SOLUTIONS

SAMPLE QUESTION PAPER - 4

Solved

Time : 3 Hours **Maximum Marks : 90**

SECTION 'A'

1. (C) $(a+\sqrt{b})(a-\sqrt{b}) = a^2-(\sqrt{b})^2$

$= a^2 - b.$ **1**

2. (D) 2 **1**

3. (C) $m^3\left(1-\frac{x}{m}\right)^3 = m^3\left[1-\frac{x^3}{m^3}-3\frac{x}{m}\left(1-\frac{x}{m}\right)\right]$

So, the coefficient of $x^3 = m^3\left(-\frac{1}{m^3}\right)$

$= -1.$ **1**

4. (C) 4 **1**

SECTION 'B'

5. $(4x+\sqrt{5}y)(4x-\sqrt{5}y) = [(4x)^2-(\sqrt{5}y)^2]$ **1**

$= 16x^2 - 5y^2$ **[CBSE Marking Scheme, 2012] 1**

6. Remainder $= p(2)$ **½**

$p(2) = 2^4 - 3(2)^2 + 7 \times 2 - 10$ **½**

$= 16 - 12 + 14 - 10$ **½**

$= 8$ **½**

7. $\left(x+\frac{1}{x}\right)\left(x-\frac{1}{x}\right)\left(x^2+\frac{1}{x^2}\right)\left(x^4+\frac{1}{x^4}\right) = \left(x^2-\frac{1}{x^2}\right)\left(x^2+\frac{1}{x^2}\right)\left(x^4+\frac{1}{x^4}\right),$ **½**

as $(a+b)(a-b) = a^2 - b^2$ **½**

$= \left(x^4-\frac{1}{x^4}\right)\left(x^4+\frac{1}{x^4}\right)$ **½**

$= \left(x^8-\frac{1}{x^8}\right).$ **[CBSE Marking Scheme, 2011, 12] ½**

8.

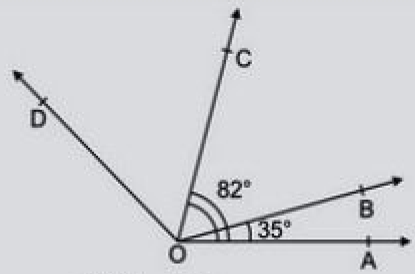
Let $AC = BC = x$ units.

Also, $$AC + CB = AB$$ ½

$$2AC = AB$$ ($\because AC = BC$) **1**

$$AC = \frac{1}{2}(AB)$$ **Proved. ½**

9. Given,

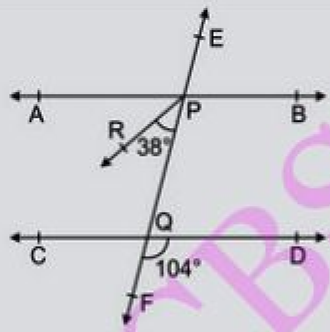


$$\angle COA = 82°$$
$$\angle COB + \angle BOA = 82°$$
$$\angle COB + 35° = 82°$$ ($\therefore \angle BOA = 35°$)
$$\angle COB = 82° - 35° = 47°$$ **1**

Similarly, $$\angle DOB = \angle DOC + \angle COB$$
$$87° = \angle DOC + 47°$$
$$\angle DOC = 87° - 47° = 40°.$$ **1**

[CBSE Marking Scheme, 2012]

OR



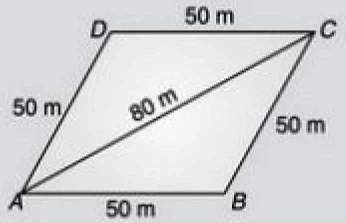
$$\angle APR = \angle RPQ = 38°$$ (Angle bisector) ½
$$\angle APQ = 38° + 38° = 76°$$
$$\angle FQD = \angle PQC = 104°,$$ (Vertically opposite angles) ½
$$\angle APQ + \angle PQC = 76° + 104°$$ ½
$$= 180°$$

Since, sum of interior angles is 180°.

$\therefore$ $$AB \parallel CD.$$ ½

[CBSE Marking Scheme, 2012]

10.



	Perimeter of rhombus	$= 4 \times \text{sides} = 200$	
	side	$= 50$ m	½
For ΔABC,	s	$= \dfrac{50+50+80}{2}\ 90\ m$	
	Area of ΔABC	$= \sqrt{s(s-a)(s-b)(s-c)}$	
		$= \sqrt{90\times(90-50)(90-50)(90-80)}$	
		$= \sqrt{90\times40\times40\times10}$	½
		$= 1200\ m^2$.	½
	Area of rhombus	$= 2 \times$ Area of ΔABC	
		$= 2 \times 1200 = 2400\ m^2$.	½

[CBSE Marking Scheme, 2012]

SECTION 'C'

11.

$$\left(\sqrt{2}+\sqrt{3}\right)^2 = \left(\sqrt{2}\right)^2 + \left(\sqrt{3}\right)^2 + 2\sqrt{6}$$

$$= 5 + 2\sqrt{6} \qquad 1$$

$$\left(\sqrt{5}-\sqrt{2}\right)^2 = \left(\sqrt{5}\right)^2 + \left(\sqrt{2}\right)^2 - 2\sqrt{10}$$

$$= 7 - 2\sqrt{10} \qquad 1$$

$$\Rightarrow \quad \left(\sqrt{2}+\sqrt{3}\right)^2 + \left(\sqrt{5}-\sqrt{2}\right)^2 = 5+2\sqrt{b}+7-2\sqrt{10}$$

$$= 12+2\sqrt{6}-2\sqrt{10}$$

$$= 2[6+\sqrt{6}-\sqrt{10}]$$

$$= 2[6+\sqrt{2}(\sqrt{3}-\sqrt{5})]$$

OR

$$\frac{\left(x^{\frac{1}{2}}\right)^{\frac{-2}{3}}\left(y^4\right)^{\frac{1}{2}}}{x^{\frac{-1}{4}}y^{\frac{-1}{4}}} = x^{\frac{-1}{3}} \times y^2 \times x^{\frac{1}{4}} \times y^{\frac{1}{4}} \qquad 1$$

$$= x^{\frac{-1}{12}} \times x^{\frac{9}{4}}$$

$$= \frac{y^{\frac{9}{4}}}{x^{\frac{1}{12}}} \qquad 1$$

[CBSE Marking Scheme, 2012]

12.

$$\left(\frac{a}{b}\right)^{x-1} = \left(\frac{b}{a}\right)^{2x-8}$$

$$\left(\frac{a}{b}\right)^{x-1} = \left(\frac{a}{b}\right)^{-[2x-8]} \qquad 1$$

$\Rightarrow \quad x - 1 = -[2x - 8]$ **1**

$\Rightarrow \quad 3x = 9$

$x = 3$ **1**

[CBSE Marking Scheme, 2012]

13. Factors of 12= (±1, ± 2, ± 3, ± 4, ± 6, ± 12) ½

$$p(x) = 6x^3 - 25x^2 + 32x - 12$$

$$p(2) = 6(2)^3 - 25(2)^2 + 32 \times 2 - 12$$

$$= 48 - 100 + 64 - 12$$ ½

$$= 112 - 112 = 0$$ ½

$\therefore x = 2$ is a zero of $p(x)$ or $(x - 2)$ is a factor of $p(x)$ ½

$$6x^3 - 25x^2 + 32x - 12 = 6x^2(x - 2) - 13x(x - 2) + 6(x - 2)$$ ½

$$= (x - 2)\,(6x^2 - 13x + 6)$$

$$= (x - 2)\,(6x^2 - 9x - 4x + 6)$$ ½

$$= (x - 2)\,[3x(2x - 3) - 2(2x - 3)]$$

$$= (x - 2)\,(2x - 3)\,(3x - 2).$$ ½

OR

$(x + y + 2z)\,[x^2 + y^2 + (2z)^2 - xy - 2yz - 2xz)$,

By the identity, $(a + b + c)\,(a^2 + b^2 + c^2 - ab - bc - ac)$ **1**

$$= a^3 + b^3 + c^3 - 3abc$$

$$= x^3 + y^3 + (2z)^3 - 3(x)\,(y)\,(2z)$$ **1**

$$= x^3 + y^3 + 8z^3 - 6xyz$$ **1**

14. $(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$ ½

$$= 280 + 2 \times \frac{9}{2}$$ ½

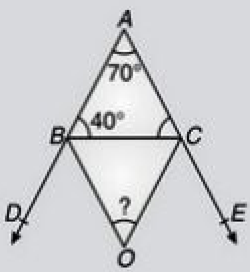
$$(a + b + c)^2 = 280 + 9 = 289$$

$$a + b + c = \sqrt{289} = 17$$ **1**

$$(a + b + c)^3 = 17^3 = 4913.$$ **1**

[CBSE Marking Scheme, 2012]

15.



$\angle DBC = 180° - 40° = 140°$ (Linear pair)

$\angle CBO = \frac{1}{2}\angle DBC = \frac{1}{2} \times 140°$

$= 70°$ **1**

$\angle ACB = 180° - (70° + 40°) = 70°$ ½

$\angle BCE = 180° - 70° = 110°$

(Linear pair)

$\angle BCO = \frac{1}{2} \times 110° = 55°$ (Angle bisector) ½

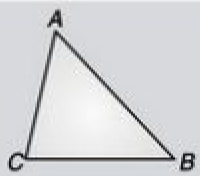
$\angle BOC = 180° - (\angle CBO + \angle BCO)$

$= 180° - (70° + 55°)$ **1**

$= 55°.$

[CBSE Marking Scheme, 2011, 2012]

OR



In ΔABC, as AB is the greatest side

$\Rightarrow$ $AB > BC \Rightarrow \angle C > \angle A$. ..(1)

$AB > AC \Rightarrow \angle C > \angle B$ ...(2) **1**

On adding (1) and (2), we get

$$2\angle C > \angle A + \angle B$$

$$2\angle C + \angle C > \angle A + \angle B + \angle C$$ **1**

$$3\angle C > 180^\circ$$

$$\angle C > 60^\circ.$$ **1**

[CBSE Marking Scheme, 2012]

16. In triangles APB and AQB,

$\angle APB = \angle AQB = 90^\circ$

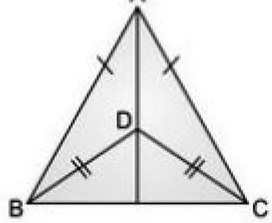
$\angle PAB = \angle QAB$ (Given) **1**

$AB = AB$ ½

$\Delta APB \cong \Delta AQB$ ½

$BP = BQ$ (C.P.C.T) **1**

17.



$\angle DBC = \angle DCB$ (Given)

$\therefore$ $DC = DB$ (Sides opp. to equal angles are equal) ...(i) ½

In ΔABD and ΔACD, $AB = AC$ (Given)

$BD = CD$ [from (i)] ½

$AD = AD$ (Common)

$\Delta ABD \cong \Delta ACD$ (*SSS*) **1**

$\angle BAD = \angle CAD$ y c.p.c.t.) ½

Hence, AD is the bisector of $\angle BAC$. **Proved.** ½

18. In ΔPQS, $PQ + QS > PS$ ...(1) **1**

(Sum of any two sides is greater than the third side)

In ΔPSR, $PR + SR > PS$ ...(2)

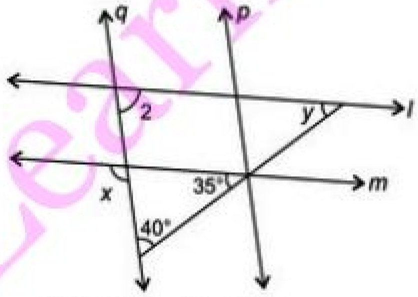
Adding (1) & (2), **1**

$$PQ + QS + PR + SR > 2PS$$

$$PQ + QR + RP > 2PS.$$ **1**

[CBSE Marking Scheme, 2012]

19.



½

$40^\circ + 35^\circ + \angle 1 = 180^\circ$ (Sum of angles of Δ) ½

$75^\circ + \angle 1 = 180^\circ$

$\angle 1 = 105^\circ$

$x + \angle 1 = 180^\circ$ (L.P.) ½

$$x = 180° - 105° = 75°$$ ½

$$\angle 2 = \angle 1 = 105°$$ (Alternate angles)

$$40° + 105° + y = 180°$$ ½

$$145° + y = 180°$$

$$y = 180° - 145°$$ ½

$$= 35°$$ ½

20. $$s = \frac{15+14+13}{2} = \frac{42}{2} = 21 \text{ cm}$$ ½

$$\text{Area of } \Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$$ ½

$$= \sqrt{21 \times 6 \times 7 \times 8}$$

$$= 84 \text{ cm}^2$$ ½

$$\text{Area of parallelogram} = \text{Area of } \Delta ABC$$

$$= 84 \text{ cm}^2$$ ½

Let the height of parallelogram be h

$\therefore$ $$h \times 14 = 84$$ ½

$$h = \frac{84}{14} = 6 \text{ cm.}$$ ½

[CBSE Marking Scheme, 2012]

SECTION 'D'

21. $$\frac{7\sqrt{3}}{\sqrt{10}+\sqrt{3}} = \frac{7\sqrt{3}}{\sqrt{10}+\sqrt{3}} \times \frac{\sqrt{10}-\sqrt{3}}{\sqrt{10}-\sqrt{3}}$$ ½

$$= \frac{7\sqrt{30}-21}{7}$$

$$= \frac{7(\sqrt{30}-3)}{7} = \sqrt{30}-3$$ ½

$$\frac{3\sqrt{2}}{\sqrt{15}+3\sqrt{2}} = \frac{3\sqrt{2}}{\sqrt{15}+3\sqrt{2}} \times \frac{\sqrt{15}-3\sqrt{2}}{\sqrt{15}-3\sqrt{2}}$$ ½

$$= \frac{3\sqrt{30}-9\sqrt{4}}{15-18}$$

$$= \frac{3\sqrt{30}-18}{-3} = 6-\sqrt{30}$$ ½

$$\frac{2\sqrt{5}}{\sqrt{6}+\sqrt{5}} = \frac{2\sqrt{5}}{\sqrt{6}+\sqrt{5}} \times \frac{\sqrt{6}-\sqrt{5}}{\sqrt{6}-\sqrt{5}}$$ ½

$$= \frac{2\sqrt{30}-10}{6-5} = 2\sqrt{30} - 10$$ ½

Now, $$\frac{7\sqrt{3}}{\sqrt{10}+\sqrt{3}} - \frac{3\sqrt{2}}{\sqrt{15}+3\sqrt{2}} - \frac{2\sqrt{5}}{\sqrt{6}+\sqrt{5}} = \sqrt{30} - 3 - 6 + \sqrt{30} - 2\sqrt{30} + 10$$ ½

$$= 1$$ ½

OR

(i)

$$\begin{array}{r|l} & 0.714285... \\ 7) & 5.000000 \\ & -49 \\ & 10 \\ & -7 \\ & 30 \\ & -28 \\ & 20 \\ & -14 \\ & 60 \\ & -56 \\ & 40 \\ & -35 \\ & 5 \end{array}$$

$\therefore$ $$\frac{5}{7} = 0.\overline{714285}$$ **1 + 1 = 2**

and

$$\begin{array}{r|l} & 0.\overline{81} \\ 11) & 9.00 \\ & -88 \\ & 20 \\ & -11 \\ & 9 \end{array}$$

$\therefore$ $$\frac{9}{11} = 0.\overline{81}$$ ½

Thus, three different irrational numbers between $\frac{5}{7}$ and $\frac{9}{11}$ are 0.75075007500075000075......, 0.767076700767000767..... and 0.80800800080000..... ½

(ii) Number system ½

(iii) Those who are irrational in three approach get full in their efforts. ½

22. $$\left(\frac{9}{4}\right)^{-3/2} = \left(\left(\frac{3}{2}\right)^2\right)^{-3/2} = \left(\frac{2}{3}\right)^3 = \frac{8}{27}$$ 1

$$\left(\frac{125}{27}\right)^{-2/3} = \left(\left(\frac{5}{3}\right)^3\right)^{-2/3} = \left(\frac{3}{5}\right)^2 = \left(\frac{9}{25}\right)$$ 1

$$\left(\frac{3}{5}\right)^{-2} = \left(\frac{5}{3}\right)^2 = \frac{25}{9}$$ ½

$$\left(\sqrt{2}\right)^4 = \left(2^{-1/2}\right)^4 = 2^2 = 4$$ ½

According to question, $$\frac{\left(\frac{9}{4}\right)^{-3/2} \times \left(\frac{125}{27}\right)^{-2/3} \times \left(\frac{3}{5}\right)^{-2}}{\left(\sqrt{2}\right)^4}$$ ½

$$= \frac{\frac{8}{27} \times \frac{9}{25} \times \frac{25}{9}}{4} = \frac{2}{27}$$ ½

23. Let, $$p(x) = x^4 - ax^3 + b$$
$$x^2 - 3x + 2 = x^2 - 2x - x + 2$$
$$= x(x-2) - 1(x-2)$$
$$= (x-1)(x-2)$$
$\therefore$ $(x-1)$ and $(x-2)$ are the factors of polynomial $p(x)$. **1**

$\therefore$ $$p(1) = 1^4 - a(1)^3 + b = 0$$
$$= 1 - a + b = 0 \quad ...(1)$$
$$p(2) = 2^4 - a(2)^3 + b = 0 \quad \mathbf{1}$$
$$= 16 - 8a + b = 0 \quad ...(2)\ \mathbf{1}$$

Subtracting (2) from (1), we get
$$-15 + 7a = 0$$
$$a = \frac{15}{7}$$

Put the value of a in equation (1)
$$1 - \frac{15}{7} = -b \quad \mathbf{½}$$
$$\frac{-8}{7} = -b$$
$\Rightarrow$ $$b = \frac{8}{7} \quad \mathbf{½}$$

24. $$x^2 + \frac{1}{x^2} = 23$$
$\therefore$ $$\left(x+\frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} + 2x \times \frac{1}{x}$$
$$\left(x+\frac{1}{x}\right)^2 = 23 + 2$$
$$\left(x+\frac{1}{x}\right) = \sqrt{25} = 5 \quad \mathbf{2}$$
$$\left(x+\frac{1}{x}\right)^3 = x^3 + \frac{1}{x^3} + 3x^2 \times \frac{1}{x} + 3x\left(\frac{1}{x^2}\right)$$
$$(5)^3 = x^3 + \frac{1}{x^3} + 3\left(x + \frac{1}{x}\right)$$
$$125 = x^3 + \frac{1}{x^3} + 3 \times 5$$
$$110 = x^3 + \frac{1}{x^3} \quad \mathbf{2}$$

25. $$a^2 + b^2 + c^2 - ab - bc - ca = 0$$ (Multiply by 2 both sides)
$$2a^2 + 2b^2 + 2c^2 - 2ab - 2bc - 2ca = 0$$
$$(a^2 + b^2 - 2ab) + (b^2 + c^2 - 2bc) + (c^2 + a^2 - 2ca) = 0 \quad \mathbf{1}$$
$$(a-b)^2 + (b-c)^2 + (c-a)^2 = 0 \quad \mathbf{1}$$
$\therefore$ a, b, c are real numbers,

So $$(a-b)^2 = 0, (b-c)^2 = 0 \text{ and } (c-a)^2 = 0$$
$\Rightarrow$ $$a - b = 0, b - c = 0 \text{ and } c - a = 0 \quad \mathbf{1}$$

$\Rightarrow$	$a = b$, $b = c$, and $c = a$	1
$\Rightarrow$	$a = b = c$	

26. The points $A(-3, 0)$, $B(5, 0)$ and $C(0, 4)$ can be plotted as shown below :

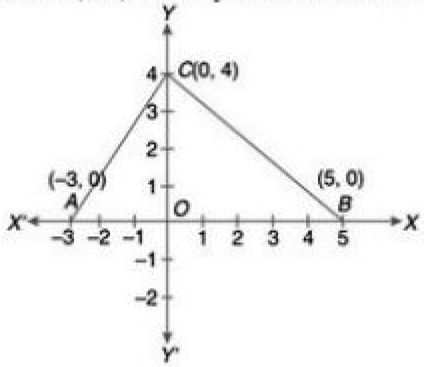


Figure formed is a triangle ABC.

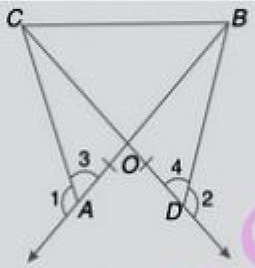
$$\text{Area of } \Delta ABC = \frac{1}{2} \times AB \times OC$$

$\therefore$ $$\text{Area} = \frac{1}{2} \times 8 \times 4$$

As, $AB = 8$ units and $OC = 4$ units

$\therefore$ Area = 16 sq. units. **2 + 2**

27.



$\because$	$\angle 1 + \angle 3 = 180°$	(Linear pair)
$\because$	$\angle 2 + \angle 4 = 180°$	(Linear pair)
$\therefore$	$\angle 1 + \angle 3 = \angle 2 + \angle 4$	
But,	$\angle 1 = \angle 2$	(Given)
$\Rightarrow$	$\angle 3 = \angle 4$	
In ΔOAC and ΔODB,		1
	$\angle 3 = \angle 4$	
	$\angle AOC = \angle DOB$	(V.O.A.) ½
	$OA = OD$	(Given) ½
$\therefore$	$\Delta OAC \cong \Delta ODB$	(*ASA*) 1
	$OC = OB$	(c.p.c.t.) ½

$\Rightarrow$ ΔOCB is an isosceles triangle ½

[CBSE Marking Scheme, 2012]

28. Given :

$$AB = PQ$$
$$BC = QR$$
$$AM = PN$$

To prove : $\Delta ABC \cong \Delta PQR$

Proof : $BC = QR$ (Given)

$$\frac{1}{2}BC = \frac{1}{2}QR$$

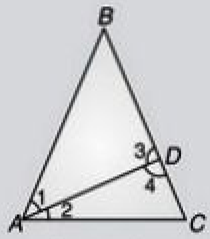
($\therefore$ M & N are the mid points of sides BC & QR, resp.) **1**

Now, In ΔABM & ΔPQN,	$AB = PQ$	(Given)
	$AM = PN$	1
	$BM = QN$	(from eq. (1))
$\therefore$	$\Delta ABM \cong \Delta PQN$	(By *SSS* rule)
$\therefore$	$\angle 1 = \angle 2$	....(2) (By c.p.c.t.)
Now, In ΔABC & ΔPQR,	$AB = PQ$	(Given)
	$\angle 1 = \angle 2$	(From eq. (2) **1**
	$BC = QR$	(Given)
$\therefore$	$\Delta ABC \cong \Delta PQR$	(By *SAS* rule) **1**

OR

Construction : Take $BD = AB$. Join AD **1**

Proof : In ΔABD,



$AB = BD \Rightarrow \angle 1 = \angle 3$ and $\angle 4 > \angle 1 \Rightarrow \angle 4 > \angle 3$ **1**

In ΔADC,	$\angle 3 > \angle 2$	
$\therefore$	$\angle 4 > \angle 3 > \angle 2$	
$\Rightarrow$	$\angle 4 > \angle 2$	**1**
$\Rightarrow$	$AC > DC$	
$\Rightarrow$	$AC > BC - BD$	$(BC = BD + CD)$ **1**
	$AC > BC - AB$	$(BD = AB)$
$\therefore$	$BC - AB < AC$	
Similarly,	$AC - AB < BC$	
and	$BC - AC < AB$	**[CBSE Marking Scheme, 2012] 1**

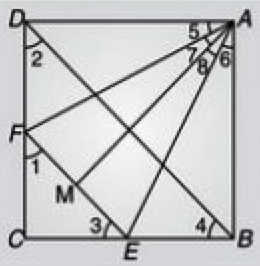
29. ROS is a straight line

$\therefore$	$4b + 75° + b = 180°$	
	$5b = 180° - 75°$	**1**
$\Rightarrow$	$5b = 105°$	
	$b = 21°$	**1**

POQ is a straight line

$\therefore$	$75° + b + a = 180°$	
$\Rightarrow$	$a + b = 105°$	
$\Rightarrow$	$a = 84°$	**1**
Also,	$a + 2c = 180°$	
$\Rightarrow$	$2c = 180° - 84°$	
$\Rightarrow$	$c = 48°$	**1**

30.



(i) $EF \parallel BD \Rightarrow \angle 1 = \angle 2, \angle 3 = \angle 4$ (Corresponding angles)

But, $\angle 2 = \angle 4$

$\therefore$ $\angle 3 = \angle 1$ ½

$\Rightarrow$ $FC = EC$ (Side opp. to equal angle)

$CD - FC = CB - CE$

$\Rightarrow$ $DF = BE$ **Proved.**

$AD = AB,$

$\angle D = \angle B = 90°$ ½

$\therefore$ $\Delta ADF \cong \Delta ABE$ (*SAS*) **1**

$\Rightarrow$ $AF = AE, \angle 5 = \angle 6$ (By c.p.c.t) **1**

(ii) In ΔAMF and ΔAME,

$AF = AE,$

$AM = AM$ (Common)

$FM = EM$ (Given)

$\Delta AMF \cong \Delta AME$ (*SSS*)

$\therefore$ $\angle 7 = \angle 8$ (By c.p.c.t)

$\angle 7 + \angle 3 = \angle 8 + \angle 6$

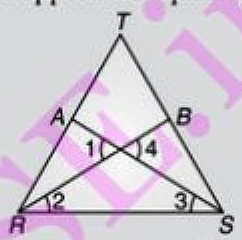
$\Rightarrow$ $\angle MAD = \angle MAB$

$\Rightarrow$ AM bisects $\angle BAD$. **1**

[CBSE Marking Scheme, 2012]

31. In ΔRTS, $RT = ST$

$\angle TSR = \angle TRS$ (Angles opposite to equal sides are equal) ...(1) ½



$\angle 1 = \angle 4$ (V.O.A)

$\Rightarrow$ $2\angle 2 = 2\angle 3$

$\Rightarrow$ $\angle 2 = \angle 3$ ...(2) ½

Subtract eqn. (2) from eqn. (1), we get

$\angle TRS - \angle 2 = \angle TSR - \angle 3$

$\angle TRB = \angle TSA$ **1**

In ΔRBT and ΔSAT, $\angle RTB = \angle STA$ (Common angles)

$RT = ST$ (Given)

$\angle TRB = \angle TSA$ (Proved) **1**

$\therefore$ $\Delta RBT \cong \Delta SAT$ (By *ASA* congruence)

$\Rightarrow$ $RB = AS$ (By c.p.c.t.) **1**

[CBSE Marking Scheme, 2012]

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