SAMPLE
QUESTION PAPER - 2
Solved

Time: 3 Hours **Maximum Marks: 90**

SECTION — A

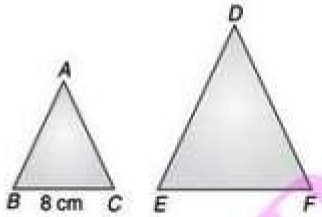
1. The smallest prime number is 2 and the smallest composite number is 2^2. ½

Hence, required $HCF\ (2^2, 2) = 2$ ½

2. The number of zeros of $p(x)$ is 1. 1

3. Given $2AB = DE$

and $BC = 8$ cm



So, $$\frac{AB}{BC} = \frac{DE}{EF}$$

$$\frac{AB}{8} = \frac{2AB}{EF}$$

$\Rightarrow$ $$EF = 2 \times 8$$
$$= 16 \text{ cm}$$ 1

4. According to the question

$$\text{Mode} - \text{Mean} = k(\text{Median} - \text{Mode})$$

$$k = \frac{\text{Mode} - \text{Mean}}{\text{Median} - \text{Mode}}$$

$$= \frac{(3\text{ Median} - 2\text{ Mean}) - \text{Mean}}{\text{Median} - \text{Mean}}$$

$$= \frac{3(\text{Medain} - \text{Mean})}{\text{Median} - \text{Mean}} = 3$$ 1

4SECTION — B

5. Here, $$f(x) = 2x^2 - 7x + 3$$

$$\text{Sum of roots} = p + q = -\frac{\text{Coeff. of } x}{\text{Coeff. of } x^2}$$

$$= -\left(\frac{-7}{2}\right) = \frac{7}{2} \qquad \tfrac{1}{2}$$

$$\text{Product of roots} = pq = \frac{\text{Constant}}{\text{Coeff. of } x^2} = \frac{3}{2} \qquad \tfrac{1}{2}$$

We know that $(p + q)^2 = p^2 + q^2 + 2pq$ ½

$\Rightarrow$ $p^2 + q^2 = (p + q)^2 - 2pq$ ½

$$= \left(\frac{7}{2}\right)^2 - 3 = \frac{49}{4} - \frac{3}{1} = \frac{37}{4}$$

(CBSE Marking Scheme, 2012)

6. $99x + 101y = 499$ ...(i)

$101x + 99y = 501$ ...(ii) ½

Adding eqn. (1) and (2), we get

$200x + 200y = 1000 \Rightarrow x + y = 5$ ...(iii) ½

Subtracting equation (2) from eqn. (1), we get

$-2x + 2y = -2$

$\Rightarrow$ $x - y = 1$ ...(iv) ½

Adding equations (3) and (4), we get $2x = 6 \Rightarrow x = 3$ ½

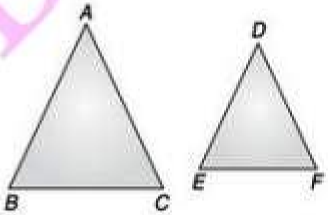
Substituting the value of x in eqn (3), we get $y = 2$.

7. In ΔABC and ΔDEF

$$\frac{AB}{DE} = 3; \frac{AC}{DF} = 3;$$

$$P_1 = 3P_2 \Rightarrow BC = 3EF$$

$\Rightarrow$ $$\frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF} = 3$$



$\Rightarrow$ $\Delta ABC \sim \Delta DEF$ 1

$\Rightarrow$ $$\frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta DEF)} = \left(\frac{AB}{DE}\right)^2 = (3)^2 = 9. \qquad 1$$

8. $$\cos(A - B) = \frac{\sqrt{3}}{2} = \cos 30^\circ$$

$\Rightarrow$ $A - B = 30^\circ$ ...(i) ½

$$\sin(A + B) = \frac{\sqrt{3}}{2}$$

$$= \sin 60^\circ$$

$\Rightarrow$ $A + B = 60^\circ$ ...(ii) ½

Adding equations (i) and (ii), we get

$$2A = 90^\circ$$

$\Rightarrow$ $A = 45^\circ$ ½

From eqn. (ii), $B = 60^\circ - A = 60^\circ - 45^\circ = 15^\circ$ ½

(CBSE Marking Scheme, 2012)

9. Here, $\sqrt{3}\sin\theta - \cos\theta = 0$ and $0^\circ < \theta < 90^\circ$

$\Rightarrow$ $\sqrt{3}\sin\theta = \cos\theta$

$\Rightarrow$ $\dfrac{\sin\theta}{\cos\theta} = \dfrac{1}{\sqrt{3}}$ 1

$\Rightarrow$ $\tan\theta = \dfrac{1}{\sqrt{3}}$

$= \tan 30^\circ$ $\left(\because \tan\theta = \dfrac{\sin\theta}{\cos\theta}\right)$

$\Rightarrow$ $\theta = 30^\circ$ 1

10.

Class Interval	Frequency	Less than *c.f.*
0 – 10	4	4
10 – 20	4	8
20 – 30	8	16
30 – 40	10	26
40 – 50	12	38
50 – 60	8	46
60 – 70	4	50
Total	$N = 50$	

1

Here, $\dfrac{N}{2} = \dfrac{50}{2} = 25$

Hence, the median class is 30 – 40. 1

SECTION — C

11. Let p be a prime number and if possible, let $\sqrt{p}$ be a rational number.

So, we can write $\sqrt{p} = \dfrac{m}{n}$, where m and n are integers having no common factor other than 1 and $n \neq 0$. ½

Then, $\sqrt{p} = \dfrac{m}{n}$

Squaring both sides, we get

$$\frac{(\sqrt{p})^2}{1} = \left(\frac{m}{n}\right)^2$$

$\Rightarrow$ $\dfrac{p}{1} = \dfrac{m^2}{n^2}$

$\Rightarrow$ $pn^2 = m^2$ ...(i) 1

$\therefore$ p divides m^2. [$\because$ p divides pn^2.]

and p divides m. [$\because$ p is prime and p divides $m^2 \Rightarrow p$ divides m]

Let $m = pq$ for some integer q.

On putting $m = pq$ in eq. (i), we get

$$pn^2 = p^2q^2$$

$\Rightarrow$ $n^2 = pq^2$

$\Rightarrow$ p divides n^2 [$\because$ p divides pq^2] **1**

$\therefore$ p divides n. [$\because$ p is prime and p divides $m^2 \Rightarrow p$ divides n]

Thus, p is a common factor of m and n but this contradicts the fact that m and n have no common factor other than 1.

The contradiction arises by assuming that $\sqrt{p}$ is rational. Hence, $\sqrt{p}$ is irrational. ½

12. The number of rooms will be minimum if each room accomodates maximum number of participants. In each room, the same number of participants are to be seated and all of them must be of the same subject. Therefore, the number of paricipants in each room must be the *HCF* of 60, 84 and 108. The prime factorisation of 60, 84 and 108 are as under **1**

$$60 = 2^2 \times 3 \times 5$$
$$84 = 2^2 \times 3 \times 7$$
$$108 = 2^2 \times 3^3$$

Hence, $HCF = 2^2 \times 3 = 12$ **1**

Therefore, in each room 12 participants can be seated.

$$\therefore \quad \text{Number of rooms required} = \frac{\text{Total number of participants}}{12}$$

$$= \frac{60+84+108}{12} = \frac{252}{12}$$

$$= 21$$ **1**

13. For zeroes of the polynomial, we get $p(x) = 5x^3 + 8x - 4 = 0$

$\Rightarrow$ $5x^2 + 10x - 2x - 4 = 0$

$\Rightarrow$ $5x(x + 2) - 2(x + 2) = 0$

$\Rightarrow$ $(x + 2)(5x - 2) = 0 \Rightarrow x = -2, x = \frac{2}{5}$ **1**

Hence, zeroes of the quadratic polynomial $5x^2 + 8x - 4$ are -2 and $2/5$.

Verification: Sum of zeroes $= -2 + \frac{2}{5} = \frac{-8}{5}$

Product of zeroes $= (-2) \times \left(\frac{2}{5}\right) = \frac{-4}{5}$ ½

Again sum of zeroes $= \frac{\text{coeff. of } x}{\text{coeff. of } x^2} = \frac{-8}{5}$ ½

Product of zeroes $= \frac{\text{constant}}{\text{coeff. of } x^2} = \frac{-4}{5}$ ½

Thus, the relationship is verified. ½

14. Given equations are:

$2x + 3y = 7$ and $2\alpha x + (\alpha + \beta)y = 28$.

We know that the condition for a pair of linear equations to be consistent and having infinite number of solution is

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$ **1**

$$\underset{\text{I}}{\frac{2}{2\alpha}} = \underset{\text{II}}{\frac{3}{\alpha+\beta}} = \underset{\text{III}}{\frac{7}{28}}$$

From I and III, $\frac{2}{2\alpha} = \frac{7}{28}$ ½

$\Rightarrow$ $\alpha = 4$ ½

From II and III, $\frac{3}{\alpha+\beta} = \frac{7}{28}$

$$\alpha + \beta = 12$$
$$\beta = 12 - \alpha$$
$$\beta = 12 - 4$$

$\Rightarrow$ $\beta = 8$

Hence $\alpha = 4$, and $\beta = 8$. 1

15. Let $\angle A = x; \angle B = y$ 1

Then $\angle C = 3\angle B \Rightarrow \angle C = 3y$ (in degree measures)

$\therefore$ $3\angle B = 2(\angle A + \angle B) \Rightarrow 3y = 2(x + y)$

$\Rightarrow$ $y = 2x \Rightarrow 2x - y = 0$ ...(i) 1

Since $\angle A, \angle B, \angle C$ are angles of a triangle.

$\therefore$ $\angle A + \angle B + \angle C = 180^\circ$

$\Rightarrow$ $x + y + 3y = 180$ ...(ii) 1

Putting $y = 2x$ in (ii), we get

$$x + 8x = 180$$

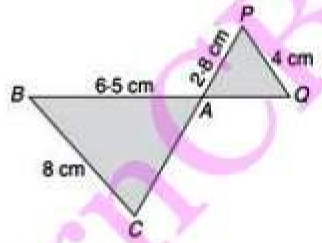
$\Rightarrow$ $9x = 180 \Rightarrow x = 20^\circ$

$\therefore$ $y = 2x = 40^\circ$

Hence $\angle A = 20^\circ, \angle B = 40^\circ$

$\angle C = 3y^\circ = 120^\circ$ 1

16. In ΔABC and ΔAPQ



$\therefore$ $BC \parallel PQ$ (Given)

$\angle CBA = \angle AQP$, (Alternate angles)

$\angle BAC = \angle PAQ$, (Vertically opposite angles)

$\therefore$ $\Delta ABC \sim \Delta AQP$ (AA Similarity) 1

$\therefore$ $\frac{AB}{AQ} = \frac{BC}{QP} = \frac{AC}{AP}$ 1

$\Rightarrow$ $\frac{6\cdot5}{AQ} = \frac{8}{4} = \frac{AC}{2\cdot8}$

$\Rightarrow$ $AQ = \frac{6\cdot5}{2} = 3\cdot25$ cm, $AC = 2 \times 2\cdot8 = 5\cdot6$ cm. 1

17. L.H.S. $= \frac{\text{cosec}^2\theta}{\text{cosec}\theta - 1} - \frac{\text{cosec}^2\theta}{\text{cosec}\theta + 1} = \text{cosec}^2\theta \left[\frac{1}{\frac{1}{\sin\theta} - 1} - \frac{1}{\frac{1}{\sin\theta} + 1}\right]$ **1**

$$= \text{cosec}^2\theta \left[\frac{\sin\theta}{1 - \sin\theta} - \frac{\sin\theta}{1 + \sin\theta}\right]$$

$$= \frac{1 \times \sin\theta}{\sin^2\theta}\left[\frac{(1 + \sin\theta) - (1 - \sin\theta)}{(1 - \sin\theta)(1 + \sin\theta)}\right]$$ **1**

$$= \frac{1}{\sin\theta}\left[\frac{2\sin\theta}{1 - \sin^2\theta}\right]$$

$$= \frac{2}{\cos^2\theta} = 2\sec^2\theta$$ **1**

= R.H.S.

18. Given, $x \sin\theta = y \cos\theta$

$\Rightarrow$ $x = \frac{y\cos\theta}{\sin\theta}$ ...(i) ½

and $x \sin^3\theta + y \cos^3\theta = \sin\theta \cos\theta$ ...(ii)

Eliminating x from (i) and (ii),

$$\frac{y\cos\theta}{\sin\theta} \times \sin^3\theta + y\cos^3\theta = \sin\theta\cos\theta$$

$\Rightarrow$ $y\cos\theta \sin^2\theta + y\cos^3\theta = \sin\theta\cos\theta$

$\Rightarrow$ $y\cos\theta(\sin^2\theta + \cos^2\theta) = \sin\theta\cos\theta$

$\Rightarrow$ $y = \sin\theta$ ....(iii) 1

Substituting this value of y in (iii),

$x = \cos\theta$ ....(iv) 1

Squaring and adding (iii) and (iv)

$x^2 + y^2 = \cos^2\theta + \sin^2\theta = 1$ ½

19.

Class Interval	x_i (Class marks)	f_i	$f_i x_i$	Cum. Frequency
0 – 10	5	8	40	8
10 – 20	15	16	240	24
20 – 30	25	36	900	60
30 – 40	35	34	1190	94
40 – 50	45	6	270	100
		$\Sigma f_i = 100$	$\Sigma f_i x_i = 2640$	

Mean $= \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{2640}{100} = 26{\cdot}4$ **1**

Here, Median Class = 20 – 30 **1**

Median $= 20 + \frac{50 - 24}{36} \times 10$

$= 20 + 7{\cdot}22 = 27{\cdot}22$ **1**

20. Let assumed mean, $a = 35$ and $h = 10$.

x_i (Class Marks)	$v_i = \frac{x_i - a}{h}$	f_i	$f_i v_i$
5	−3	5	−15
15	−2	13	−26
25	−1	20	−20
35	0	15	0
45	1	7	7
55	2	5	10
Total		$\Sigma f_i = 65$	$\Sigma f_i v_i = -44$

1½

$\therefore$ Mean, $\bar{x} = a + \frac{\Sigma f_i x_i}{\Sigma f_i} \times h = 35 + \frac{-44}{65} \times 10 = 35 - 6{\cdot}76$ 1

$= 28{\cdot}27$ ½

SECTION — D

21. (a) Maximum number of parallel rows of each class

$= HCF$ of 104 and 96

Now, $104 = 2 \times 2 \times 2 \times 13$

$96 = 2 \times 2 \times 2 \times 12$

Hence, $HCF\ (104, 9) = 8.$ 1

Number of students of class X in 1 row $= \frac{104}{8} = 13$ 1

Number of students of class IX in 1 row $= \frac{96}{8} = 12$ 1

(b) Mathematical concept used is *HCF* of numbers 104 and 96.

(c) To minimise the tendency of the students to copy and to teach them the value of honesty. 1

22. Polynomial $g(x) = x^4 - 5x^3 + 2x^2 + 10x - 8$ and $p(x) =$

$x^2 - 2)\ x^4 - 5x^3 + 2x^2 + 10x - 8\ (\ x^2 - 5x + 4$

$x^4 \quad - 2x^2$

$- \quad +$

$-5x^3 + 4x^2 + 10x$

$-5x^3 \quad + 10x$

$+ \quad -$

$4x^2 - 8$ 2

$4x^2 - 8$

$- \quad +$

0

$x^2 - 5x + 4 = (x - 4)(x - 1)$ 1

Hence, other zeroes are 4 and 1. 1

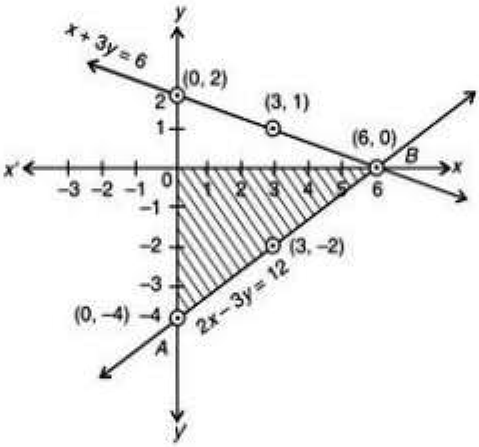
23. $x + 3y = 6 \Rightarrow y = \frac{6 - x}{3}$ ...(1)

x	3	6	0
y	1	0	2

$2x - 3y = 12 \Rightarrow y = \frac{2x - 12}{3}$ ...(2) ½

x	0	6	3
y	−4	0	−2

Plotting the above points and drawing line joining them, we get the graph of the equations $x + 3y = 6$ and $2x - 3y = 12$.



1

Clearly, the two lines intersect at point B (6, 0).

Hence, $x = 6$ and $y = 0$ is the solution of the system. 1

Again $AOBA$ is the region bounded by the line $2x - 3y = 12$ and both the coordinate axes. 1

24.

$$\frac{5}{x-1} + \frac{1}{y-2} = 2 \quad \text{...(i)}$$

$$\frac{6}{x-1} - \frac{3}{y-2} = 1 \quad \text{...(ii)} \; 1$$

Let $\frac{1}{x-1} = p, \; \frac{1}{y-2} = q$

Then the equations (i) and (ii) will be

$$5p + q = 2 \quad \text{...(iii)}$$

$$6p - 3q = 1 \quad \text{(iv)} \; 1$$

Multiplying eq. (iii) by 3 and then adding in eq. (iv), we get

$$21p = 7 \Rightarrow p = \frac{7}{21} = \frac{1}{3} \quad 1$$

Putting this value of p in eq. (iii),

$$q = 2 - 5p = 2 - 5 \times \frac{1}{3} = \frac{1}{3}$$

Now $p = \frac{1}{x-1} = \frac{1}{3}$

$\Rightarrow$ $x - 1 = 3$

$\therefore$ $x = 4$

and $q = \frac{1}{y-2} = \frac{1}{3}$

$\Rightarrow$ $y - 2 = 3$

$y = 5$ 1

$\therefore$ Solution is $x = 4, y = 5$.

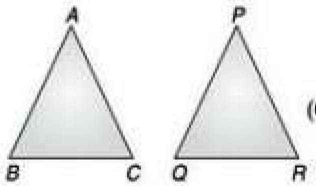
(CBSE Marking Scheme, 2012)

25. Given: Two triangles ABC and PQR such that

$$\Delta ABC \sim \Delta PQR,$$

and $$\text{ar } \Delta ABC = \text{ar } \Delta PQR$$

To prove: $$\Delta ABC \cong \Delta PQR$$



Proof: $$\Delta ABC \sim \Delta PQR \quad \text{(Given)}$$

$\Rightarrow$ $$\frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta PQR)} = \frac{AB^2}{PQ^2} = \frac{BC^2}{QR^2} = \frac{CA^2}{RP^2} \quad ...(1)$$

Also $$\text{ar }(\Delta ABC) = \text{ar }(\Delta PQR) \quad \text{(Given)}$$

$\Rightarrow$ $$\frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta PQR)} = 1 \quad 1$$

From equation (1), we have $$\frac{AB^2}{PQ^2} = \frac{BC^2}{QR^2} = \frac{CA^2}{RP^2} = 1 \quad 1$$

$\Rightarrow$ $$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP} = 1 \quad 1$$

$\Rightarrow$ $$AB = PQ$$
$$BC = QR$$
$$CA = RP$$

$\therefore$ By SSS congruency, $$\Delta ABC \cong \Delta PQR. \quad 1$$

26. Let the number of red balls be x and white balls by y.
According to the question,

$$\frac{1}{2}y = \frac{1}{3}x \text{ or } 2x - 3y = 0 \quad ....(i)$$

and $$3(x+y) - 7y = 6 \quad 1$$

or, $$3x - 4y = 6 \quad ....(ii)$$

Multiplying eq. (i) by 3 and eq. (ii) by 2 and then subtracting, we get

$$6x - 9y = 0 \quad 1$$
$$6x - 8y = 12$$
$$- \quad + \quad -$$
$$-y = -12$$

$\Rightarrow$ $$y = 12$$

$\therefore$ $$2x - 36 = 0 \Rightarrow x = 18 \quad 1$$
$$x = 18, y = 12$$

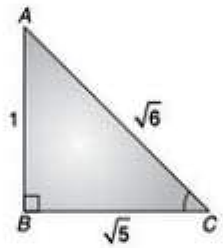
Hence, number of red balls = 18
and number of white balls = 12 1

(CBSE Marking Scheme, 2012)

27. $$\tan\theta = \frac{AB}{BC} = \frac{1}{\sqrt{5}}$$

In ΔABC, $$AC^2 = AB^2 + BC^2 = 1 + 5 = 6 \quad 1$$

$\Rightarrow$ $$AC = \sqrt{6}$$



(i) $$\frac{\text{cosec}^2\theta - \sec^2\theta}{\text{cosec}^2\theta + \sec^2\theta} = \frac{(\sqrt{6})^2 - \left(\frac{\sqrt{6}}{\sqrt{5}}\right)^2}{(\sqrt{6})^2 + \left(\frac{\sqrt{6}}{\sqrt{5}}\right)^2} = \frac{6 - \frac{6}{5}}{6 + \frac{6}{5}}$$

$$= \frac{24}{36} = \frac{2}{3}.$$ 1½

(ii) $$\sin^2\theta + \cos^2\theta = \left(\frac{1}{\sqrt{6}}\right)^2 + \left(\frac{\sqrt{5}}{\sqrt{6}}\right)^2$$

$$= \frac{1}{6} + \frac{5}{6} = 1.$$ 1½

28. $$\text{L.H.S.} = \sqrt{\frac{\sec\theta - 1}{\sec\theta + 1}} + \sqrt{\frac{\sec\theta + 1}{\sec\theta - 1}}$$

$$= \frac{(\sec\theta - 1) + (\sec\theta + 1)}{\sqrt{(\sec\theta + 1)(\sec\theta - 1)}}$$ 1

$$= \frac{2\sec\theta}{\sqrt{\sec^2\theta - 1}} = \frac{2\sec\theta}{\tan\theta}$$ 1

$$= 2 \times \frac{1}{\cos\theta} \times \frac{\cos\theta}{\sin\theta}$$ 1

$$= 2 \times \frac{1}{\sin\theta}$$

$$= 2\,\text{cosec}\,\theta$$ 1

(CBSE Marking Scheme, 2012)

29. $$15\tan^2\theta + 4\sec^2\theta = 23$$

$$15\tan^2\theta + 4(\tan^2\theta + 1) = 23$$ 1

$$15\tan^2\theta + 4\tan^2\theta + 4 = 23$$

$$19\tan^2\theta = 19$$

$\Rightarrow$ $$\tan\theta = 1 = \tan 45°$$

$\Rightarrow$ $$\theta = 45°$$ 1

Now, $$(\sec\theta + \text{cosec}\,\theta)^2 - \sin^2\theta = (\sec 45° + \text{cosec}\,45°)^2 - \sin^2 45°$$

$$= \left(\sqrt{2} + \sqrt{2}\right)^2 - \left(\frac{1}{\sqrt{2}}\right)^2$$ 1

$$= \left(2\sqrt{2}\right)^2 - \frac{1}{2} = 8 - \frac{1}{2} = \frac{15}{2}$$ 1

30.

Class Interval	Frequency	Cumuative Frequency
0 – 10	5	5
10 – 20	f_1	$5 + f_1$
20 – 30	20	$25 + f_1$
30 – 40	15	$40 + f_1$
40 – 50	f_2	$40 + f_1 + f_2$
50 – 60	5	$45 + f_1 + f_2$
Total	$N = 60$	

2

Hence, $$45 + f_1 + f_2 = 60$$

$\Rightarrow$ $$f_1 + f_2 = 15 \quad \text{...(i)}$$

and $$N = 60$$

Median = 28·5, $\therefore$ Median class is 20 – 30.

$$\text{Median} = l + \frac{\frac{N}{2} - c.f}{f} \times h$$

$$28{\cdot}5 = 20 + \frac{30 - 5 - f_1}{20} \times 10$$ 1

$\Rightarrow$ $8{\cdot}5 \times 2 = 25 - f_1$

$\Rightarrow$ $f_1 = 25 - 17 = 8$

From (1), $f_2 = 15 - f_1 = 15 - 8 = 7$

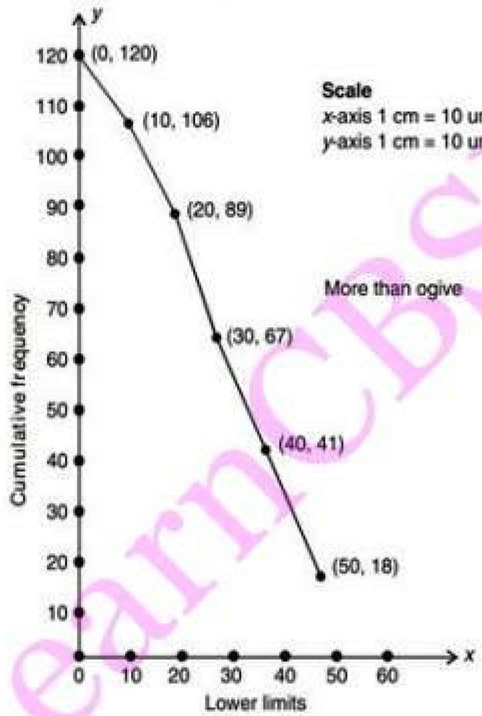
$\therefore$ $f_1 = 8$ and $f_2 = 7$ 1

31.

Weight (in kg)	Cumulative Frequency
More than or equal to 0	120
More than or equal to 10	106
More than or equal to 20	89
More than or equal to 30	67
More than or equal to 40	41
More than or equal to 50	18

2

Plotting the points, we get the following ogive.



2