SAMPLE
QUESTION PAPER - 5
Solved

Time: 3 Hours **Maximum Marks: 90**

SECTION — A

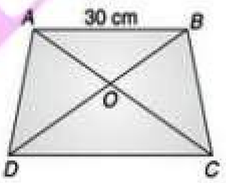
1. The least number that is divisible by all numbers from 1 to 10.

$$= \text{LCM of 1 to 10}$$
$$= 1 \times 2 \times 3 \times 2 \times 5 \times 7 \times 2 \times 3$$
$$= 2520$$ **1**

2. Given: $\alpha = \sqrt{3}$ $\beta = -\sqrt{3}$.

So, $$\text{Polynomial} = (x - \alpha)(x - \beta)$$
$$= (x - \sqrt{3})(x + \sqrt{3})$$
$$= x^2 - 3$$ **1**

3. In the triangle AOB and DOC, $\frac{AO}{OC} = \frac{BO}{OD} = \frac{1}{2}$ and $AB = 3$ cm



$\because$ $$\frac{AO}{OC} = \frac{BO}{OD}$$

$\therefore$ $$\angle AOB = \angle DOC$$

Hence, both triangles are similar

$$\frac{AO}{OC} = \frac{BO}{OD} = \frac{AB}{DC}$$

(In similar triangle corresponding sides are proportional.)

$$DC = 2AB$$
$$DC = 2 \times 3 = 6 \text{ cm}$$ **1**

4. If the curves for more than ogive and less than ogive of a grouped data meet at (30, 45), then median of the data is 30. **1**

SECTION — B

5. Let $$f(x) = 2x^2 - 5x - 3$$

Let the zeroes of polynomial are α and β, then

$$\text{Sum of zeroes, } \alpha + \beta = \frac{5}{2}$$

$$\text{product of zeroes, } \alpha\beta = -\frac{3}{2}$$

According to the quesion, zeroes of $x^2 + px + q$ are 2α and 2β.

$$\text{Sum of zeroes} = -\frac{\text{Coeff. of } x}{\text{Coeff. of } x^2} = \frac{-p}{1}$$

$$-p = 2\alpha + 2\beta = 2(\alpha + \beta)$$

$$-p = 2 \times \frac{5}{2} = 5 \text{ term}$$ 1

$$\text{Product of zeroes} = \frac{\text{contant}}{\text{coeff. of } x^2} = \frac{q}{1}$$

$$q = 2\alpha \times 2\beta = 4\alpha\beta$$

$$q = 4\left(-\frac{3}{2}\right) = -6$$

$$p = -5 \text{ and } q = -6.$$ 1

(CBSE Marking Scheme, 2012)

6. Yes. For justification, we have for the equation

$$2x + 3y = 9$$

$$a_1 = 2, b_1 = 3 \text{ and } c_1 = -9$$ ½

and for the equation, $4x + 6y = 18$

$$a_2 = 4, b_2 = 6 \text{ and } c_2 = -18$$ ½

Here, $\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}$

$$\frac{b_1}{b_2} = \frac{3}{6} = \frac{1}{2}$$

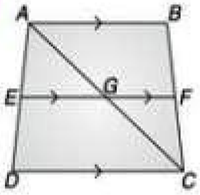
and $\frac{c_1}{c_2} = \frac{-9}{-18} = \frac{1}{2}$ ½

From above, it is clear that $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ ½

Hence, system of linear equations is consistent.

7. Draw AC intersecting EF at G.

In ΔCAB, $GF \parallel AB$



$\Rightarrow$ $\frac{AG}{CG} = \frac{BF}{FC}$ ...(i) 1

In ΔADC, $EG \parallel DC$

$\Rightarrow$ $\frac{AE}{ED} = \frac{AC}{CG}$ ...(ii)

From equations (i) and (ii), $\frac{AE}{ED} = \frac{BF}{FC}$. 1

8. Given: $4 \cos \theta = 11 \sin \theta$

$\Rightarrow$ $\cos \theta = \frac{11}{4} \sin \theta$

Now, $\frac{11\cos\theta - 7\sin\theta}{11\cos\theta + 7\sin\theta} = \frac{11 \times \frac{11}{4}\sin\theta - 7\sin\theta}{11 \times \frac{11}{4}\sin\theta + 7\sin\theta}$ 1

$$= \frac{\sin\theta\left(\frac{121}{4}-7\right)}{\sin\theta\left(\frac{121}{4}+7\right)}$$

$$= \frac{121-28}{121+28} = \frac{93}{149}.$$ 1

9. In ΔABP, $\sin 30° = \frac{AB}{AP}$

$\frac{1}{2} = \frac{50}{AP} \Rightarrow AP = 100$ cm ½

In ΔAQD, $\sin 30° = \frac{AD}{AQ}$ ½

$\Rightarrow$ $\frac{1}{2} = \frac{20}{AQ} \Rightarrow AQ = 40$ cm ½

Now, the length of $(AP + AQ)$ $= 100 + 40$

$= 140$ cm ½

10.

Heights	No. of girls
120 and more	50
130 and more	48
140 and more	40
150 and more	28
160 and more	8

2

SECTION — C

11. Len n by any +ve integer, then $n = 3q + r, r = 0, 1, 2$ ½

$n = 3q$ or $3q + 1$ or $3q + 2$ ½

Case I:

When $r = 0$, $n = 3q$, which is divisible by 3. ½

$n + 2 = 3q + 2$, which is not divisible by 3.

$n + 4 = 3q + 4$, which is not divisible by 3.

Case II:

When $r = 1$, $n = 3q + 1$, which is not divisible by 3. ½

$n + 2 = 3q + 1 + 2 = 3q + 3$, which is divisible by 3.

$n + 4 = 3q + 1 + 4 = 3q + 5$, which is not divisible by 3. ½

Case III:

When $r = 2$, $n = 3q + 2$, which is not divisible by 3.

$n + 2 = 3q + 1 + 2 = 3q + 4$, which is not divisible by 3.

$n + 4 = 3q + 2 + 4 = 3q + 6$, which is divisible by 3. ½

Case I, II and III $\Rightarrow$ One and only one out of n, $n + 2$ or $n + 4$ is divisible by 3.

12. Let the number of columns be x.

x is the largest number, which should divide both 104 and 96.

$104 = 96 \times 1 + 8$ 1

$96 = 8 \times 12 + 0$ 1

$\therefore$ HCF of 104 and 96 is 8.

Hence, 8 columns are required. 1

13. $3x^3 + 4x^2 + 5x - 13 = (3x + 10)\, g(x) + (16x - 43)$ ½

$\Rightarrow$ $$\frac{3x^3 + 4x^2 - 11x + 30}{3x + 10} = g(x)$$ (Adding (16x – 43) to the divided)½

$$
\begin{array}{r|l}
3x + 10 & 3x^3 + 4x^2 - 11x + 30 \;(\; x^2 - 2x + 3 \\
 & 3x^3 + 10x^2 \\
 & \underline{-\quad -} \\
 & \quad -6x^2 - 11x \\
 & \quad -6x^2 - 20x \\
 & \quad \underline{+\quad +} \\
 & \qquad 9x - 30 \\
 & \qquad 9x - 30 \\
 & \qquad \underline{-\quad -} \\
 & \qquad\quad 0
\end{array}
$$ 1

Hence, $g(x) = x^2 - 2x + 3.$ 1

14. $7x - 4y = 49$ ...(i)

On comparing with the equation

$a_1x + b_1y = c_1$

$a_1 = 7, b_1 = -4, c_1 = 49$

Again, $5x - 6y = 57$ ...(ii)

On comparing with the equation

$a_2x + b_2y = c_2$

$a_2 = 5, b_2 = -6, c_2 = 57$

Since, $\frac{a_1}{a_2} = \frac{7}{5}$ and $\frac{b_1}{b_2} = \frac{4}{6}$

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ 1

So, system has unique solution,

Multiply eqn. (i) by 5 and eqn. (ii) by 7 and subtract,

$35x - 20y = 245$

$35x - 42y = 399$ 1

$\underline{-\quad +\quad\quad -}$

$22y = -154$

$\Rightarrow$ $y = -7$

Put the value of y in eqn. (i),

$5x - 6(-7) = 57$

$\Rightarrow$ $5x = 57 - 42 = 15$

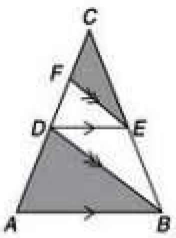
$\Rightarrow$ $x = 3$

Hence, $x = 3$ and $y = -7$ 1

15. According to the question,

$DE \parallel AB$

$\therefore$ $\frac{CD}{AD} = \frac{CE}{EB}$ (By BPT) ...(i) ½



Again since $FE \parallel DB$,

$\therefore$ $\frac{CE}{EB} = \frac{CF}{FD}$ (By BPT) ...(ii) ½

From equations (i) and (ii),

$$\frac{CD}{AD} = \frac{CF}{FD} \Rightarrow \frac{AD}{CD} = \frac{FD}{CF}$$

$\Rightarrow$ $$1+\frac{DA}{CD} = \frac{FD}{CF}+1$$ 1

$\Rightarrow$ $$\frac{CD+DA}{CD} = \frac{FD+CF}{CF}$$

$\Rightarrow$ $$\frac{AC}{CD} = \frac{CD}{CF}$$

$\Rightarrow$ $$CD^2 = AC \cdot CF \Rightarrow DC^2 = CF \times AC$$ 1

16. To Prove: $\frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta DBC)} = \frac{AO}{DO}$

Construction: Draw $AE \perp BC$ and $DF \perp BC$.

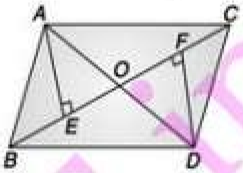
Proof:

In ΔAOE and ΔDOF, $\angle AOE = \angle DOF$ (Vertically opposite angles)

$\angle AEO = \angle DFO = 90°$ (Construction)

$\Rightarrow$ $\Delta AOE \sim \Delta DOF$ (By AA similarity)

$\therefore$ $$\frac{AO}{DO} = \frac{AE}{DF}$$ ...(i) 1



Now, $$\frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta DBC)} = \frac{\frac{1}{2}\times BC \times AE}{\frac{1}{2}\times BC \times DF} = \frac{AE}{DF}$$ 1

$$= \frac{AO}{DO}$$ [From eqaution (i)] 1

17.

$$\text{L.H.S.} = \frac{\cos A}{1-\tan A} + \frac{\sin A}{1-\cot A}$$

$$= \frac{\cos A}{1-\left(\frac{\sin A}{\cos A}\right)} + \frac{\sin A}{1-\left(\frac{\cos A}{\sin A}\right)}$$ 1

$$= \frac{\cos^2 A}{\cos A - \sin A} + \frac{\sin^2 A}{\sin A - \cos A}$$

$$= \frac{\cos^2 A}{\cos A - \sin A} - \frac{\sin^2 A}{\cos A - \sin A}$$ 1

$$= \frac{\cos^2 A - \sin^2 A}{\cos A - \sin A}$$

$$= \frac{(\cos A - \sin A)(\cos A + \sin A)}{(\cos A - \sin A)}$$

$$= \cos A + \sin A$$

$$= \text{R.H.S.}$$ **Hence Proved. 1**

18. Let $AB = x$

$\because$ $AC - AB = 1$

$\Rightarrow$ $AC = x + 1$

$\because$ $AC^2 = AB^2 + BC^2$

$\therefore$ $(x+1)^2 = x^2 + (5)^2$

$\Rightarrow \quad x^2 + 2x + 1 = x^2 + 25$

$\Rightarrow \quad 2x = 24 \Rightarrow x = \frac{24}{2} = 12$ cm **1**

Hence, $AB = 12$ cm, $AC = 13$ cm

$$\sin C = \frac{AB}{AC} = \frac{12}{13}$$

$$\cos C = \frac{BC}{AC} = \frac{5}{13}$$ **1**

Now $$\frac{1+\sin C}{1+\cos C} = \frac{1+\frac{12}{13}}{1+\frac{5}{13}} = \frac{\frac{25}{13}}{\frac{18}{13}} = \frac{25}{18}$$ **1**

19. $$\text{Mode} = l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$$ **1**

Here, Modal class = 30 – 40,

$l = 30, f_1 = 25, f_0 = 20, f_2 = 12, h = 10$ **½**

$$\text{Mode} = 30 + \frac{25-20}{50-20-12} \times 10$$ **1**

$$= 30 + \frac{5 \times 10}{18}$$

$= 30 + 2{\cdot}77 = 32{\cdot}77$ (Modal age) **½**

20.

x_i	f_i	f_ix_i
10	12	120
30	15	450
50	32	1600
70	p	$70p$
90	13	1170
Total	$\Sigma f_i = 72 + p$	$\Sigma f_ix_i = 3340 + 70p$

1

$\therefore$ Mean, $$\bar{x} = \frac{\Sigma f_ix_i}{\Sigma f_i}$$ **1**

$\Rightarrow \quad 53 = \frac{3340+70p}{72+p}$

$\Rightarrow \quad 3340 + 70p = 53(72 + p)$

$\Rightarrow \quad 3340 + 70p = 3816 + 53p$

$\Rightarrow \quad 70p + 53p = 3816 - 3340$

$\Rightarrow \quad 17p = 476$

$$p = \frac{476}{17} = 28$$ **1**

SECTION — D

21. Let $\sqrt{2}$ be a rational number.

$\sqrt{2} = \frac{a}{b}$, (a, b are co-prime integers and $b \neq 0$)

$$a = \sqrt{2}\, b$$

Squaring, $a = 2b^2$ **1**

$\Rightarrow$ 2 divides a^2.

$\Rightarrow$ 2 divides a^2.

So, we can write $a = 2c$ for same integer c. Substitute for a, $2b^2 = 4c^2$, $b^2 = 2c^2$

This means 2 divides b^2, so 2 divides b.

$\therefore$ a and b have at least '2' as a common factor. **1**

But this contradicts that a, b have no common factor other than 1.

$\therefore$ Our assumption is wrong.

Hence, $\sqrt{2}$ is irrational.

Let $\frac{3}{\sqrt{2}} = a$, where a is rational

$$a\sqrt{2} = 3$$

$$\sqrt{2} = \frac{3}{a}$$ **1**

$\frac{3}{a}$ is rational but $\sqrt{2}$ is not rational.

$\therefore$ Our assumption is wrong.

$\therefore$ $\frac{3}{\sqrt{2}}$ is an irrational number. **1**

22. Since α and β are the zeroes of polynomial $3x^2 + 2x + 1$.

Hence, $\alpha + \beta = -\frac{2}{3}$

and $\alpha\beta = \frac{1}{3}$ **1**

Now for the new polynomial,

$$\text{Sum of the zeroes} = \frac{1-\alpha}{1+\alpha} + \frac{1-\beta}{1+\beta}$$

$$= \frac{(1-\alpha+\beta-\alpha\beta)+(1+\alpha-\beta-\alpha\beta)}{(1+\alpha)(1+\beta)}$$

$$= \frac{2-2\alpha\beta}{1+\alpha+\beta+\alpha\beta} = \frac{2-\frac{2}{3}}{1-\frac{2}{3}+\frac{1}{3}}$$

$$\text{Sum of the zeroes} = \frac{\frac{4}{3}}{\frac{2}{3}} = 2$$ **1**

$$\text{Product of zeroes} = \left[\frac{1-\alpha}{1+\alpha}\right]\left[\frac{1-\beta}{1+\beta}\right] = \frac{(1-\alpha)(1-\beta)}{(1+\alpha)(1+\beta)}$$

$$= \frac{1-\alpha-\beta+\alpha\beta}{1+\alpha+\beta+\alpha\beta} = \frac{1-(\alpha+\beta)+\alpha\beta}{1+(\alpha+\beta)+\alpha\beta}$$

$$\text{Product of zeroes} = \frac{1+\frac{2}{3}+\frac{1}{3}}{1-\frac{2}{3}+\frac{1}{3}} = \frac{\frac{6}{3}}{\frac{2}{3}} = 3 \quad 1$$

Hence, required polynomial $= x^2 - (\text{sum of zeroes})x + \text{Product of zeroes}$

$= x^2 - 2x + 3.$ 1

(CBSE Marking Scheme, 2012)

23. Let the speed of the car I from $A = x$ km/hr.

Speed of the car II from $B = y$ km/hr.

Same direction:

Distance covered by car I = 150 + (distance covered by car II)

$\Rightarrow \quad 15x = 150 + 15y$

$\Rightarrow \quad 15x - 15y = 150$

$\Rightarrow \quad x - y = 10$ ...(i) 1

Opposite direction:

Distance covered by car I + distance covered by car II

= 150 km

$x + y = 150$ ....(ii) 1

Adding eq. (i) and (ii), we get $\quad 2x = 160$

$\Rightarrow \quad x = 80$

Putting $x = 80$ in eq. (i), $\quad y = 70$ 1

$\therefore$ Speed of the car I = 80 km/hr

and speed of the car II = 70 km/hr 1

(CBSE Marking Scheme, 2012)

24. Let the two digits number be $10x + y$

According to the question, $\quad 8(x + y) - 5 = 10x + y$

$\Rightarrow \quad 2x - 7y + 5 = 0$ ...(i)

and $\quad 16(x - y) + 3 = 10x + y$

$6x - 17y + 3 = 0$ ...(ii) 1

Solving equation (i) and (ii) by cross-multiplication method, we get

$$\frac{x}{(-7)(3)-(-17)(5)} = \frac{y}{(5)(6)-(2)(3)} = \frac{1}{(2)(-17)-(6)(-7)} \quad 1$$

$$\Rightarrow \quad \frac{x}{-21+85} = \frac{y}{30-6} = \frac{1}{-34+42}$$

$$\Rightarrow \quad \frac{x}{64} = \frac{y}{24} = \frac{1}{8}$$

$$\Rightarrow \quad \frac{x}{8} = \frac{y}{3} = 1 \quad 1$$

Hence, $\quad x = 8, y = 3$ 1

So, the required number = $10 \times 8 + 3 = 83$.

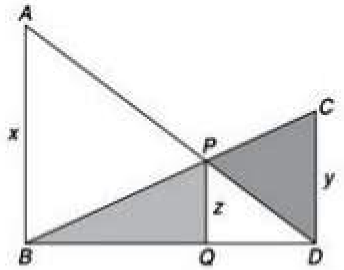
25. $\because \quad AB \parallel PQ$

$\therefore \quad \angle ABQ = \angle PQD$ (Corresponding angles) 1

In ΔADB and ΔPDQ,

$$\angle ADB = \angle PDQ \quad \text{(Common)}$$

$$\angle ABQ = \angle PQD$$



By AA similarity,

$$\Delta ADB \sim \Delta PDQ$$

$$\therefore \quad \frac{DQ}{DB} = \frac{PQ}{AB} \quad 1$$

$$\frac{DQ}{DB} = \frac{z}{x} \quad \text{...(i)}$$

Similarly, $\Delta PBQ \sim \Delta CBD$

and $$\frac{BQ}{DB} = \frac{z}{y} \quad \text{...(ii)}$$

Adding (i) and (ii), we get

$$\frac{z}{x}+\frac{z}{y} = \frac{DQ+BQ}{DB}=\frac{BD}{BD} \quad 1$$

$$\Rightarrow \quad \frac{z}{x}+\frac{z}{y} = 1 \quad ½$$

$$\therefore \quad \frac{1}{x}+\frac{1}{y} = \frac{1}{z}. \quad ½$$

26. (i) Let the initial position of the man be at O and his final position be B. Since the man goes to 10 m due. east and then 24 m due north. Therefore, ΔAOB is a right triangle right angled at A such that $OA = 10$ m and AB = 24 m. ½

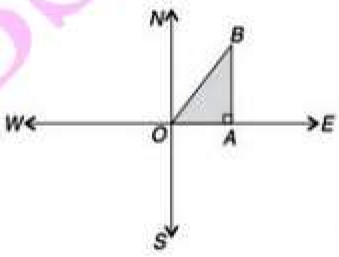
½

By Pythagoras theorem, $$OB^2 = OA^2 + AB^2$$

$$\Rightarrow \quad OB^2 = (10)^2 + (24)^2 \quad 1$$

$$= 100 + 576 = 676$$

$$\Rightarrow \quad OB = \sqrt{676} = 26 \text{ m} \quad 1$$



Hence, the man is at a distance of 26 m from the starting point.

(ii) Pythagoras theorem in Right-angled triangle. ½

(iii) Slow and steady wins the race. ½

27. $$\text{L.H.S.} = \frac{\tan\theta+\sec\theta-1}{\tan\theta-\sec\theta+1}$$

$$= \frac{\tan\theta+\sec\theta-(\sec^2\theta-\tan^2\theta)}{\tan\theta-\sec\theta+1} \quad [\because 1 = \sec^2\theta - \tan^2\theta]$$

$$= \frac{(\tan\theta+\sec\theta)-[(\sec\theta+\tan\theta)(\sec\theta-\tan\theta)]}{\tan\theta-\sec\theta+1} \quad 1$$

$$= (\tan\theta+\sec\theta)\frac{[1-\sec\theta+\tan\theta]}{\tan\theta-\sec\theta+1} \quad 1$$

$$= (\tan\theta+\sec\theta)\frac{[1-\sec\theta+\tan\theta]}{[1-\sec\theta+\tan\theta]} \quad 1$$

$$= \frac{\sin\theta}{\cos\theta} + \frac{1}{\cos\theta}$$

$$= \frac{1+\sin\theta}{\cos\theta}$$

$= \text{R.H.S.}$ **Hence Proved. 1**

28. $$\text{L.H.S.} = \frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta}$$

$$= \frac{\tan\theta}{1-\frac{1}{\tan\theta}} + \frac{\frac{1}{\tan\theta}}{1-\tan\theta}$$

$$= \frac{\tan^2\theta}{\tan\theta-1} + \frac{1}{\tan\theta(1-\tan\theta)}$$ **1½**

$$= \frac{\tan^3\theta-1}{(\tan\theta-1)\tan\theta}$$ **1**

$$= \frac{(\tan\theta-1)(\tan^2\theta+\tan\theta+1)}{(\tan\theta-1)(\tan\theta)}$$ **1**

$= \tan\theta + 1 + \cot\theta$

$= 1 + \tan\theta + \cot\theta = \text{R.H.S.}$ **½**

29. Given: $\operatorname{cosec}\theta = \sqrt{5}$

$\Rightarrow$ $\sin\theta = \frac{1}{\sqrt{5}}$

So, $\cos^2\theta = 1 - \sin^2\theta = 1 - \frac{1}{5} = \frac{4}{5}$

$\therefore$ $\cos\theta = \frac{2}{\sqrt{5}}$ **1**

(i) $$\cot\theta - \operatorname{cosec}\theta = \frac{\cos\theta}{\sin\theta} - \operatorname{cosec}\theta$$

$$= \frac{2/\sqrt{5}}{1/\sqrt{5}} - \sqrt{5} = 2 - \sqrt{5}$$ **1½**

(ii) $$\sin^2\theta + \cos^2\theta = \left(\frac{1}{\sqrt{5}}\right)^2 + \left(\frac{2}{\sqrt{5}}\right)^2$$

$$= \frac{1}{5} + \frac{4}{5} = 1$$ **1½**

30.

Height (in cm)	Number of girls (f_i)	x_i	$u_i = \frac{x_i - a}{h}$	$f_i u_i$
120 – 130	2	125	– 2	– 4
130 – 140	8	135	– 1	– 8
140 – 150	12	145	0	0
150 – 160	20	155	1	20
160 – 170	8	165	2	16
Total	$\Sigma f_i = 50$			$\Sigma f_i u_i = 24$

Let assumed mean, $a = 145$ and $h = 10$

$$\text{Mean} = a + h \times \frac{\Sigma f_i u_i}{\Sigma f_i} \qquad 2$$

$$= 145 + 10 \times \frac{24}{50} \qquad 1$$

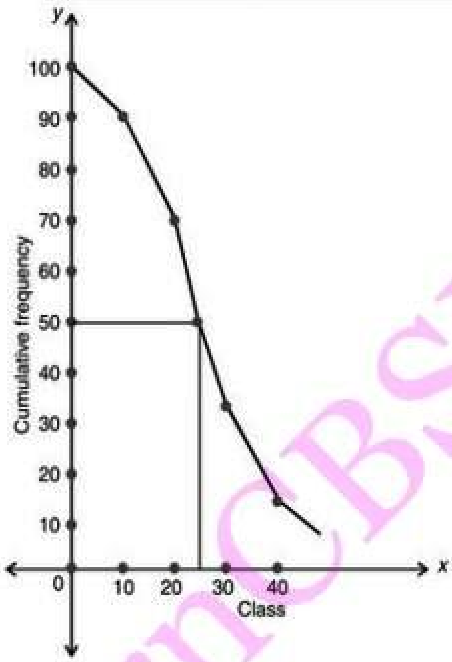
$$= 145 + 4{\cdot}8$$

$$= 149{\cdot}8 \qquad 1$$

31.

More than	c.f.
0	100
10	90
20	72
30	32
40	12

1



2

From graph, $\frac{N}{2} = \frac{100}{2} = 50$

Hence, Median = 25 1