SOLUTIONS

SAMPLE QUESTION PAPER - 3

Solved

Time : 3 Hours **Maximum Marks : 90**

SECTION 'A'

1. (A) $\left((2^2)^{\frac{1}{3}}\right)^{\frac{1}{4}} = 2^{\frac{1}{6}}$ **1**

2. (C)
$$p(x) = x^2 + 11x + k$$
$$p(-4) = 0$$
$\Rightarrow$ $(-4)^2 + 11 \times (-4) + k = 0$
$$16 - 44 + k = 0$$ **1**
$$k = 28.$$

3. (D) The maximum number of cubic polynomials 3. **1**

4. (B)
$$x^2 + 8x + 15 = x^2 + 5x + 3x + 15$$
$$= x(x + 5) + 3(x + 5)$$
$$= (x + 3)(x + 5)$$
$$x^2 + 3x - 10 = x^2 - 2x + 5x - 10$$
$$= x(x - 2) + 5(x - 2)$$
$$= (x - 2)(x + 5)$$
Hence common factor is $(x + 5)$.

SECTION 'B'

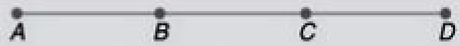
5. 0.5101001000100001....... and 0.502002000200002..... . **2**

[CBSE Marking Scheme, 2012]

6.
$$f(x) = 2x^3 - 3x^2 + 7x - 6$$
Put, $x - 1 = 0$ or $x = 1$ in $f(x)$ ½
Thus, $f(1) = 2 \times 1^3 - 3 \times 1^2 + 7 \times 1 - 6$ **1**
$= 2 - 3 + 7 - 6 = 0$ ½
Hence, $(x - 1)$ is a factor of $f(x)$. **[CBSE Marking Scheme, 2012]**

7.
$103^3 = (100 + 3)^3$ ½
$= 100^3 + 3^3 + 3 \times 100 \times 3\,(100 + 3)$ ½
$= 1000000 + 27 + 900 \times 103$
$= 1000000 + 27 + 92700$ ½
$= 1092727.$ **[CBSE Marking Scheme, 2012]** ½

8.



$\therefore$

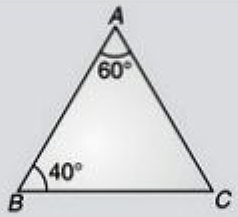
$$AC = BD \quad \text{(given)}$$

$$AB + BC = BC + CD \quad 1$$

$$AB = CD. \quad 1$$

[CBSE Marking Scheme I, 2012 , 2014]

9.



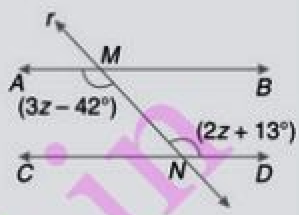
$$\angle C = 180^\circ - (60^\circ + 40^\circ)$$

$$= 180^\circ - 100^\circ \quad ½$$

$$= 80^\circ \Rightarrow AC \text{ is the smallest side} \quad ½$$

Reason : Side opposite to smaller angle is shorter. **[CBSE Marking Scheme, 2012] 1**

OR



$$3z - 42^\circ = 2z + 13^\circ$$

(Alternate interior angles)

$$z = 42^\circ + 13^\circ \quad ½$$

$$z = 55^\circ \quad ½$$

$$\angle DNM = 2z + 13^\circ = 110^\circ + 13^\circ$$

$$= 123^\circ \quad ½$$

$$\angle CNM = 180^\circ - \angle DNM$$

$$= 180^\circ - 123^\circ = 57^\circ. \quad ½$$

[CBSE Marking Scheme, 2012]

10. Perimeter of an equilateral triangle $= 3a = 60$

$$a = 20 \text{ cm} \quad ½$$

$$\text{Area} = \frac{\sqrt{3}}{4}a^2 = \frac{\sqrt{3}}{4} \times 20 \times 20 \quad ½$$

$$= 100\sqrt{3} \text{ cm}^2.$$

[CBSE Marking Scheme, 2012] 1

SECTION 'C'

11.

$$a^7 + ab^6 = a(a^6 + b^6) \quad 1$$

$$= a[(a^2)^3 + (b^2)^3]$$

$$= a(a^2 + b^2)\,[(a^2)^2 + (b^2)^2 - a^2 \times b^2] \quad 1$$

$$= a(a^2 + b^2)\,(a^4 + b^4 - a^2b^2). \quad 1$$

[CBSE Marking Scheme, 2012]

12.

$$\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} = \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} \times \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}} \quad 1$$

$$= \frac{8+2\sqrt{15}}{2} = 4 + \sqrt{15} \quad 1$$

$$a + b\sqrt{15} = 4 + \sqrt{15} \Rightarrow a = 4, b = 1 \quad 1$$

[CBSE Marking Scheme, 2012]

13. To factorise, $\left(5a+\frac{2}{3}\right)^2-\left(2a-\frac{1}{3}\right)^2$

$$a^2-b^2 = (a+b)(a-b) \quad 1$$
$$= \left(5a+\frac{2}{3}+2a-\frac{1}{3}\right)\left(5a+\frac{2}{3}-2a+\frac{1}{3}\right) \quad 1$$
$$= \left(7a+\frac{1}{3}\right)(3a+1) \quad 1$$

[CBSE Marking Scheme, 2012]

OR

$$a^6-b^6 = (a^3)^2-(b^3)^2 \quad ½$$
$$= (a^3-b^3)(a^3+b^3) \quad ½$$
$$= (a-b)(a^2+b^2+ab)(a+b)(a^2+b^2-ab) \quad 1$$
$$= (a-b)(a+b)(a^2+b^2+ab)(a^2+b^2-ab). \quad 1$$

[CBSE Marking Scheme, 2012]

14. $p(x) = 3x^2-mx-nx$

If $(x-a)$ is a factor of $p(x)$, then $p(a) = 0$ 1

$3(a)^2-m\times a-n\times a = 0$ ½

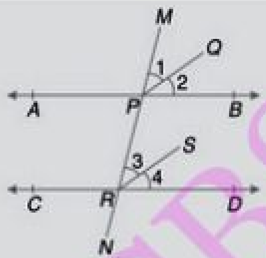
$a[3a-m-n] = 0, a\neq 0$ ½

Since, $3a-m-n = 0$ ½

$\therefore$ $a = \frac{m+n}{3}$. ½

[CBSE Marking Scheme, 2012]

15.



1

Given, $PQ \parallel RS$ ½

Given, $\angle 1 = \angle 2$ and $\angle 3 = \angle 4$

But, $\angle 1 = \angle 3$ (Corr. angles)

$\therefore$ $2\angle 1 = 2\angle 3$ ½

$\therefore$ $\angle MPB = \angle PRD$

But they are corresponding angles. 1

Hence, $AB \parallel CD$.

[CBSE Marking Scheme, 2011, 2012, 2013]

OR

$EF \perp CD \Rightarrow \angle CEF = 90°$ 1

$90° + z = \angle CFG$

$z = 130° - 90°$

$= 40°$ 1

$x = \angle CFG$ (Alt. int. angles)

$= 130°$

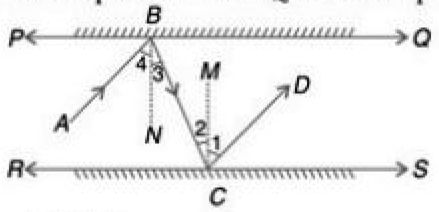
$x + y = 180°$ (Linear pair)

$$130° + y = 180°$$
$$y = 180° - 130°$$
$$= 50° \quad 1$$

[CBSE Marking Scheme, 2013, 12]

16. **(i)** Two plane mirrors PQ and RS are placed parallel to each other *i.e.* $PQ \parallel RS$. An incident ray AB after reflection takes the path BC and CD.

BN and CM are the normals to the plane mirrors PQ and RS respectively. ½



Since $BN \perp PQ$, $CM \perp RS$ and $PQ \parallel RS$

$\therefore BN \perp RS \Rightarrow BN \parallel CM$ ½

Thus, BN and CM are two parallel lines and transversal BC cuts them at B and C respectively.

$\therefore$ $\angle 2 = \angle 3$

But, $\angle 1 = \angle 2$ and $\angle 3 = \angle 4$

$\therefore$ $\angle 1 + \angle 2 = \angle 2 + \angle 2$

and $\angle 3 + \angle 4 = \angle 3 + \angle 3$

$\Rightarrow$ $\angle 1 + \angle 2 = 2(\angle 2)$

and $\angle 3 + \angle 4 = 2(\angle 3)$

$\Rightarrow$ $\angle 1 + \angle 2 = \angle 3 + \angle 4$ ½

$\Rightarrow$ $\angle ABC = \angle BCD$.

Thus, lines AB and CD are intersected by transversal BC, such that

$$\angle ABC = \angle BCD$$

i.e. alternate interior angles are equal.

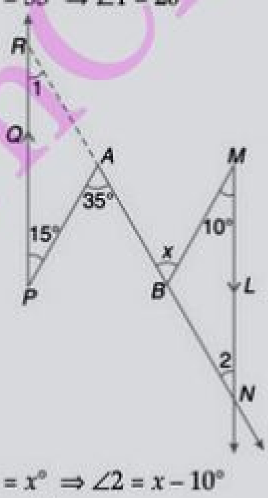
Therefore, $AB \parallel CD$ ½

(ii) Lines and angles. ½

(iii) Similarity leads to unanimity. ½

17. Extend PQ, AB and ML, so that lines intersect.

In ΔPRA and ΔMNB, $\angle 1 + 15° = 35° \Rightarrow \angle 1 = 20°$ 1



1

$$\angle 2 + 10° = x° \Rightarrow \angle 2 = x - 10°$$
$$\angle 1 = \angle 2$$

(Alternate interior angles) ½

$\Rightarrow$ $x - 10 = 20°$

$x = 30°$. **[CBSE Marking Scheme, 2012, 2013]** ½

18. $x = 50°$ (V.O.A.) ½

$y + 130° = 180°$ (Linear pair) ½

$\Rightarrow$ $y = 50°$ ½

$x = y = 50°$ ½

But x and y are alternate angles.

$\therefore$ $l \parallel m$. 1

[CBSE Marking Scheme, 2010, 2011, 2012]

19. $\angle BDC + \angle CDA = 180°$ (Linear pair)

$\angle BDC + x = 180°$

$\angle BDC = 180° - x$ ½

Similarly, $\angle BEA = 180° - y°$ ½

Since, $x = y$ (Given)

$\therefore$ $\angle BDC = \angle BEA$ ½

$\angle B = \angle B$ (Common)

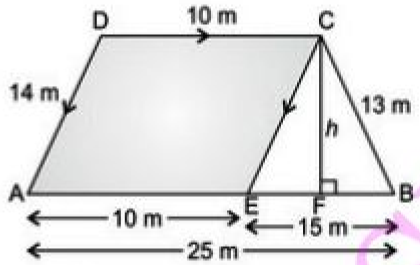
$AB = BC$ 1

$\therefore$ $\Delta BAE \cong \Delta BCD$ (ASA) ½

$\therefore$ $AE = CD$ (c.p.c.t.)

[CBSE Marking Scheme, 2012]

20. Let $ABCD$ be the given field in the form of trapezium in which $AB = 25$ m, $CD = 10$ m, $BC = 13$ m, $AD = 14$ m and $DC \parallel AB$



½

Through C, draw $CE \parallel DA$ and let it meet AB at E.

Let h metres (CP) be the height of the trapezium.

$DC \parallel AE$

and $CE \parallel DA$

$\therefore$ $AECD$ is a parallelogram.

$AE = DC = 10$ m

and $CE = DA = 14$ m

In ΔCEB, $CB = 13$ m, $CE = 14$ m

and $BE = AB - AE$

$= 25 - 10 = 15$ m

Let $a = 14$ m, $b = 13$ m and $c = 15$ m

Then, $s = \dfrac{a+b+c}{2}$

$= \dfrac{14+13+15}{2} = 21$ m ½

Area of $\Delta CEB = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{21(21-14)(21-13)(21-15)}$ ½

$= \sqrt{21 \times 7 \times 8 \times 6}$

$$= \sqrt{7 \times 3 \times 7 \times 2 \times 4 \times 3 \times 2}$$

$$= 84 \text{ m}^2 \qquad ½$$

Also, Area of $\Delta CEB = \frac{1}{2} \times \text{base} \times \text{height}$

$$= \frac{1}{2} \times 15 \times h$$

$$\frac{1}{2} \times 15 \times h = 84 \text{ m}^2$$

$$h = \frac{84 \times 2}{15} = \frac{56}{5} \text{ m}$$

Area of parallelogram $AECD = 10 \times \frac{56}{5} = 112 \text{ m}^2$ ½

Area of trapezium $= 84 + 112 = 196 \text{ m}^2$ ½

SECTION 'D'

21.

$$x = \frac{\sqrt{3}+1}{\sqrt{3}-1} = \frac{\sqrt{3}+1}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} = \frac{4+2\sqrt{3}}{2}$$

$$= 2 + \sqrt{3} \qquad 1$$

$$y = \frac{\sqrt{3}-1}{\sqrt{3}+1} = \frac{\sqrt{3}-1}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1} = \frac{4-2\sqrt{3}}{2}$$

$$= 2 - \sqrt{3} \qquad 1$$

$$xy = \frac{\sqrt{3}+1}{\sqrt{3}-1} \times \frac{\sqrt{3}-1}{\sqrt{3}+1} = 1 \qquad 1$$

$$x^2 + y^2 + xy = \left(2+\sqrt{3}\right)^2 + \left(2-\sqrt{3}\right)^2 + 1 = 15 \qquad 1$$

[CBSE Marking Scheme, 2012]

OR

Since, $x = 3-2\sqrt{2}$ 1

So, $\frac{1}{x} = \frac{1}{3-2\sqrt{2}} = \frac{1}{3-2\sqrt{2}} \times \frac{3+2\sqrt{2}}{3+2\sqrt{2}} = \frac{3+2\sqrt{2}}{1}$

$$x - \frac{1}{x} = 3-2\sqrt{2} - 3-2\sqrt{2}$$

$$= -4\sqrt{2} \qquad 1$$

$\therefore$ $\left(x-\frac{1}{x}\right)^3 = x^3 - \frac{1}{x^3} - 3\left(x-\frac{1}{x}\right)$ 1

$\Rightarrow$ $-128\sqrt{2} = x^3 - \frac{1}{x^3} - 3\left(-4\sqrt{2}\right)$

$\Rightarrow$ $x^3 - \frac{1}{x^3} = -128\sqrt{2} - 12\sqrt{2}$

$$= -140\sqrt{2} \qquad 1$$

22.

$$\frac{1}{\sqrt{4}+\sqrt{5}} = \frac{1}{(\sqrt{5}+\sqrt{4})}\times\frac{(\sqrt{5}-\sqrt{4})}{(\sqrt{5}-\sqrt{4})} = \frac{\sqrt{5}-\sqrt{4}}{5-4} = \sqrt{5}-\sqrt{4} \qquad \tfrac{1}{2}$$

$$\frac{1}{\sqrt{5}+\sqrt{6}} = \frac{1}{(\sqrt{6}+\sqrt{5})}\times\frac{(\sqrt{6}-\sqrt{5})}{(\sqrt{6}-\sqrt{5})} = \frac{\sqrt{6}-\sqrt{5}}{6-5} = \sqrt{6}-\sqrt{5} \qquad \tfrac{1}{2}$$

$$\frac{1}{\sqrt{6}+\sqrt{7}} = \frac{1}{(\sqrt{7}+\sqrt{6})}+\frac{(\sqrt{7}-\sqrt{6})}{(\sqrt{7}-\sqrt{6})} = \frac{\sqrt{7}-\sqrt{6}}{7-6} = \sqrt{7}-\sqrt{6} \qquad \tfrac{1}{2}$$

$$\frac{1}{\sqrt{7}+\sqrt{8}} = \frac{1}{(\sqrt{8}+\sqrt{7})}\times\frac{(\sqrt{8}-\sqrt{7})}{(\sqrt{8}-\sqrt{7})} = \frac{\sqrt{8}-\sqrt{7}}{8-7} = \sqrt{8}-\sqrt{7} \qquad \tfrac{1}{2}$$

$$\frac{1}{\sqrt{8}+\sqrt{9}} = \frac{1}{(\sqrt{9}+\sqrt{8})}\times\frac{(\sqrt{9}-\sqrt{8})}{(\sqrt{9}-\sqrt{8})} = \frac{\sqrt{9}-\sqrt{8}}{9-8} = \sqrt{9}-\sqrt{8} \qquad \tfrac{1}{2}$$

$$LHS = \sqrt{5}-\sqrt{4}+\sqrt{6}-\sqrt{5}+\sqrt{7}-\sqrt{6}+\sqrt{8}-\sqrt{7}+\sqrt{9}-\sqrt{8} \qquad 1$$

$$= -\sqrt{4}+\sqrt{9} = -2+3 = 1 = \text{RHS} \qquad \textbf{Proved.}\ \tfrac{1}{2}$$

[CBSE Marking Scheme, 2012]

23.

Factors of 20 $= (\pm 1, \pm 2, \pm 4, \pm 5, \pm 10, \pm 20)$ 1

$p(x) = x^3 + 13x^2 + 32x + 20$

$p(-1) = (-1)^3 + 13(-1)^2 + 32(-1) + 20$

$= -1 + 13 - 32 + 20$

$= 33 - 33 = 0$ ½

$\therefore x = -1$ is a zero of $p(x)$, and $(x + 1)$ is a factor of $p(x)$ ½

Then, $x^3 + 13x^2 + 32x + 20 = x^2(x + 1) + 12x(x + 1) + 20(x + 1)$ ½

$= (x + 1)(x^2 + 12x + 20)$ ½

$= (x + 1)[x(x + 10) + 2(x + 10)]$

$= (x + 1)(x + 2)(x + 10)$ 1

[CBSE Marking Scheme, 2014]

24.

$(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$ 1

$(6)^2 = a^2 + b^2 + c^2 + 2 \times 11$ ½

$a^2 + b^2 + c^2 = 36 - 22 = 14$ ½

$a^3 + b^3 + c^3 - 3abc = (a + b + c)[a^2 + b^2 + c^2 - (ab + bc + ca)]$ 1

$= 6 \times (14 - 11) = 6 \times 3 = 18.$ 1

[CBSE Marking Scheme, 2014]

25. **(i)** The point $(-2, 4)$ lies in the II quadrant.

(ii) The point $(3, -1)$ lies in the IV quadrant.

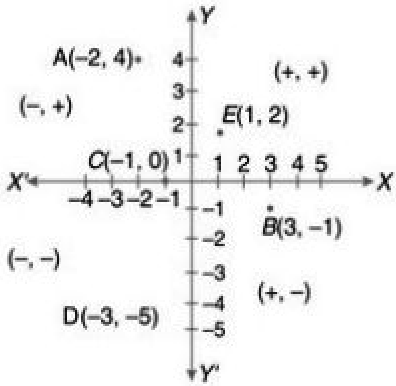
(iii) The point $(-1, 0)$ lies on the negative x-axis.

(vi) The point $(-3, -5)$ lies in the III quadrant.

(v) The point $(1, 2)$ lies in the I quadrant.

Locations of these points are shown in the figure. 2

These points are respectively represented by A, B, C, D and E, which clearly verify their location. 2



26. Let, $f(x) = ax^3 - 3x^2 + 4$

and $g(x) = 2x^3 - 5x + a$

When divided by $(x - 2)$,

$f(2) = p$ and $g(2) = q$ ½

$f(2) = a \times 2^3 - 3 \times 2^2 + 4$ ½

$p = 8a - 12 + 4$

$p = 8a - 8$ ...(1) ½

$g(2) = 2 \times 2^3 - 5 \times 2 + a$

$q = 16 - 10 + a$ ½

$q = 6 + a$ ...(2) ½

$p - 2q = 4,$ (given)

$8a - 8 - 12 - 2a = 4$

$6a - 20 = 4$

$a = 4.$ 1

[CBSE Marking Scheme, 2010, 2011]

27. Proof : We are given two triangles ABC and PQR in which

$\angle B = \angle Q, \angle C = \angle R$

and $BC = QR$

We need to prove that $\Delta ABC \cong \Delta PQR$ 1

There are three cases.

Case I : Let $AB = PQ$

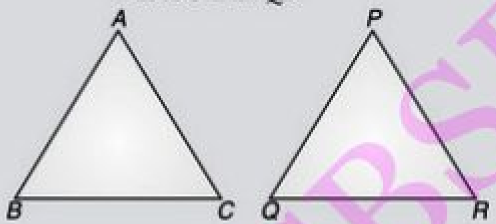
In ΔABC and ΔPQR,

$\angle B = \angle Q$ (Given)

$BC = QR$ (Given)

$AB = PQ$ (Assumed)

$\therefore$ $\Delta ABC \cong \Delta PQR$ (By *SAS* rule) 1



Case II : Suppose $AB \neq PQ$ and $AB < PQ$

Take a point S on PQ such that

$QS = AB$

Join RS.

In ΔABC and ΔSQR,

$AB = SQ$ (By construction)

$BC = QR$ (Given)

$\angle B = \angle Q$ (Given) 1

$\therefore$ $\Delta ABC \cong \Delta SQR$ (By *SAS* rule)

$\angle ACB = \angle QRS$ (By c.p.c.t.)

But, $\angle QRP = \angle ACB$

$\Rightarrow$ $\angle QRP = \angle QRS$

which is impossible unless ray RS coincides with RP.

$\therefore$ AB must be equal to PQ. ½

So, $\Delta ABC \cong \Delta PQR$

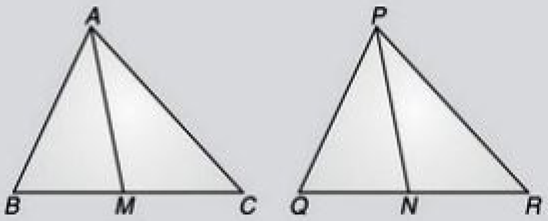
Case III : If $AB > PQ$.

We can choose a point T on AB such that $TB = PQ$ and repeating the arrangements as given in Case II, we can conclude that $AB = PQ$ and so,

$\Delta ABC \cong \Delta PQR$ ½

[CBSE Marking Scheme, 2012, 2013]

28.



(i) In ΔABM and ΔPQN,

$AB = PQ$ (Given)

$BC = QR$ (Given)

$\Rightarrow$ $\frac{1}{2}BC = \frac{1}{2}QR$ ½

i.e., $BM = QN$ ½

$AM = PN$ (Given)

$\therefore$ $\Delta ABM \cong \Delta PQN$ (*SSS*) ½

$\Rightarrow$ $\angle ABM = \angle PQN$ (c.p.c.t.)

i.e., $\angle ABC = \angle PQR$ **1**

(ii) Now, in ΔABC and ΔPQR,

$AB = PQ$ (Given)

$\angle ABC = \angle PQR$ (Proved)

$BC = QR$ (Given)

$\therefore$ $\Delta ABC \cong \Delta PQR.$ (*SSS*) **1½**

[CBSE Marking Scheme, 2012]

OR

$4b + 75° + b = 180°$ (Lateral pair) ½

$5b = 180° - 75° = 105°$

$b = \frac{105°}{5} = 21°$ ½

$4b = 4 \times 21° = 84°$ **1**

$a = 4b = 84°$ (V.O.A.) ½

$2c + a = 180°$ (Lateral pair) ½

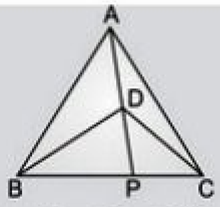
$2c = 180 - 84° = 96°$

$\Rightarrow$ $c = 48°.$ **1**

29. (i) $AB = AC, BD = CD, AD = DA$ ½

$\Delta ABD \cong \Delta ACD$ (By SSS)

$\therefore$ $\angle BAD = \angle CAD$ (By c.p.c.t.) ...(i) ½



(ii) $AB = AC, \angle BAP = \angle CAP, AP = AP$

$\Delta ABP \cong \Delta ACP$ (By SAS) ½

$\therefore$ $BP = CP$ (By c.p.c.t.) ½

(iii) From (i), $\angle BAD = \angle CAD$ ½

$\Rightarrow AP$ is the bisector of $\angle A$. ½

$BD = CD, BP = CP, DP = DP$ ½

$\Delta BDP \cong \Delta CDP \Rightarrow \angle BDP = \angle CDP$

DP is the bisector of $\angle D$. **[CBSE Marking Scheme, 2012]** ½

30. **Given :** $ABCD$ is a square. X and Y are points on sides AD and BC respectively such that $AY = BX$.

To prove : $BY = AX$ and $\angle BAY = \angle ABX$.

Proof : $ABCD$ is a square. 1

$\therefore$ $\angle A = \angle B = 90°$, ($\because$ Each angle of a square is a right angle)

Now, in right ΔABY and right ΔBAX,

$\because$ Hyp. AY = Hyp. BX (Given) 1

Side AB = Side BA, (Common)

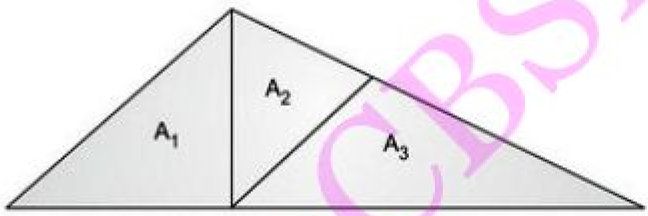
$\therefore$ $\Delta ADY = \Delta BAX$, (R.H.S.) 1

Hence, $BY = AX$ 1

and $\angle BAY = \angle ABX$. (c.p.c.t.)

Proved.

31.



The first field A_1 is a triangle with sides 25, 52 and 63 m. Now

$$s = \frac{25+52+63}{2}$$

$$= 70$$ 1

By Heron's formula,

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{70(70-25)(70-52)(70-63)}$$

$$= \sqrt{70 \times 45 \times 18 \times 7}$$

$$= 7 \times 9 \times 10$$ 1

$$= 630 \text{ sq. m}$$

Rana grows wheat in 630 sq. m. area.

The second field $(A_2 + A_3)$ has sides 25, 101 and 114 m.

$$\therefore \quad s = \frac{25+101+114}{2} = 120$$

By Heron's formula,

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{120(120-25)(120-101)(120-114)}$$

$$= \sqrt{120 \times 95 \times 19 \times 6}$$

$$= 19 \times 5 \times 12$$ 1

$$= 1140$$

But triangles A_2 and A_3 have same base (mid point) and same height, so their areas are equal.

Hence, area of A_2 = area of A_3 = $\frac{1140}{2}$

$$= 570 \text{ sq. m.}$$

Therefore, both rice and vegetables are grown in 570 sq. m. area. 1