SAMPLE
QUESTION PAPER - 4
Solved

Time: 3 Hours **Maximum Marks: 90**

SECTION — A

1. Any prime number greater than 3 is of the form $6k \pm 1$, where k is a natural number.

$$(6k \pm 1)^2 = 36k^2 \pm 12k + 1$$
$$= 6k(6k \pm 2) + 1$$

$\therefore$ Required remainder = 1 **1**

2. Let the zeroes be α, β, $\alpha = -5, \beta = 4$

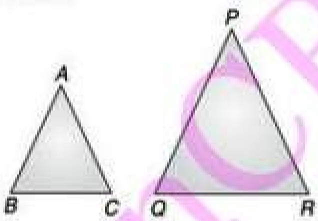
So, the required polynomial $= (x - \alpha)(x - \beta)$

$$= (x + 5)(x - 4)$$
$$= x^2 - 4x + 5x - 20$$
$$= x^2 + x - 20$$ **1**

3. Given,

Perimeter of ΔABC = 32 cm
Perimeter of ΔPQR = 48 cm

For similar triangles, we know that



$$\frac{AB}{PQ} = \frac{AC}{PR} = \frac{BC}{QR} = \frac{\text{Perimeter of } \Delta ABC}{\text{Perimeter of } \Delta PQR}$$ ½

$\Rightarrow$ $$\frac{AC}{6} = \frac{32}{48}$$

$\Rightarrow$ $$AC = \frac{6 \times 32}{48} = 4 \text{ cm.}$$ ½

4. $$\text{Median} = \frac{1}{3}\text{ Mode} + \frac{2}{3}\text{ Mean}$$

$$= \frac{1}{3} \times (8) + \frac{2}{3} \times (8)$$ ½

$$= \frac{8+16}{3} = \frac{24}{3} = 8$$ ½

SECTION — B

5. Since α and β are the zeroes of the polynomial, then

$$\alpha + \beta = -\frac{\text{coeff. of } x}{\text{coeff. of } x^2}$$

$\Rightarrow$ $$\alpha + \beta = \left(\frac{-1}{1}\right) = 1 \quad ...(i)$$

Given, $\alpha - \beta = 9$ ...(ii) **1**

From (i) and (ii) $\alpha = 5, \beta = -4$

Again $\alpha\beta = -k$

$(5)(-4) = -k$

$\Rightarrow$ $k = 20$ **1**

6. Pair of equations $kx - 4y = 3$

$6x - 12y = 9$

Condition for infinite number of solutions

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \quad \textbf{1}$$

$$\frac{k}{6} = \frac{-4}{-12} = \frac{3}{9} \quad \textbf{1}$$

$\Rightarrow$ $k = 2$

7. In ΔCAB, $\angle A = \angle B$ (Given)

$\therefore$ $AC = CB$ (By isosceles triangle property) **1**

But, $AD = BE$ (Given) ...(i)

$\therefore$ $AC - AD = CB - BE$

$CD = CE$ ...(ii)

Dividing equation (ii) by (i), $\frac{CD}{AD} = \frac{CE}{BE}$

By converse of *BPT*, $DE \parallel AB$. **1**

8.
$$\text{L.H.S.} = -1 + \frac{\sin A \sin(90° - A)}{\cot(90° - A)}$$

$$= -1 + \frac{\sin A \cos A}{\tan A} \quad ½$$

$$= 1 + \sin A \cos A \times \cot A \quad ½$$

$$= -1 + \sin A \cos A \times \frac{\cos A}{\sin A} \quad ½$$

$$= -1 + \cos^2 A = -(1 - \cos^2 A) \quad ½$$

$$= -\sin^2 A = \text{R.H.S.}$$ **Hence Proved.**

(CBSE Marking Scheme, 2012)

9. Given : $5 \operatorname{cosec} \theta = 7$

$\Rightarrow$ $$\operatorname{cosec} \theta = \frac{7}{5}$$

$\Rightarrow$ $$\sin \theta = \frac{5}{7} \quad \left[\because \operatorname{cosec}\theta = \frac{1}{\sin\theta}\right] \textbf{ 1}$$

$$\sin\theta + \cos^2\theta - 1 = \sin\theta - (1-\cos^2\theta)$$
$$= \sin\theta - \sin^2\theta$$
$$= \frac{5}{7} - \left(\frac{5}{7}\right)^2 = \frac{35-25}{49} = \frac{10}{49}$$ 1

(*CBSE Marking Scheme, 2012*)

10. Here; Modal class = 35 – 40

$l = 35, f_1 = 50, f_2 = 42, f_0 = 34, h = 5$ ½

$$\text{Mode} = l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$$ ½

$$= 35 + \frac{50-34}{100-34-42} \times 5$$ ½

$$= 35 + \frac{16 \times 5}{24} = 38{\cdot}33.$$ ½

(*CBSE Marking Scheme, 2012*)

SECTION — C

11. Let $x = 0{\cdot}3\overline{178}$

$x = {\cdot}3178178178...$ 1

$10000\,x = 3178{\cdot}178178...$

$10\,x = 3{\cdot}178178...$

Subtracting, $9990\,x = 3175$ 1

$$x = \frac{3175}{9990} = \frac{635}{1998}$$ 1

12. We have $117 = 65 \times 1 + 52$

$65 = 52 \times 1 + 13$

$52 = 13 \times 4 + 0$ 1

Hence, $HCF = 13$

$\therefore$ $65m - 117 = 13$

$\Rightarrow$ $65m = 117 + 13 = 130$

$\therefore$ $m = \frac{130}{65} = 2$ 1

Now, $65 = 13 \times 5$

$117 = 3^2 \times 13$

$\text{LCM} = 13 \times 5 \times 3^2 = 585$ 1

13. $p(x) = 6y^2 - 7y + 2$

$$\alpha + \beta = -\left(-\frac{7}{6}\right) = \frac{7}{6}$$

$$\alpha\beta = \frac{2}{6}$$

Now, $$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta} = \frac{\frac{7}{6}}{\frac{2}{6}} = \frac{7}{2}$$ 1

$$\frac{1}{\alpha}\times\frac{1}{\beta} = \frac{1}{\alpha\beta}=3$$ **1**

The required polynomial is

$$y^2-\frac{7}{2}y+3 = \frac{1}{2}[2y^2-7y+6]$$ **1**

14. Given, $x-5y = 6 \Rightarrow x = 6+5y$

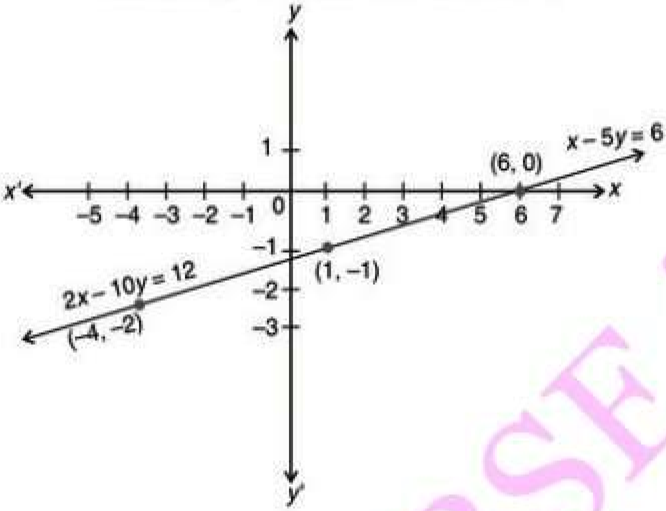
y	0	−1	−2
x	6	1	−4

1

$2x-10y = 12 \Rightarrow x = 5y+6$

y	0	−1	−2
x	6	1	−4

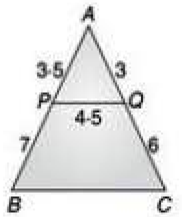
1



Since the lines are coincident, so the system of linear equations is cosistent with infinite solutions. **1**

15.

$$\frac{AP}{AB} = \frac{3{\cdot}5}{10{\cdot}5}=\frac{1}{3}$$



$\Rightarrow$ $$\frac{AQ}{AC} = \frac{3}{9}=\frac{1}{3}$$ **1**

In ΔABC $$\frac{AP}{AB} = \frac{AQ}{AC}$$ and $\angle A$ is common **1**

$\Rightarrow$ $\Delta APQ \sim \Delta ABC$ (SAS)

$\therefore$ $$\frac{AP}{AB} = \frac{PQ}{BC}$$

$$\frac{1}{3} = \frac{4{\cdot}5}{BC} \Rightarrow BC = 13{\cdot}5 \text{ cm}$$ **1**

16. Given: $DB \perp BC$, $DE \perp AB$ and $AC \perp BC$.

To prove: $$\frac{BE}{DE} = \frac{AC}{BC}$$ **1**

Proof: In ΔABC,

$\angle 1 + \angle 2 = 90°$ $[\angle C = 90°]$ **1**

But $\angle 2 + \angle 3 = 90°$ (Given)

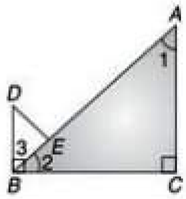
$\Rightarrow$ $\angle 1 = \angle 3$

In ΔABC and ΔBDE, $\angle 1 = \angle 3$ (Proved)

$\angle ACB = \angle DEB = 90^\circ$ (Given)

$\therefore$ $\Delta ABC \sim \Delta BDE$ (*AA*) **1**

$\Rightarrow$ $\dfrac{AC}{BC} = \dfrac{BE}{DE}$.



17. $\dfrac{\sec^2(90^\circ-\theta)-\cot^2\theta}{2(\sin^2 25^\circ+\sin^2 65^\circ)}-\dfrac{2\cos^2 60^\circ \tan^2 28^\circ \tan^2 62^\circ}{3(\sec^2 43^\circ-\cot^2 47^\circ)}$

$= \dfrac{(\text{cosec}^2\theta-\cot^2\theta)}{2(\sin^2 25^\circ+\cos^2 25^\circ)}-\dfrac{2\times\frac{1}{2}\times\frac{1}{2}\tan^2 28^\circ\times\cot^2 28^\circ}{3[\sec^2 43^\circ-\tan^2 43^\circ]}$ **1**

$= \dfrac{1}{2\times(1)}-\dfrac{\frac{1}{2}\times\tan^2 28^\circ\times\frac{1}{\tan^2 28^\circ}}{3}$ **1**

$= \dfrac{1}{2}-\dfrac{1}{6}=\dfrac{1}{3}$ **1**

(CBSE Marking Scheme, 2012)

18. L.H.S. $= \dfrac{1}{\text{cosec}A-\cot A}-\dfrac{1}{\sin A}$

$= \dfrac{1}{\text{cosec}A-\cot A}\times\left(\dfrac{\text{cosec}A+\cot A}{\text{cosec}A+\cot A}\right)-\text{cosec }A$ ½

$= \dfrac{\text{cosec}A+\cot A}{\text{cosec}^2A-\cot^2 A}-\text{cosec}A$ **1**

$= \dfrac{\text{cosec}A+\cot A}{1}-\text{cosec}A=\cot A$ ½

Now, R.H.S. $= \dfrac{1}{\sin A}-\dfrac{1}{\text{cosec}A+\cot A}$

$= \text{cosec}A-\dfrac{1}{\text{cosec}A+\cot A}\times\left(\dfrac{\text{cosec}A-\cot A}{\text{cosec}A-\cot A}\right)$ ½

$= \text{cosec}A-\dfrac{(\text{cosec}A-\cot A)}{\text{cosec}^2A-\cot^2 A}$

$= \text{cosec}A-\dfrac{(\text{cosec}A-\cot A)}{1}$

$= \cot A$ ½

$\therefore$ L.H.S. = R.H.S. **Hence Proved.**

19.

Class Interval	f_i	*c.f.*
0 – 100	15	15
100 – 200	17	32
200 – 300	f	$32+f$
300 – 400	12	$44+f$
400 – 500	9	$53+f$
500 – 600	5	$58+f$
600 – 700	2	$60+f$

From table, $n = 60 + f \Rightarrow \frac{n}{2} = \frac{60+f}{2}$ **1**

Since, Median = 240

$\therefore$ Median class = 200 – 300 **½**

$$\text{Median} = l + \left[\frac{\frac{n}{2} - c.f.}{f}\right] \times h$$ **½**

$$\Rightarrow \quad 240 = 200 + \left[\frac{\frac{60+f}{2} - 32}{f}\right] \times 100$$

$$\Rightarrow \quad 40 = \left[\frac{60 + f - 64}{2f}\right] 100$$ **½**

$\Rightarrow \quad 8f = 10f - 40$

$\Rightarrow \quad 2f = 40$

$f = 20$ **½**

20.

Class Interval	Frequency
0 – 10	8
10 – 20	12
20 – 30	25
30 – 40	13
40 – 50	12
Total	**70**

1

Here, modal class 20 – 30.

and $l = 20, f_1 = 25, f_2 = 13, f_0 = 12, h = 10$ **½**

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$ **½**

$$= 20 + \left(\frac{25 - 12}{50 - 12 - 13}\right) \times 10$$ **½**

$$= 20 + \frac{13}{25} \times 10 = 20 + 5{\cdot}2 = 25{\cdot}2.$$ **½**

SECTION — D

21. $n^3 - n = n(n^2 - 1)$

$= n(n + 1)(n + 1)$

$= (n - 1)\,n(n + 1)$

= product of three consecutive positive integers.

Now, we have to show that the product of three consecutive positive integers is divisible by 6.

We know that any positive integer a is of the form $3q$, $3q + 1$ or $3q + 2$ for some integer q.

Let a, $a + 1$, $a + 2$ be any three consecutive integers. **½**

Case I: If $a = 3q$.

$a(a + 1)(a + 2) = 3q(3q + 1)(3q + 2)$

$= 3q$ (even number, say $2r$)

($\because$ Product of two consecutive integers $(3q + 1)$ and $(3q + 2)$ is an even integer) which is divisible by 6.) **1**

$= 6qr$, which is divisible by 6

Case II: If $a = 3q + 1.$

$\therefore$ $a(a+1)(a+2) = (3q+1)(3q+2)(3q+3)$

$= \text{(even number, say } 2r\text{) } (3)\ (q+1)$

$= 6r(q+1),$ 1

which is divisible by 6.

Case III: If $a = 3q + 2.$

$\therefore$ $a(a+1)(a+2) = (3q+2)(3q+3)(3q+4)$

$= \text{multiple of 6 from every } q$

$= 6r\text{(say)}.$ 1

which is divisible by 6.

Hence, the product of three consecutive integers, *i.e.*, $n^3 - n$ is divisible by 6. ½

(CBSE Marking Scheme, 2012)

22. Since α and β are the zeroes of the quadratic polynomial $x^2 + 4x + 3$.

$$\alpha + \beta = -4$$
$$\alpha\beta = 3$$ 1

Sum of zeroes of the required polynomial

$$= 1 + \frac{\beta}{\alpha} + 1 + \frac{\alpha}{\beta}$$

$$= \frac{\alpha\beta + \beta^2 + \alpha\beta + \alpha^2}{\alpha\beta}$$

$$= \frac{\alpha^2 + \beta^2 + 2\alpha\beta}{\alpha\beta}$$

$$= \frac{(\alpha+\beta)^2}{\alpha\beta} = \frac{(-4)^2}{3} = \frac{16}{3}$$ 1

Product of zeroes of the required polynomial

$$= \left(1 + \frac{\beta}{\alpha}\right)\left(1 + \frac{\alpha}{\beta}\right)$$

$$= 1 + \frac{\alpha}{\beta} + \frac{\beta}{\alpha} + \frac{\alpha\beta}{\alpha\beta}$$

$$= \frac{\alpha^2 + \beta^2 + 2\alpha\beta}{\alpha\beta} = \frac{(\alpha+\beta)^2}{\alpha\beta}$$

$$= \frac{(-4)^2}{3} = \frac{16}{3}$$ 1

Thus, the required polynomial is:

x^2 – (sum of the zeroes)x + product of the zeroes

$$= x^2 - \left(\frac{16}{3}\right)x + \frac{16}{3}$$

or $$= \left(x^2 - \frac{16}{3}x + \frac{16}{3}\right)$$

$$= (3x^2 - 16x + 16)\frac{1}{3}$$ 1

23. Let the incomes of two persons be $11x$ and $7x$.
Also the expenditures of two persons by $9y$ and $5y$,

$$11x - 9y = 400 \quad \text{....(i)}$$

and

$$7x - 5y = 400 \quad \text{....(ii)}$$

Multipying eq. (i) by 5 and eq. (ii) by 9 and subtracting,

$$55x - 45y = 2000 \quad \text{....(iii)}$$

$$63x - 45y = 3600 \quad \text{....(iv)} \; 1$$

$$\underline{- \quad + \quad +}$$

On subtracting

$$-8x = -1600$$

$$x = \frac{-1600}{-8} = 200 \quad 1$$

Putting this value of x in eq. (i), we get

$$2200 - 9y = 400$$

$\Rightarrow$

$$9y = 2200 - 400 = 1800$$

$$y = \frac{1800}{9} = 200 \quad 1$$

Their monthly incomes are ₹ 2200 and ₹ 1400. 1

(CBSE Marking Scheme, 2014)

24. Let the speed of the boat be x km/hr and the speed of the stream be y km/hr.
According to the question,

$$\frac{32}{x-y} + \frac{36}{x+y} = 7$$

and

$$\frac{40}{x-y} + \frac{48}{x+y} = 9$$

Let

$$\frac{1}{x-y} = A, \frac{1}{x+y} = B,$$

$$32A + 36B = 7$$

and

$$40A + 48B = 9$$

Solving these equations, we get

$$A = \frac{1}{8}, B = \frac{1}{12} \quad 1$$

Hence

$$A = \frac{1}{8} = \frac{1}{x-y}$$

$\Rightarrow$

$$x - y = 8 \quad \text{....(i)} \; ½$$

and

$$B = \frac{1}{12} = \frac{1}{x+y}$$

$\Rightarrow$

$$x + y = 12 \quad \text{....(ii)} \; ½$$

Adding equations (i) and (ii),

$$2x = 20$$

$\Rightarrow$

$$x = 10 \quad 1$$

Putting this value of x in eq. (i), $y = x - 8 = 10 - 8 = 2$

Hence, the speed of the boat = 10 km/hr and speed of the stream = 2 km/hr. 1

(CBSE Marking Scheme, 2012)

25. Given: In the given figure. $\Delta FEC \cong \Delta GBD$ and $\angle 1 = \angle 2$.

To Prove: $\Delta ADE \sim \Delta ABC$

$\Delta FEC \cong \Delta GBD$

$\Rightarrow$ $EC = BD$ ...(1) **1**

It is given that $\angle 1 = \angle 2$

$\Rightarrow$ $AE = AD$ ...(2) **1**

From eqns. (1) and (2), $\dfrac{AE}{EC} = \dfrac{AD}{BD}$

$\Rightarrow$ $DE \parallel BC$, (converse of B.P.T.)

$\Rightarrow$ $\angle 1 = \angle 3$ and $\angle 2 = \angle 4$ **1**

Thus, in ΔADE and ΔABC, $\angle A = \angle A$

$\angle 1 = \angle 3$

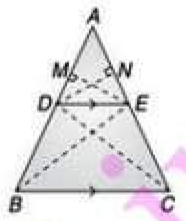
$\angle 2 = \angle 4$

So by AAA criterion of similarity, we have

$\Delta ADE \sim \Delta ABC$. **Hence Proved. 1**

26. Given : ABC is a tirangle in which $DE \parallel BC$.

To prove: $\dfrac{AD}{BD} = \dfrac{AE}{CE}$



Construction: Draw $DN \perp AE$ and $EM \perp AD$, Join BE and CD. **1**

Proof: In ΔADE, area $(\Delta ADE) = \dfrac{1}{2} \times AE \times DN$ ...(i)

In ΔDEC, area $(\Delta DCE) = \dfrac{1}{2} \times CE \times DN$ ...(ii)

Dividing eqn. (ii) by eqn. (i), we get

$$\frac{\text{area}(\Delta ADE)}{\text{area}(\Delta DEC)} = \frac{\frac{1}{2} \times AE \times DN}{\frac{1}{2} \times CE \times DN}$$

$$\frac{\text{area}(\Delta ADE)}{\text{area}(\Delta DEC)} = \frac{AE}{CE} \quad \text{...(iii) } \tfrac{1}{2}$$

Now in ΔADE, area $(\Delta ADE) = \dfrac{1}{2} \times AD \times EM$ ...(iv) ½

Now in ΔDEB, area $(\Delta DEB) = \dfrac{1}{2} \times EM \times BD$ ...(v)

Dividing equation (iv) by equation (v), we get

$$\frac{\text{area}(\Delta ADE)}{\text{area}(\Delta DEB)} = \frac{\frac{1}{2} \times AD \times EM}{\frac{1}{2} \times BD \times EM} \quad \tfrac{1}{2}$$

$$\frac{\text{area}(\Delta ADE)}{\text{area}(\Delta DEB)} = \frac{AD}{BD}. \quad \text{...(iv) } \tfrac{1}{2}$$

ΔDEB and ΔDEC lies on the same base DE and between same parallel lines DE and BC.

$\therefore$ area (ΔDEB) = area (ΔDEC)

Thus, from equation (iii), $\dfrac{\text{area}(\Delta ADE)}{\text{area}(\Delta DEB)} = \dfrac{AE}{CE}$ ...(vii)

$\dfrac{AE}{CE} = \dfrac{AD}{BD}$ **Hence Proved. 1**

27.

$$\text{L.H.S.} = \frac{\cos^2\theta}{1-\tan\theta} + \frac{\sin^3\theta}{\sin\theta - \cos\theta}$$

$$= \frac{\cos^2\theta}{1-\frac{\sin\theta}{\cos\theta}} + \frac{\sin^3\theta}{\sin\theta-\cos\theta}$$ 1

$$= \frac{\cos^3\theta}{\cos\theta-\sin\theta} - \frac{\sin^3\theta}{\cos\theta-\sin\theta}$$ 1

$$= \frac{\cos^3\theta-\sin^3\theta}{\cos\theta-\sin\theta}$$ 1

$$= \frac{(\cos\theta-\sin\theta)(\cos^2\theta+\sin^2\theta+\sin\theta\cos\theta)}{(\cos\theta-\sin\theta)}$$

$= 1 + \sin\theta\cos\theta = \text{R.H.S.}$ **Hence Proved. 1**

(CBSE Marking Scheme, 2012)

28. We have $m^2 = a^2\cos^2\theta + 2ab\sin\theta\cos\theta + b^2\sin^2\theta$ ...(i) **1**

and $n^2 = a^2\sin^2\theta - 2ab\sin\theta\cos\theta + b^2\cos^2\theta$ ....(ii) **1**

Adding equatons (i) and (ii),

$$m^2 + n^2 = a^2(\cos^2\theta + \sin^2\theta) + b^2(\cos^2\theta + \sin^2\theta)$$

$= a^2(1) + b^2(1)$ **1**

$= a^2 + b^2 = \text{R.H.S.}$ **Hence Proved. 1**

29.

$$\text{R.H.S.} = \frac{p^2-1}{p^2+1}$$

$$= \frac{(\text{cosec}\theta+\cot\theta)^2-1}{(\text{cosec}\theta+\cot\theta)^2+1}$$ 1

$$= \frac{\text{cosec}^2\theta+\cot^2\theta+2\text{cosec}\theta.\cot\theta-1}{\text{cosec}^2\theta+\cot^2\theta+2\text{cosec}\theta.\cot\theta+1}$$ 1

$$= \frac{1+\cot^2\theta+\cot^2\theta+2\text{cosec}\theta\cot\theta-1}{\text{cosec}^2\theta+\text{cosec}^2\theta-1+2\text{cosec}\theta\cot\theta+1}$$

$$= \frac{2\cot\theta(\cot\theta+\text{cosec}\theta)}{2\text{cosec}\theta(\text{cosec}\theta+\cot\theta)}$$ 1

$= \frac{\cos\theta}{\sin\theta}\times\sin\theta = \cos\theta = \text{L.H.S.}$ **Hence Proved. 1**

30.

Class Interval	f	$c.f.$
5 – 10	2	2
10 – 15	12	14
15 – 20	2	16
20 – 25	4	20
25 – 30	3	23
30 – 35	4	27
35 – 40	3	30
Total	$\Sigma f = 30 = N$	

2

Since, $\frac{N}{2} = 15.$

$\therefore$ Median class = 15 – 20

$$\text{Median} = l + \left(\frac{\frac{N}{2} - c.f.}{f}\right) \times h$$ 1

From table, $l = 15, N = 30, c.f. = 14, f = 2, h = 5$

$$\text{Median} = 15 + \left(\frac{15 - 14}{2}\right) \times 5 = 15 + 2{\cdot}5$$

$$= 17{\cdot}5$$ 1

31. (i)

No. of children (x_i)	No. of famlies (f_i)	$f_i x_i$
0	5	0
1	11	11
2	25	50
3	12	36
4	5	20
5	2	10
Total	$\Sigma f_i = 60$	$\Sigma f_i x_i = 127$

1

$$\text{Mean} = \frac{\Sigma f_i x_i}{\Sigma f_i}$$

$$= \frac{127}{60} = 2{\cdot}12 \text{ approx.}$$ 1

(ii) Mean of ungrouped data. 1

(iii) For progress, we should decrease population growth. 1