frac{18}{30}$

$\therefore \quad y = \frac{12 \times 30}{18} = 20$

Hence, time taken to fill the pool by larger pipe = 20 h and time taken to fill the pool by smaller pipe = 30 h **[1]**

OR

Sol. Let monthly salary = ₹ x

and fixed increment = ₹ y

According to the question,

$x + 4y = 1500$ ...(i) **[1]**

and $x + 10y = 1800$ ...(ii) **[1]**

On subtracting Eq. (i) from Eq. (ii), we get

$$\begin{array}{r} x + 10y = 1800 \\ x + 4y = 1500 \\ - \quad - \quad\; - \\ \hline 6y = 300 \end{array}$$

$\Rightarrow \quad y = \frac{300}{6} = 50$

On putting the value of y in Eq. (i), we get

$x + 4 \times 50 = 1500$

$\Rightarrow \quad x = 1500 - 200$

$= 1300$

Hence, monthly salary = ₹ 1300

and fixed increment = ₹ 50 **[1]**

28. Let assumed mean, $A = 25$

Class interval	Number of families (f_i)	Mid value (x_i)	$u_i = \frac{x_i - A}{h}$	$u_i \times f_i$
0-10	16	5	–2	–32
10-20	12	15	–1	–12
20-30	18	25	0	0
30-40	6	35	1	6
40-50	4	45	2	8
Total	$N = 56$			$\sum f_i u_i = -30$

[2]

Here, $A = 25$, $\sum f_i = N = 56$ and $\sum f_i u_i = -30$

$h = 10$

$\therefore$ Mean, $\bar{x} = A + h\left(\frac{\sum f_i u_i}{\sum f_i}\right)$ **[1]**

$= 25 + 10\left(\frac{-30}{56}\right)$

$= 25 - 5.36$

$= 19.64$ **[1]**

29. Let $P(x) = 3x^2 - 2kx + 2m$

Put $x = 2$, then

$P(2) = 3(2)^2 - 2k(2) + 2m$

$= 12 - 4k + 2m$

$\Rightarrow \quad 12 - 4k + 2m = 0$ ...(i) **[1]**

Also, $P(3) = 3(3)^2 - 2k \times 3 + 2m$

$\Rightarrow \quad 27 - 6k + 2m = 0$ ...(ii) **[1]**

On subtracting Eq. (i) from Eq. (ii), we get

$27 - 6k + 2m = 0$

$\underline{12 \mp 4k \pm 2m = 0}$

$15 - 2k = 0$

$\Rightarrow \quad -2k = -15$

$\Rightarrow \quad k = \frac{15}{2}$ **[1]**

On putting $k = \frac{15}{2}$ in Eq. (i), we get

$12 - 4 \times \frac{15}{2} + 2m = 0$

$\Rightarrow \quad 12 - 30 + 2m = 0$

$\Rightarrow \quad -18 + 2m = 0$

$\Rightarrow \quad 2m = 18$

$\therefore \quad m = 9$

Hence, $k = \frac{15}{2}$ and $m = 9$ **[1]**

OR

Sol. Given, α and β are the roots of $x^2 - 6x + a$.

$\therefore \quad \alpha\beta = a$ ...(i) **[1/2]**

and $\quad \alpha + \beta = -(-6)$

$\Rightarrow \quad \alpha + \beta = 6$ ...(ii) **[1/2]**

Also, $\quad 3\alpha + 2\beta = 20$ ...(iii) **[1/2]**

On multiplying Eq. (ii) by 2, then subtracting from Eq. (iii), we get

$3\alpha + 2\beta = 20$

$\underline{2\alpha \pm 2\beta = 12}$ **[1]**

$\alpha = 8$

On putting $\alpha = 8$ in Eq. (ii), we get

$8 + \beta = 6$

$\Rightarrow \quad \beta = 6 - 8 = -2$ **[1/2]**

Now, putting $\alpha = 8$ and $\beta = (-2)$ in Eq. (i), we get

$8(-2) = a$

$\Rightarrow \quad a = -16$ **[1]**

30. LHS $= \frac{\cos A}{1 - \sin A} + \frac{\sin A}{1 - \cos A} + 1$

$= \frac{\left[\begin{array}{r}\cos A(1 - \cos A) + \sin A(1 - \sin A) \\ + (1 - \sin A)(1 - \cos A)\end{array}\right]}{(1 - \sin A)(1 - \cos A)}$

$= \frac{\left[\begin{array}{r}\cos A - \cos^2 A + \sin A - \sin^2 A + 1 - \cos A \\ - \sin A + \sin A \cos A\end{array}\right]}{(1 - \sin A)(1 - \cos A)}$ **[1]**

$= \frac{-(\cos^2 A + \sin^2 A) + 1 + \sin A \cos A}{(1 - \sin A)(1 - \cos A)}$ **[1]**

$= \frac{-1 + 1 + \sin A \cos A}{(1 - \sin A)(1 - \cos A)}$ $[\because \sin^2 A + \cos^2 A = 1]$

$= \frac{\sin A \cos A}{(1 - \sin A)(1 - \cos A)}$

LHS = RHS **[1]**

Hence proved.

31. We have,

$6x + 3y = 8x + 9y - 5 = 10x + 12y - 8$

$6x + 3y = 8x + 9y - 5$

$\Rightarrow \quad 6x + 3y - 8x - 9y = -5$

$\Rightarrow \quad -2x - 6y = -5$

$\Rightarrow \quad 2x + 6y = 5$ ...(i) **[1]**

and $\quad 8x + 9y - 5 = 10x + 12y - 8$

$\Rightarrow \quad 8x + 9y - 10x - 12y = -8 + 5$

$\Rightarrow \quad -2x - 3y = -3$

$\Rightarrow \quad 2x + 3y = 3$ ...(ii) **[1]**

On subtracting Eq. (ii) from Eq. (i), we get

$2x + 6y = 5$

$\underline{2x + 3y = 3}$

$3y = 2$

$\Rightarrow \quad y = \frac{2}{3}$ **[1]**

On putting $y = \frac{2}{3}$ in Eq. (i), we get

$2x + 6 \times \frac{2}{3} = 5$

$\Rightarrow \quad 2x = 5 - 4$

$\therefore \quad x = \frac{1}{2}$

Hence, $x = \frac{1}{2}$ and $y = \frac{2}{3}$ **[1]**

32. (i) Let the number of T-shirt be x and the number of shoes be y.

According to the question,

$$y = 4x - 4 \quad ...(i)$$

and $$y = 8x - 8 \quad ...(ii)$$ **[1]**

From Eq. (i) and (ii), we get

$$8x - 8 = 4x - 4$$

$\Rightarrow$ $8x - 4x = 4 + 8$

$\Rightarrow$ $4x = 4$

$\Rightarrow$ $x = 1$

On putting $x = 1$ in Eq. (i),

$$y = 4(1) - 4$$
$$= 4 - 4 = 0$$ **[1]**

Hence, the number of T-shirts, he purchase is 1 and he did not by any shoes.

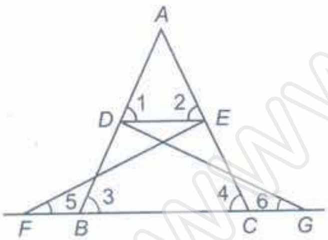
(ii) Polynomial [pair of linear equation in two variable] **[1]**

(iii) Obediently and fond of shopping. **[1]**

33. Given $\Delta FEC \cong \Delta GBD$

and $\angle 1 = \angle 2$

To prove $\Delta ADE \sim \Delta ABC$ **[1/2]**



[1/2]

Since, $\Delta FEC \cong \Delta GBD$

$\Rightarrow$ $DG = EF$ $\angle EFG = \angle DGB$ i.e., $\angle 5 = \angle 6$

$\Rightarrow$ $BD = EC$ $\angle DBC = \angle ECB$ i.e., $\angle 3 = \angle 4$ **[1]**

and $BG = FC$ $\angle BDC = \angle CEF$

But also $\angle 1 = \angle 2$

$\Rightarrow$ $AD = AE$ [by property of triangle] **[1]**

In ΔADE and ΔABC,

$\because$ $AD = AE$

Also $BD = EC$

$$\therefore \quad \frac{AD}{BD} = \frac{AE}{EC}$$

$\therefore$ $\Delta ADE \sim \Delta ABC$[by proportionality theorem] **[1]**

34. Let assumed mean, $A = 60$, $h = 20$

Class interval	f_i	x_i	$u_i = \frac{x_i - A}{h}$	$f_i \times u_i$
10-30	5	20	–2	–10
30-50	8	40	–1	–8
50-70	f_1	60	0	0
70-90	20	80	1	20
90-110	f_2	100	2	$2f_2$
110-130	2	120	3	6
	$\Sigma f_i = 50$			$\Sigma f_i u_i = 2f_2 + 8$

Hence, $\Sigma f_i = 50$, mean = 65.6 [given] **[1]**

and $\Sigma f_i u_i = 2f_2 + 8$

$$\therefore \quad \text{Mean, } \bar{x} = A + h\left(\frac{\Sigma f_i u_i}{\Sigma f_i}\right)$$ **[1]**

$\Rightarrow$ $$65.6 = 60 + 20 \times \frac{(2f_2 + 8)}{50}$$

$\Rightarrow$ $$65.6 - 60 = \frac{2(2f_2 + 8)}{5}$$

$\Rightarrow$ $$5.6 = \frac{4f_2 + 16}{5}$$

$\Rightarrow$ $4f_2 + 16 = 28$

$\Rightarrow$ $4f_2 = 28 - 16$

$\therefore$ $$f_2 = \frac{12}{4} = 3$$ **[1]**

Also, $35 + f_1 + f_2 = 50$

$\Rightarrow$ $35 + f_1 + 3 = 50$

$\Rightarrow$ $f_1 = 50 - 38$
$= 12$

$\therefore$ $f_1 = 12$, $f_2 = 3$ **[1]**